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T. M. Kubiak

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For Darlene, my wife:

The world is no longer as it was and will no longer be as it is. The only constant and consolation I find and know is you. I no longer know a time before you. We have always been; we act as one; we think as one. An arched brow, a brief glance, a thought conveyed. Even apart, we remain together. Our past lives together; our current lives together; and our future lives together. For there will never be a goodbye, there will only be . . . us.

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Preface

am pleased to present the first edition of *The Certified Six Sigma Master Black Belt Handbook*. It has been created to reflect the most current thinking among key Six Sigma leaders regarding what should be contained in the ASQ Master Black Belt Body of Knowledge that was first released in 2010.

As with all ASQ certification–based handbooks, the primary audience for this work is the individual who plans to prepare to sit for the Six Sigma Master Black Belt certification examination. Therefore, this book assumes the individual has the necessary educational background and experience in quality and Lean Six Sigma. Concepts are dealt with briefly, but supplemented with practical examples. I have intentionally avoided theoretical discussion unless such a discussion was necessary to communicate a concept. As always, readers are encouraged to use additional sources when seeking much deeper levels of discussion. The citations provided in the references will be helpful in this regard.

Undoubtedly, practicing Master Black Belts will find much of this material new, particularly in Part IV Advanced Measurement Methods and Tools. Some of the tools discussed in this part are simply not part of most Master Black Belt curriculums. Therefore, many of you will discover the Body of Knowledge to be challenging. However, it will provide the opportunity to expand your learning horizons and hopefully grow as Master Black Belts.

A secondary audience for the handbook is the quality and Six Sigma professional who would like a relevant Lean Six Sigma reference book. With this audience in mind, some material has been expanded beyond what the Body of Knowledge requires. I am confident that readers will find these additions useful.

As you might expect, chapter and section numbering follows the same method used in the ASQ Six Sigma Master Black Belt Body of Knowledge. This has made for some awkward placement of discussions (for example, project selection, prioritization, and alignment, and various project management concepts), and in some cases, redundancy of discussion still exists. Whenever possible, though, I have tried to reference the main content in the handbook and refer the reader there for the primary discussion. I thought the ease of access for readers, who might be struggling with some particular point in the BOK, would more than offset these disadvantages.

Further, to minimize redundancy, I have designed this handbook to serve as a companion volume to *The Certified Six Sigma Black Belt Handbook*. Consequently, I have assumed the reader is familiar with that book and refer to it many times throughout this book. However, I have also duplicated many of the appendices from *The Certified Six Sigma Black Belt Handbook* in this book to reinforce its usefulness as a stand-alone reference.

As you read this book, you may wonder why I have chosen to use "Lean Six Sigma" in place of "Six Sigma" in most instances. The answer is simple! The Black Belt and Master Black Belt bodies of knowledge discuss Lean, and the reality is that a practitioner must be competent in both. Further, the terminology also reflects the integrated nature of these tools and techniques.

Readers have often sent me e-mails questioning why I use only Minitab for my examples. It's because the preponderance of users use Minitab. For those who do not, data have been provided for every example, thus allowing users to parallel examples using their software of choice.

As you can imagine, much of the material dealing with soft skills and related topics can be quite voluminous. Consequently, it was important that I considered either the most commonly used or most well known tools, techniques, or theories. In some cases, the material was so diverse that I found I needed to summarize key thoughts or characteristics. Therefore, you may not find your favorite tool, technique, or theory. However, if a significant omission exists, please feel free to contact me at authors@asq.org.

Please note that in some cases, your answer where a numerical solution is required may not completely agree with those given in the examples. In many instances, I found that discrepancies could be attributed to the following: use of computers with different bits, the number of significant digits accounted for by the software used, the sequence in which the arithmetic was performed, and the propagation of errors due to rounding or truncation. Therefore, I urge the reader to carefully consider the above points as the examples are worked. However, I do recognize that errors occasionally occur and thus have established a LinkedIn page that will permit readers to recommend suggestions, additions, corrections, or deletions, as well as to seek out any corrections that may have been found and published. The LinkedIn address is http://www.linkedin.com/groups/ Certified-Six-Sigma-Master-Black-4320830.

On occasion, I have received complaints from readers regarding relevance of the examination questions. Usually, these are from those who have not passed the examination. I believe it is critically important to note that I do not have an awareness of the examination questions. This is by design and is intended to maintain the integrity of the ASQ certification process. Those who have a role in developing the examination questions are barred from writing ASQ handbooks for two years. This is part of the arms-length process. Furthermore, questions are added to the examination database at prescribed intervals by new question developers. Additionally, the question set varies from examination to examination. The questions I provide are those that I see as reasonably potential questions based on the subject matter provided in this handbook.

Finally, the following is a short description of the supplementary material included on a CD with the print book and at the end of the e-book versions:

• A practice problem set covering each part of the book. I suggest the reader study a particular part and then do the supplementary problems for that part.

• A simulated examination that has problems distributed among parts according to the scheme published in the Body of Knowledge. After attaining success with all parts, the reader may complete the simulated examination to confirm mastery of the ASQ Six Sigma Master Black Belt Body of Knowledge.

Good luck!

—T. M. Kubiak

Acknowledgments

This was a difficult project, and I could not have completed it successfully without much-needed support.

First and foremost, there is my wife, Darlene. Darlene served as my proofreader for the entire project. I am deeply appreciative of her patience and of her keen eyes. She could find an extra space or missing period at fifty paces. Of course, there was always a smile on her face as she eagerly pointed out my mistakes, in red ink no less.

Special thanks go to Jim Bossert, Joe Basala, and Greg Watson for their support in interpreting the body of knowledge. Their input brought clarity to me. I, in turn, hope that I have successfully captured that clarity and transferred it to the book you are now holding. Also, I would like to thank Jim, in particular, for his ongoing show of support and encouragement.

Many thanks go to Bruce Sheridan for materials he provided regarding the use of hoshin kanri. Bruce is an expert in hoshin kanri and the author of the Quality Press book, *Policy Deployment: The TQM Approach to Long-Range Planning*.

I would like to express my deepest appreciation to Minitab Inc. for providing me with the use of Minitab 16 and Quality Companion 3 software, and for permission to use several examples and graphics from these software programs. This software was instrumental in creating and verifying examples used throughout the book.

Also, I would like to thank Wolfram Research for their support in providing Mathematica, software that has unbelievable computing capability.

Finally, I would like to thank the Quality Press management staff for their outstanding support and exceptional patience while I completed this project.

—T. M. Kubiak

Part I

Enterprise-Wide Planning and Deployment

| Chapter 1 | Strategic Plan Deployment |
|-----------|-------------------------------------|
| Chapter 2 | Strategic Plan Alignment |
| Chapter 3 | Deployment of Six Sigma Systems |
| Chapter 4 | Six Sigma Methodologies |
| Chapter 5 | Opportunities for Improvement |
| Chapter 6 | Risk Analysis of Projects and the |
| | Pipeline |
| Chapter 7 | Organizational Design |
| Chapter 8 | Organizational Commitment |
| Chapter 9 | Organizational Finance and Business |
| | Performance Metrics |
| | |

Part I

Chapter 1 Strategic Plan Deployment

Describe strategic planning tools and methods (hoshin kanri, SWOT, PEST, etc.) and their utilization in developing enterprise planning. (Apply)

Body of Knowledge I.A

estcott (2006) defines *strategic planning* as the process by which an organization establishes long-term goals and objectives and the actions necessary to achieve them. Strategic planning is fundamental to the success of any organization. It provides direction and unifies the organization and its workforce with a common focus. Without it, organizations often wander about aimlessly expending significant human and capital resources before eventually failing.

There are various methods and processes used to accomplish enterprise-wide strategic planning. Some of these are effective; some are not. This chapter will discuss several key tools used as input to the strategic planning process. These tools, including the SWOT and PEST analyses, are used to force an organization to look deeply within itself and to understand the external factors that influence its ability to be successful.

In addition, several strategic planning methodologies will be presented. These methodologies include hoshin kanri and the traditional approach used by most organizations. No one approach is the best. Organizations must determine which is best suited for both their environment and culture.

SWOT

A SWOT analysis is an effective strategic planning tool applicable to an organization or project objective. SWOT stands for: *strengths, weaknesses, opportunities,* and *threats.* It is attributed to Albert Humphrey from Stanford University during the 1950s and 1960s.

Strengths and weaknesses are identified with respect to the internal capabilities of an organization. They provide an introspective view. Strengths and weaknesses should not be considered opposites. This will become apparent in the example below. *Strengths* address what the organization does well. This might be: research and development, production, marketing, acquisitions, partnering, and so on. Strengths need to be assessed fairly and objectively to bring credibility to the SWOT analysis.

Weaknesses address the organization in terms of skills, capabilities, and competencies. For example, an organization continually fails to deliver its products to

EXAMPLE 1.1

A private higher-education institution has experienced declining enrollment for four consecutive years, particularly from the two major cities in the state where it is located. This declining enrollment has impacted profitability. The president is feeling pressure from the board of directors, which wants answers as to why the enrollment levels are declining. The industrial engineering department has proposed a Lean Six Sigma project aimed at improving the enrollment rate for students from these two cities. As the department prepares to make probability estimates for its risk analysis, it first produces the SWOT analysis shown in Table 1.1. Analysis of the SWOT matrix provides insight for the president and board of directors that was not previously known. Viewed collectively in the SWOT format, it serves as valuable input to the institution's upcoming strategic planning process.

| | Strengths | Weaknesses | |
|----------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|
| Internal | High academic entrance requirements Strong general education program High percentage of full-time faculty Fundraising goals are being met each year Well-designed internship and study abroad opportunities | Print communications for prospective students are bland and generic Recruitment staff is too passive in contacting prospective students Uneven implementation of assessment and program review methods Student-centered customer focus is not recognized throughout the institution | |
| | Opportunities | Threats | |
| External | Metropolitan area location has population advantages | • The number of traditional-age prospective students is expected to decline | |
| | Area corporations offer tuition | • With the cost of education escalating, | |
| ixtern | reimbursementsReputable business program with a global | community colleges have recently become viable competitors | |

Table 1.1SWOT analysis matrix for Example 1.1.

its customers on time. This might be due to a lack of skill, incapable processes, or poor training of employees. Note: A weakness should never be improved at the expense of a strength!

On the other hand, *opportunities* and *threats* provide the organization with an extrospective view. Quite simply, we are trying to identify opportunities for

EXAMPLE 1.2

Connie is the owner of the two stores that comprise the Brides on a Budget Boutique. She met with her two store managers to commence their first strategic planning process. After an intense three-hour meeting, the SWOT analysis depicted in Table 1.2 was developed.

| Table 1.2 | SWOT | analysis | matrix fc | or Example | 1.2. |
|-----------|------|-----------|-----------|--------------|------|
| rabie na | 0 | annanyono | | - Lindinipie | |

| | Strengths | Weaknesses | |
|----------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|
| | • We are trend leaders in both cities in which the stores are located | Connie is the only person who understands the billing system | |
| Internal | Both locations are ideal for their market niche (high traffic, multi-store malls) Customer retention indicates extreme loyalty and a high referral rate Customer surveys and media coverage acknowledge the superb personalized customer service provided by personnel of both stores | Because of taking advantage of a manufacturers' "closeout," the business currently shows a negative cash flow Inventory obsolescence tends to be higher than others in the industry | |
| | Opportunities | Threats | |
| External | Assess the products carried to determine which products are most profitable and which should be dropped Examine the potential of new approaches to marketing Explore alliance opportunities with other mall-store owners Consider presenting a "fashion show" Consider opening an outlet store for "previously owned" bridal wear | Malls have both been bought from original owners, and a rent increase is rumored Refurbishing of downtown areas is attracting customers away from the malls Malls are becoming hangouts for unruly teenagers and could be a distraction for brides-to-be An increasing number of crimes committed in the malls is affecting the attraction of potential buyers | |

Adapted from the *Certified Manager of Quality/Organizational Excellence: Module 2, Exhibit 2-5, ASQ's Foundations in Quality Learning Series.* 2005. Milwaukee: ASQ.

the organization to grow to new levels, and threats to the organization that will impede its progress. Threats are often barriers to an organization's growth and can represent a competitive disadvantage.

When analyzing a SWOT, the question to ask is, "How can the organization leverage its strengths and improve its weaknesses to take advantage of the opportunities while mitigating or eliminating the threats?"

PEST

An analysis similar to SWOT is the *PEST analysis,* which brings together four environmental scanning perspectives that serve as useful input into an organization's strategic planning process:

- *Political*. This perspective looks at political stability, threats of wars, regulatory issues, tax considerations, labor problems, laws, and so forth. For example, as this book is being written, we have:
 - *Political stability.* Mid-term congressional elections have shifted the dominating party from Democratic to Republican in the U.S. House of Representatives.
 - Threat of wars. The war in Afghanistan continues; arguably, the war in Iraq is over; an unstable ruler of North Korea recently died and leadership was assumed by his hardliner son; Iran is clamoring for its own nuclear weapons and attempting to threaten the world's oil supply; and terrorist bombings are rampant.
 - *Regulatory issues*. The recent financial meltdown has resulted in strengthening the power and authority of regulatory agencies.
 - *Tax considerations*. In an attempt to stimulate the economy, the federal government has proposed special depreciation considerations designed to motivate organizations to purchase capital equipment.
 - *Labor problems*. Eight and a half percent unemployment continues to strain our government welfare systems, and public pension legacy issues continue to mount.
 - *Laws*. Broad-sweeping healthcare and financial reform laws were passed that will have significant impact to both organizations and individuals.
- *Economic*. This perspective looks at currency exchange rates, market conditions, unemployment rates, inflation factors, interest rates, and so forth. Current examples include: the U.S. dollar against other major international currencies, rates of unemployment in the United States and in other major countries of the world, almost zero percent inflation, low GDP growth rates in the developed world, outsourcing to low-cost nations such as India, China, and those of Southeast Asia, and, in some cases, negative interest rates on bonds.

- *Social.* This perspective looks at education considerations, demographics, health factors, cultural implications, and so forth. Current examples include: the widening educational gap between the United States and other first-world countries, and shifting demographics such that "baby boomers" are now the largest group, and consequently place the biggest strain on the U.S. healthcare system.
- *Technological*. This perspective looks at new technology developments, rate of technological change, cost impact of technology, and so forth.

EXAMPLE 1.3

A nonprofit organization seeks to improve quality of life for citizens in a neighborhood by upgrading storm sewers. Its PEST analysis would look similar to Table 1.3. Once these perspectives have been identified and conveniently summarized in the PEST matrix, they can be systematically addressed.

Table 1.3PEST analysis matrix for Example 1.3.

| Political | Economic |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| The proposal will be competing with other projects for public money The neighborhood must have adequate representation on the governing bodies Other neighborhoods will be improved as a result of the proposal, so they should support it The current situation allows storm water to infiltrate the sanitary sewer system, which impacts all users | An increase in real estate taxes for two years will be necessary to cover the costs of the upgrade Environmental costs will be associated with increased stream and river pollution that occurs from the ponded storm water |
| Social | Technological |
| People in the affected neighborhood tend to feel disenfranchised and mistreated by the government decision makers, which may impact attitudes toward others Lack of an adequate storm sewer system has led to an increasing trend of gastrointestinal illnesses as the population increases and more demand is placed on the existing sewer system | Several studies have been commissioned by the various stakeholders and have yielded conflicting results New pipe-TV techniques are now available |

Current examples include stem cell research; animal cloning; genetic manipulation of corn, beets, soybeans, and so on; and the introduction of 3D television with a steeper price reduction curve than previously seen for plasma and LCD television sets.

HOSHIN KANRI

Hoshin kanri, a Japanese term, is a top-down, bottom-up, systematic and structured strategic planning process that engages all levels of the organization while creating measurable and aligned goals that imbue the concept of continuous improvement through use of the *plan-do-check-act cycle*. (Note: Some sources will cite the *plan-do-study-act* [PDSA] cycle.)

The term *hoshin kanri* has been translated many different ways. For example, Akao (2004) decomposes "hoshin kanri" into its four basic syllables to help explain its meeting:

- Ho—direction
- Shin—focus
- Kan—alignment
- Ri—reason

He further indicates that "ho + shin" translates to "shining metal," "compass," or "point the direction." Similarly, "kan + ri" means "management" or "control." Perhaps the loose translation is that it aligns an organization toward achieving a common goal.

Although King (1989) agrees that "hoshin" means "shining metal" and "pointing the direction," he does not mention, "compass." However, he states that "hoshin" is translated as "policy" or "target and means." Likewise, he agrees that "kanri" means "management" or "control" and translates it as "planning." King suggests another literal translation for "hoshin kanri" as "target and means management." He further notes that "hoshin kanri" is sometimes referred to as "policy management," "policy control," or "management by policy."

As is evident from the above, "hoshin kanri" is known by many different names:

- Hoshin planning
- Policy deployment
- Policy management
- Policy control
- Management by policy

And perhaps a few others we have yet to uncover.

The fundamental processes, principles, and concepts of hoshin kanri differ among authors such that it became necessary to synthesize these into the elements of a common process and to extract the basic principles and concepts that most authors agree on. Figure 1.1 represents the underlying principles and concepts upon which hoshin kanri is built. Most authors agree that the process focuses on the development of both long-term and mid-term plans, while some authors suggest a focus on short-term plans as well. Long-term plans are usually 2–5 years; mid-term plans are from 1–2 or 3 years; and short-term plans are generally 1 year long and are commonly called the *annual* or *operational plan*.

Another key aspect of Figure 1.1 worth highlighting is the idea of "alignment/ focus." Hoshin kanri is, essentially, all about organizational alignment. It creates both vertical and horizontal alignment, thus establishing focus throughout the entire organization.

Figure 1.2 depicts an example of the hierarchy of plan deployment in an organization. Notice that plans may be deployed either through individuals or organizational structures. In fact, plans may be deployed to cross-functional teams as well, thus creating horizontal alignment.

Figure 1.3 illustrates a generalized hoshin kanri process flow. Key inputs to the planning process are the SWOT and PEST analyses discussed earlier in this chapter. These analyses help provide both internal and external perspectives. Also, customer, employee, and shareholder input is specifically sought. The weight of these inputs can carry significant influence on the planning process. Additional stakeholder (for example, community, suppliers, partners) input may be added, as the organization deems appropriate.

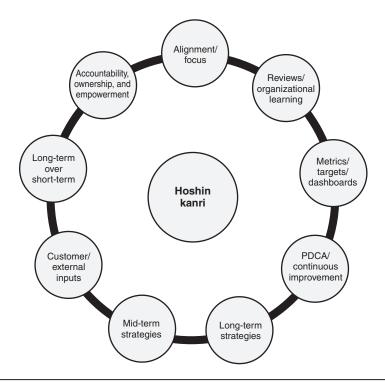


Figure 1.1 Fundamental principles and concepts of hoshin kanri.

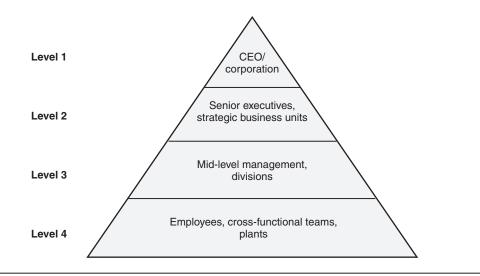


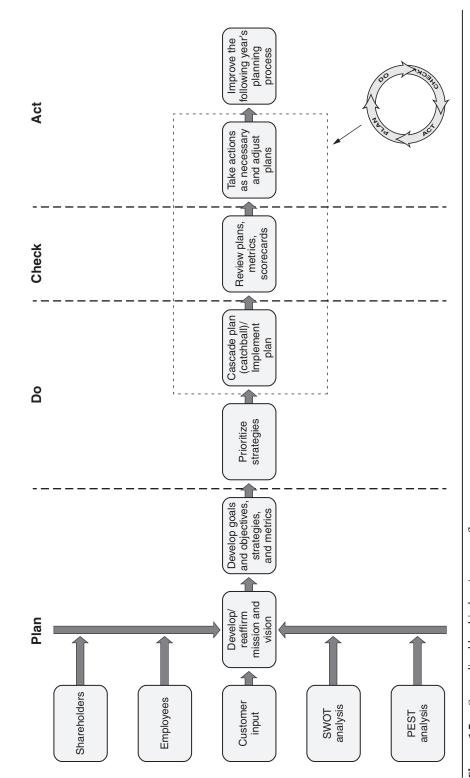
Figure 1.2 The hierarchy of plan deployment.

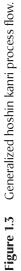
Figure 1.3 also illustrates that strategies are measured at the strategy level. This will become clearer in later figures. Strategies are also prioritized. This explicitly recognizes that not all strategies are created equal. Prioritizing strategies helps resolve issues later when strategies are assigned human resources and funding. Plans are cascaded downward throughout the organization in a systematic manner that also will be illustrated in a later figure. Further, plans are reviewed systematically, actions taken when results fail to meet plans, and plans adjusted as necessary. The "adjusting of strategic plans in mid-cycle" is typically viewed as a corporate sin; but with the hoshin kanri process, it is an integral part. Finally, the entire planning process is reviewed, and lessons are gathered and incorporated as improvements into the following year's planning process. This closed-loop planning process created the PDCA loop mentioned previously.

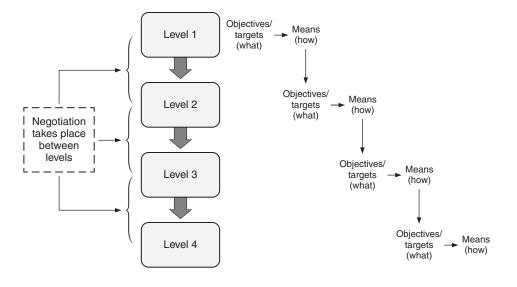
Recall King's (1989) translation of hoshin kanri as "target and means management." Figure 1.4 illustrates how this concept applies within the hoshin kanri framework. As objectives/targets are established, these are translated into the means. Objectives/targets are considered the "whats" while the "means" are considered the "hows." The "means" at one level of the organization are the "objectives/targets" at the next lower level. Negotiation occurs between these two levels before the "objectives/targets" and/or "means" are finalized. This process continues down through the organization until the "objectives/targets" and "means" are deployed at the lowest level appropriate. This deployment activity also recognizes the need for horizontal coordination as well.

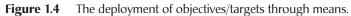
As discussed previously, one of the key principles and concepts associated with the hoshin kanri process is the application of measurement directly to strategies. However, Figures 1.4 and 1.5 relate this idea directly to goals and objectives, although Figure 1.3 alluded to the idea of quantifying strategies directly. Fortunately, Table 1.4 provides us with a common hoshin planning matrix that

Part I.A









| | Objectives/ targets | Means | |
|----------------------------------------|----------------------------------------------|-----------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Executive management | Reduce customer order lead time by 50% | Production control by Sales by 40% Manufacturing by 20% | |
| Department | | Objectives/ targets | Means |
| Production control | | Reduce by 50% | Implement an online order acceptance and control system |
| • Sales | | Reduce by 50% | Improve the order form Provide additional training |
| • Manufacturing | | Reduce by 50% | Implement a product-focused cell for product ABC Reduce setup times to 10 minutes or less Increase machine availability to 95% by having operators perform routine maintenance |
| Engineering | | | Redesign product ABC to be modular so that customer orders can be built to order in final assembly |
| Human resources | | | Finalize new work rules with the union to allow operators to perform routine maintenance |

Figure 1.5 Target/means deployment process for Example 1.4. *Source:* Adapted from Collins and Huge (1993).

EXAMPLE 1.4

Top management of a manufacturing company has decided that the need exists to "reduce customer order lead time by 50%." Figure 1.5 demonstrates via the method of "target and means management" how such an "objective/target" might be deployed. Notice in Figure 1.5 that while the engineering and human resources departments do not have "means" responsibilities, both departments play supporting roles in achieving the targets.

permits a clear linkage across goals and objectives, strategies, metrics, and results. Let's examine this figure briefly:

- *G&O category.* This allows goals and objectives to be grouped logically and in a focused manner.
- *Goals and objectives*. Goals and objectives are stated quantitatively using the SMART approach defined in Kubiak and Benbow (2009). Those shown here have been abbreviated due to space limitations.
- *Strategies.* These are verbalized in a succinct manner. Each objective may have multiple strategies associated with it.
- *Primary metric.* This metric is used to measure whether or not the strategy is achieved.
- *Countermeasure*. Countermeasures are used to ensure that primary metrics are achieved, but not at the expense of something else. The countermeasure for strategy 1.1 attempts to ensure that the introduction of the "No Questions Asked Return Policy" reduces complaints, but doesn't raise the return rate. Likewise, the countermeasure for strategy 2.1 is designed to prevent the problem from shifting elsewhere. In this case, from receiving inspection to source inspection.
- *Results.* Notice that the results for strategy 1.1 are not yet being met. However, strategy 2.1 has already been achieved. Depending where the organization is in its operational cycle, management may elect to apply resources to strategy 1.1 to bring it on track with expectations. Further, it will continue to review strategy 2.1 to ensure that gains are not lost.
- *Strategy owner*. Another fundamental concept from Table 1.4 is embedded in this planning matrix, that of "ownership." For each strategy, a strategy owner is identified at the executive level. This individual would serve as the coordinator across functional organizations such as business units, plants, departments, and so on, to ensure the synchronized development, funding, resourcing, and accomplishment of this strategy. Briefly, recall Figure 1.5. In this example, a strategy owner would be required to provide coordinated effort across five departments.

| G&O category | Goals and objectives | Strategies | Primary measure | Counter- measure | Results | Strategy owner |
|-----------------|---------------------------------------------------------------------|-----------------------------------------------------------------------|--------------------------------------------------------|-----------------------------------------------------|-----------------------------------------|-------------------|
| Customer | Reduce customer complaints from 60% to 15% | 1.1 Introduce "No Questions Asked Return Policy" | % complaints received during returns | % returns | 25% reduction to-date | R. U. Done |
| | | 1.2 | | | | |
| Suppliers | Reduce PPM at receiving inspection from 500K to 300K | 2.1 Have suppliers transition from hand to wave soldering | % reduction in PPM at receiving inspection | % reduction in PPM at source inspection | 60% reduction achieved to-date | I. M. Fini |
| | | 2.2 | | | | |
| Employees | 89% satisfied | 3.1 | | | | |
| | | 3.2 | | | | |

Table 1.4 Example of a typical hoshin kanri strategy deployment form.

It should now be readily apparent that the planning matrix described in Table 1.4 can be used at all levels of an organization as depicted in Figures 1.2 and 1.4.

Let's return to Figure 1.3 for a moment. So far, we have addressed the "plan," and "do" phases of the PDCA cycle. As the hoshin kanri process continues through the organization's operational cycle, a system of reviews is established and conducted in a bottom-up manner. The process is now in the "check" phase. This system of reviews should be properly timed so that they progress rapidly up through the organization over a short period.

During these reviews, it will be determined whether strategies are failing or not. If they are, resources may be applied or strategies adjusted as internal or external organizational conditions merit. When this occurs, the process has just entered the "act" phase.

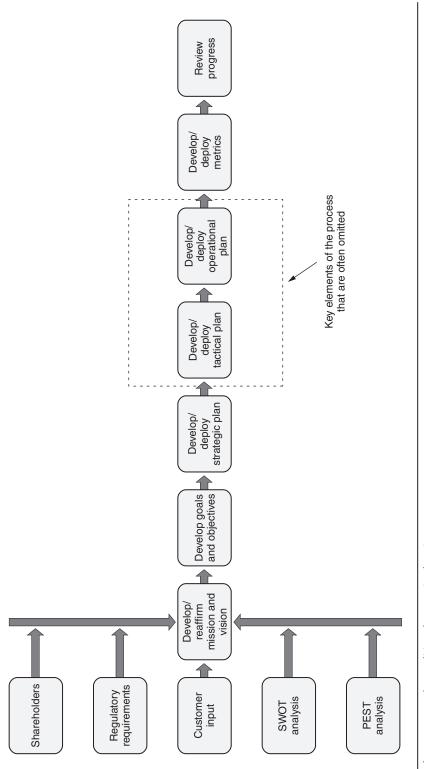
As stated previously, when the operational cycle of the organization ends, lessons are gathered and improvements made for the next planning cycle. This completes the overarching PDCA cycle. The mini PDCA cycle shown in Figure 1.3 is facilitated by the fact that adjustment to plans in mid-cycle as necessary is an integral part of the hoshin kanri process.

TRADITIONAL STRATEGIC PLANNING APPROACH

The traditional strategic planning approach employed by many organizations is illustrated in Figure 1.6. Similarly to the hoshin kanri approach, the traditional approach uses a three-tiered set of plans:

• Strategic plan. A set of long-term plans







- Tactical plan. A set of mid-term plans
- Operational plan. A set of short-term plans

Figure 1.6 depicts the traditional strategic planning process. As we can see from this figure, there are multiple inputs to the strategic plan and it is decomposed into manageable, but smaller plans (that is, tactical and operational plans). Following this, metrics are developed and plans are reviewed throughout the annual or operational cycle. However, unlike hoshin kanri, this approach is generally linear and top-down, with no clear demonstration of cycles of process improvement taking place.

Table 1.5 provides a comparative perspective of the three types of plans and identifies who is responsible for developing each plan, the time horizon, and what each plan addresses. It is important to note that the time horizons suggested are generally typical in most industries. However, they are not fixed. Instead, they should reflect the dynamics of the organizational environment. For example,

EXAMPLE 1.4

A national retailer is losing sales to another major competitor. Analysis of customer feedback suggests the primary reason is due to poor customer satisfaction associated with the company's product return policy. Table 1.6 illustrates how a strategy has been defined to combat the sale loss and how that strategy has been decomposed into lower-level tactical and operational plans.

Two key points should readily surface from a review of Table 1.6. First, there can be a one-to-many relationship between a strategy and the number of tactics. Second, there can be a one-to-many relationship between a tactic and the number of operational plans. What is not readily apparent is that a systematic planning process is never a substitute for poorly constructed plans. Hopefully, such plans are swiftly identified and changed during routine plan reviews.

| Plan | Developed by | Horizon | Addresses |
|-------------|---------------------------|----------------------------------------------------------------------------------------|---------------------------------------------------|
| Strategic | Executive management | 3–5 years or what the organization deems to be its strategic planning time frame | What is to be accomplished |
| Tactical | Mid-level management | 1 year | How it is to be accomplished |
| Operational | First-level management | Daily activities | • By whom and when it is to be accomplished |
| | | | • What resources are required |
| | | | Process focus |

Table 1.5 Comparing strategic, tactical, and operational plans.

| Strategy | Tactic | Operational plan |
|---------------------------------------------------------------|--------------------------------------------------------------------------|------------------------------------------------------------------------------------------|
| 1.0 Improve customer satisfaction by 40% within 3 years | 1.1 Implement a new return policy by the end of the second quarter | 1.1.1 Modify cash register computer system. See Gantt chart for detailed schedule. |
| | | 1.1.2 Establish budget and secure funding and resources |
| | 1.2 Train all frontline employees in conflict management | 1.2.1 Determine who is considered a frontline employee |
| | | 1.2.2 Establish budget and secure funding and resources |
| | | 1.2.3 Develop training schedule |
| | | 1.2.4 Human Resources to conduct training per schedule |
| | | 1.2.5 Conduct pilot |
| | | 1.2.6 |

| Table 1.6 | An example of | traditional | strategic | planning. |
|-----------|---------------|-------------|-----------|-----------|
|-----------|---------------|-------------|-----------|-----------|

the strategic planning horizon for an organization doing business with the military might be 10–20 years while an electronics firm that produces cellular handsets might have a strategic planning horizon of one year. In fact, their tactical planning horizon might be no longer than a quarter.

Though many organizations favor the traditional approach, many find it does not bring them closer to achieving their desired objectives. This is primarily because it lacks the rigor of hoshin kanri. Consider the following failure modes associated with the traditional approach:

- Tactical and/or operational planning is often omitted. This eliminates line of site from strategy to process (see Kubiak 2010).
- The budgeting and planning processes are not synchronized. Therefore, resources and funding in support of the operational plans may be absent.
- Lateral or horizontal planning may be ignored. There is nothing inherent in the process to ensure that such planning is accomplished.
- Metrics may be developed that are unrelated to any of the three plan types. Often, existing or convenient metrics are employed that are not representative of the process or activity being measured. Therefore, as metrics roll up to the strategy level, it may become impossible to determine whether strategies are successful or not.
- Metrics are often lagging versus leading indicators. As a result, momentum builds, and it is difficult to change plans in mid-cycle.

- Changing plans in mid-cycle is viewed as a "corporate sin." Corporations are reluctant to make any adjustments since changes may be viewed as "failures" of an individual, department, and so on.
- The planning process does not benefit from cycles of improvement. Generally, organizations tend to use the same process cycle after cycle. Organizational learning is not captured and driven back into the process in the form of improvements.
- Underlying assumptions on which plans are based change or become invalid, but the plans and their subordinate plans do not.
- Plans are made and shelved. This happens all too frequently. Organizations apply extensive effort and expend significant cost to develop plans only to let them be forgotten or obsolete. The goal of the planning activity is to check the box.
- Strategies are often confused with tactics. More specifically, they represent tactical thinking versus strategic thinking. Hence, many organizations change their strategies on a yearly basis. Consequently, this creates a whipsaw effect in organizations, which is reflected in priorities. Projects are often not completed or lose importance as new priorities are communicated. Remember, strategic plans are long-term while tactical plans are mid-term. Operational plans are shorter still. This three-tiered planning approach is necessary to provide stability to both planning and execution activities.

Remember, regardless of the planning process your organization selects, plans are plans. Plans are guidelines that help center and focus an organization whereby everyone pulls in the same direction. But they should not be considered absolutes.

Although only two types of strategic planning processes have been addressed in this chapter, other types are in use as well. The military uses its own form of planning, and in recent years "scenario" planning has had its followers. Readers interested in learning more about these processes should see Cobb (2003) and Dettmer (2003).

Chapter 2 Strategic Plan Alignment

In any organization, alignment is the key to achieving success. This means all units, departments, groups, individuals, and so on, pulling in the same direction. Alignment starts at the apex of the organization with its stated goals and objectives and is pushed downward through the organization until it impacts every plan, individual, and activity. Alignment also occurs horizontally across the organization through coordinated planning activities.

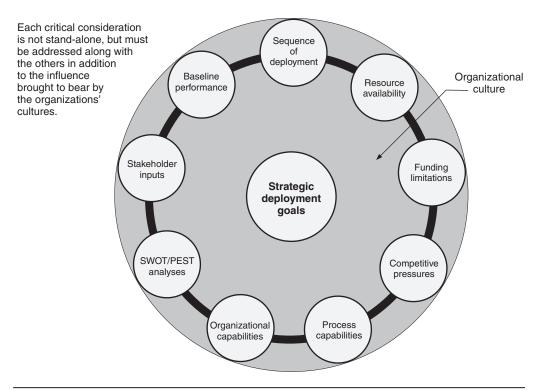
STRATEGIC DEPLOYMENT GOALS

Describe how to develop strategic deployment goals. (Apply)

Body of Knowledge I.B.1

The development of strategic, tactical, or operational plans is a wasted exercise if careful consideration is not applied to developing meaningful deployment goals or a deployment approach. More specifically, several key considerations need to be addressed to answer the question, "What is the best way to deploy the plans we developed?" Figure 2.1 identifies many of these considerations, each of which will now be discussed:

- *Sequence of deployment.* This is not a trivial consideration. The "sequence of deployment" may be broken into four categories: organizational, demographical, geographical, and products or services. However, you may think of others more suitable to your specific organization:
 - Organizational. The basic idea behind this category is to address the question as to which organization leads the deployment, which organization goes next, and so on. It also defines the level of the organization. For example, strategic business unit, division, plant, operation, product line, and so on. The output of this exercise might be quite complicated and best reflected in something like a





Gantt chart. (The Gantt chart was discussed in Chapter 17 of *The Certified Six Sigma Black Belt Handbook*.)

- Demographical. The basic idea behind this category is to address the question as to which group of employees leads the deployment, which group goes next, and so on. For example, in a production organization, we might want to deploy Lean Six Sigma to the production engineering employees in the production department because they have demonstrated a progressiveness and willingness to learn. By contrast, for example, widget engineers at the organization have historically been the most difficult to introduce new concepts and ideas to because this group comprises tenured employees who are set in their ways. It might be easier to convince widget engineers of the usefulness of Lean Six Sigma after we have demonstrated a string of successful project completions with results.
- *Geographical*. As you might expect by now, the basic idea behind this category is to address the question as to which geography leads the deployment, which geography goes next, and so on. For example, a global manufacturing organization has facilities in the United States, India, Mexico, and France. Within the United States, facilities

are located in the Northeast, Southeast, Midwest, and California. In this instance, the organization has four national cultures it must consider, and likely four subcultures within the United States. Each culture/subculture presents its own difficulties. The pros and cons of deploying to each culture/subculture must be clearly understood before constructing deployment goals based on this consideration.

 Products or services. These are in direct line of sight of the customer, be they internal or external (including intermediate and end-user) customers. Input from these various categories of customers provides valuable information regarding how a deployment should proceed. This is similar to addressing the "squeakiest wheel."

Given the above discussion of "organizational," "demographical," "geographical," and "product or service" deployment considerations, it should be noted that these four categories are not necessarily mutually exclusive. In fact, it is likely a finalized deployment goal might be stated as: "Lean Six Sigma will be deployed first to the production engineering employees in the Production Department on Product A on Product Line B in Plant C of Division D of Strategic Business Unit E located in France during the second quarter of the upcoming fiscal year." As additional statements such as these are made, the need to depict them in a systematic manner becomes more urgent. Again, the Gantt chart or similar tool is useful.

- *Resource availability.* Deployment and subsequent execution of plans requires resources. In particular, people are required. As with most organizations, such resources may vary widely across individual sub-organizational units, thus limiting the rate of deployment and execution of plans. This is particularly true when priorities are not well established. In such cases, units have difficulty diverting human resources from daily operations to driving improvement efforts, and are fighting the expectation of doing both simultaneously.
- *Funding limitations*. Like resource availability, funding limitations may vary widely as well. In addition to paralleling the resource availability issues, many organizations develop funding plans independently of their strategic planning process. Consequently, strategic plans and their subordinate plans are usually not fully funded either in total or by organizational unit. This results in less than full cooperation between departments, plants, divisions, and so on. When less than full cooperation is present, the plan may be undermined unconsciously or otherwise.
- *Competitive pressure*. Understanding the "competitive pressure" or environment is paramount when establishing deployment goals. As such, it is critical to have a comprehensive competitive analysis at hand. The risk of not having one is the possibility of losing the competitive edge or the failure to maintain parity with a competitor.

- *Process capabilities.* Strategic plans are likely to be developed without regard to capability of an organization's processes since they focus on the long-term, big picture. However, as these plans are decomposed into tactical and operational plans, process capability should surface as a key consideration. Some processes will be capable while others will not. Of course, the cost of making non-capable processes capable, along with the associated time element, is always a concern, as funding is limited.
- *Organizational capabilities.* Organizational capabilities come in many different forms, for example, limited distribution channels, choice of partnerships, employee skills and education levels, union agreements, or laws and other regulatory constraints that may limit some organizations. It may hamper their competitiveness or detract from their ability to attract and hire employees with the skills and talent they require. Although this consideration is embedded within the SWOT analysis, its level of importance is high enough to be identified separately.
- *SWOT/PEST analyses.* These analyses have been discussed previously in Chapter 1. They provide a wealth of input that is useful in establishing the sequence and timing of deployment goals.
- *Stakeholder inputs.* There are many different categories of stakeholder inputs. So far, we have mentioned shareholders, customers, employees, communities, suppliers, and partners, to name a few. Regardless of how you identify or classify your organization's stakeholders, their input is crucial to establishing meaningful deployment goals. In fact, many of these stakeholders reflect the typical organization's value chain. Any stakeholder within or along that value chain can have a significant impact on the overall success of the organization. Let's look at each of these stakeholders individually:
 - Shareholders. Recently, and likely due to the economic downturn, shareholders have voiced their concerns loudly at shareholder meetings and through the annual shareholder voting process. They have, in many instances, provided organizations with a clear communication of their unhappiness with leadership performance and strategy, among other issues. This conveyance has resulted in more than one leader losing their position and subsequent shuffling of management.
 - *Customers.* Customers (or clients) are the raison d'être of any organization. Organizations should have the proper listening posts so that the voice of the customer (VOC) is captured sufficiently and timely, and is, of course, acted on.
 - *Employees.* If customers are the raison d'être, then employees are the lifeblood. Employees make an organization work. They tend to its processes, deal with its stakeholders, and, if they choose, sabotage its ability to succeed. Organizations that listen to employees,

recognize their limitations, act on their needs, train appropriately, and communicate effectively are generally more competitive and successful over the long term.

- Communities. This stakeholder category is often ignored by many organizations, yet it's one that can have a powerful influence over an organization's future. For example, consider big-box retailers. Some communities are boycotting Wal-Mart specifically, using zoning ordinances to restrict it and/or other big-box retailers from changing the look and feel of their communities, or not permitting them to build within their jurisdictional boundaries altogether. Perhaps this has been influential in leading Wal-Mart to develop a strategy whereby it will build new stores that are significantly smaller, architecturally consistent with community standards, and reflective of the uniqueness of the market that each new community store serves.
- Suppliers/partners. Suppliers and partners can be a limiting factor in the deployment of strategic plans. All too often, organizations fail to consult with these key stakeholders only to find out that the suppliers/partners' strategic plans are not consistent with the organization's plans. For example, consider a major aerospace company. After a supplier achieved the status of "partner," the company became bound by its selection criteria to share its strategy with its partners. However, the aerospace company decided against sharing its plan or even obtaining input with its partner for "proprietary reasons." The aerospace company decided to pursue the development of product A that required component B from its "partner." Unfortunately, the partner's strategic plan called for the phase-out of component B, thus leaving the organization with a very undesirable dilemma. The company had the option of buying all the remaining stock of component B from its partner while trying to find another supplier, or it could go back and change its strategy. Once more, an unfortunate situation prevailed. The partner was a sole-source provider and the company was unwilling to change its strategy. The alternative it chose was to convince (after significant effort and capital infusion) another supplier to design and produce the product, which the company purchased in small quantities. As a result, the cost per unit of each component was extremely high, thus making the aerospace company's product noncompetitive. The moral of the story is that input from a supplier/partner to the process of setting strategic deployment goals would have alerted the company to a problem much earlier during the actual planning process when it would have been more amenable to changes. The secondary moral of the story is: stick to your process. Remember, the process called for the aerospace company to share its strategic plan with its partner. However, it chose not to do this and, therefore, it paid a heavy price for its decision.

• *Baseline performance*. This aspect is important in that it provides insight into the gap that, say, an organizational or geographical unit has to overcome. Units with larger gaps may be scheduled for earlier deployments in a fiscal period than those with smaller gaps. This is based on the idea that larger gaps require more time to close. This may or may not be true. A well-conducted gap analysis might be necessary to fully understand how this factor should influence deployment goals.

Before leaving the topic of strategy deployment, one final point must be made. The literature abounds with claims that for organizations to be successful, everyone in the organization must understand its strategy. Visibility into strategy at the lowest levels of the organization has been frequently misinterpreted. It does not mean those at the lowest level must have direct access to the strategy; what it does mean is that their actions must be guided by strategy whether those actions are manifested by operational plans, personal goals and objectives, projects, work orders, action plans (see Westcott 2006), factory-level planning boards, or any number of forms. What is important is that their actions are systematically derived from the strategy.

Hopefully, the above discussion has demonstrated that the development of strategic deployment goals is not to be taken lightly, for the consequence of poor goals can be significant. Also, it should be apparent by now that no magic formula exists. What is required, though, is a solid understanding of the key considerations depicted in Figure 2.1 and recognition that multiple deployment goal schemes are possible.

PROJECT ALIGNMENT WITH STRATEGIC PLANS

Describe how to align projects to the organizational strategic plan. (Apply)

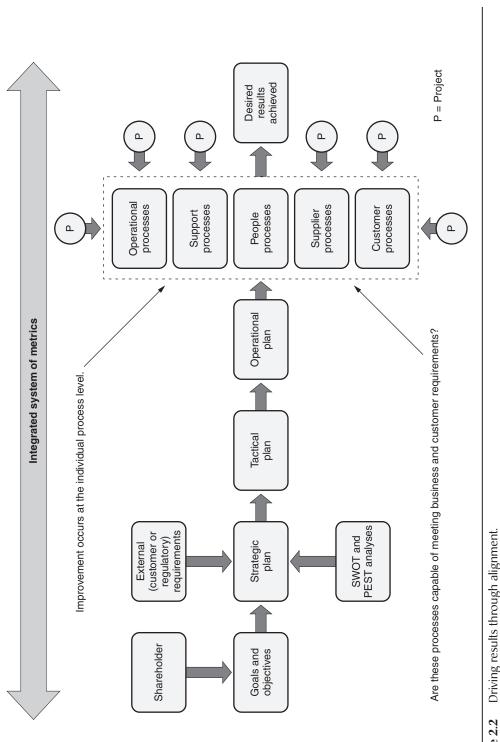
Body of Knowledge I.B.2

Projects align with an organization's strategic plan by using two complementary approaches:

- The project selection process
- The strategic planning process

The first methodology, the project selection process, will not be addressed here. However, it will be discussed in Chapter 5 when we deal with the critical topic of project identification.

The second methodology, the strategic planning process, has been discussed at great length in Chapter 1. However, how the alignment between plans and projects is created has not yet been addressed. Figure 2.2 has been created to help



Source: Adapted from T. M. Kubiak. 2010. "The Driving Force: Creating Line of Sight from Strategy to Process." Quality Progress, June, 52–54. Figure 2.2

facilitate this discussion. Notice that Figure 2.2 represents a minor modification of Figure 1.6, the traditional strategic planning process. In particular, Figure 2.2 has been modified to include:

- *The backdrop of an integrated system of metrics.* This backdrop is intended to run both horizontally and vertically throughout an organization. Such a system of metrics is needed to bring organizational alignment to bear, measure success, and make proactive course adjustments as required. Organizations functioning without such a system often find it difficult to execute their plans effectively.
- *Typical organizational processes.* The categorization of the processes depends on the specifics of the organization, though those provided in this figure are common. Some organizations may find this categorization useful while others may elect a different approach. For example, many have chosen the four perspectives of the *balanced scorecard* (for a discussion of the balanced scorecard, see Chapter 9).
- *Projects.* Processes are executed. While this is self-evident to quality and Lean Six Sigma professionals, often it is not apparent to other disciplines and professionals. When processes are executed, work is accomplished, and results are achieved. When processes are improved through meaningful projects, better results are achieved. And, as we know, Lean Six Sigma projects drive improvement at the individual process level.
- *The desired results.* When it is all said and done, this is what it's all about. We know we have achieved our desired results because we have a comprehensive and integrated system of metrics to tell us so. Unfortunately, there are multiple reasons why an organization may fail to accomplish its goals. Perhaps the planning process was flawed, metrics were improper, resources were insufficient, and last, but not least, either poor projects were selected or they did not improve processes sufficiently. After all, a systematic and rigorous planning process doesn't ensure stellar results and is not a substitute for good judgment.

While the flow from strategy to project in Figure 2.2 is straight, it is complex, and planning must be systematically decomposed into manageable elements (for example, tactical and operational planning) to create visibility and subsequent alignment to projects. Often, the tactical and operational planning is abbreviated or completely forgotten. When this occurs, the separation between strategy and projects is too great, and the line of sight is broken as illustrated in Figure 2.3. Therefore, it is likely that projects will not align to strategy or even to one another. As a result, it is possible projects may duplicate effort or be in conflict with one another, thus wasting precious time, energy, and cost.

However, when these detailed plans are created, rigorous alignment can be achieved. Meaningful projects can be identified, selected, and prioritized for execution. Furthermore, this alignment between strategy and projects creates a driving force for business success.

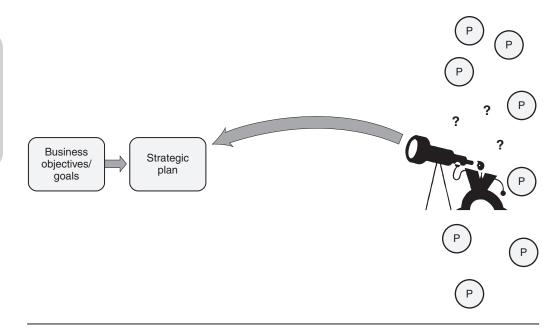


Figure 2.3 Breaking the alignment between strategy and projects.

PROJECT ALIGNMENT WITH BUSINESS OBJECTIVES

Describe how projects are aligned with business objectives. (Apply)

Body of Knowledge I.B.3

In general terms, when we speak of *business objectives* we mean *organizational objectives*. Therefore, we will try to use the more general term going forward, whenever possible.

We have just seen in the previous section how projects are aligned to strategy through an intermediate (that is, tactical and operational plans) planning structure and organizational processes. For the sake of simplicity, this discussion has been summarized and is now reflected in Figure 2.4a.

In Chapter 1, we discussed two different strategic planning methodologies. Regardless of the methodology chosen, goals and objectives were the necessary input for strategies to exist. Put another way, strategies without goals and objectives are without merit. Hence, they are always paired, thus creating alignment. This is reflected in Figure 2.4b.

Figures 2.4a and 2.4b taken together produce Figure 2.4c. Consequently, our alignment from organizational objectives to projects is complete. While the

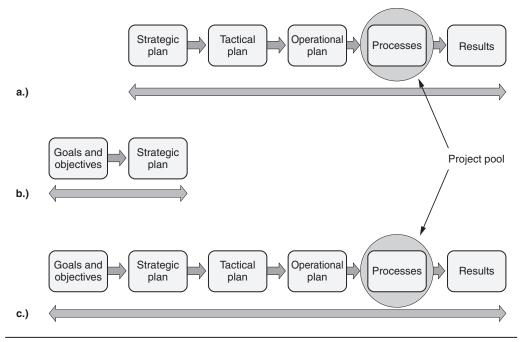


Figure 2.4 Achieving alignment from projects to strategies to objectives.

separation between objectives and projects is greater than the separation between strategy and projects (see Figure 2.3), the alignment is strong both horizontally and vertically. Remember, alignment results from following a rigorous planning process. Skipping or shortcutting steps weakens that alignment and undermines the organization's ability to achieve its objectives.

Chapter 3

Deployment of Six Sigma Systems

There are many elements necessary to ensure a successful deployment. Key among them are the following:

- Governance
- Assessment
- Resource planning
- Resource development
- Execution
- Measure and improve the system

Each of these elements will now be discussed in detail.

GOVERNANCE

Part I.C.1

Describe the following key deployment element: Governance (quality councils or process leadership teams). (Apply)

Body of Knowledge I.C.1

Bertin and Watson (2007) state that "Governance establishes the policy framework within which business leaders will make strategic decisions to fulfill the organizational purpose as well as the tactical actions that they take at the level of operational management to deploy and execute the organization's guiding policy and strategic direction." This, of course, refers to corporate governance, in general. *Governance* in the context of Lean Six Sigma deployment includes all the processes, procedures, rules, roles, and responsibilities associated with the strategic, tactical, and operational deployment of Lean Six Sigma. It also includes the authoritative, decision-making body charged with the responsibility of providing the required governance.

The concept of governance is critical to ensure a successful deployment. Many organizations omit this crucial component and wonder why the deployment stalls or even fails.

A common governance structure within a simple organizational unit is illustrated in Figure 3.1. Note that the Lean Six Sigma deployment team, led by the Lean Six Sigma deployment leader, "reports" into the governing body. This body is known by many names, common of which are the "Quality Council" or "Lean Six Sigma Deployment Council." Also, it is customary that the quality council reports locally within the organizational unit it serves to an "Executive Steering Committee." Such a committee usually comprises the chief executive of the unit and his or her direct reports. The value of this committee is to engage the executive level, build ownership, and drive accountability.

Occasionally, the quality council will operate within a complex organizational structure such as the one shown in Figure 3.2. In this example, each chairperson of a tier 3 council would serve on his or her respective tier 2 council. Subsequently, each chairperson of a tier 2 council would serve on the top or tier 1 council. This approach facilitates both vertical and horizontal communication throughout the entire organization.

Consider the various roles of a typical quality council depicted in Figure 3.3. Let's briefly discuss each of these roles:

• *Create policies.* Policies provide an overarching framework for the deployment and set the governing principles for how the deployment will function. An example of a policy might be that the organization will only select projects that provide hard dollar bottom-line savings. All other categories of savings will not be considered.

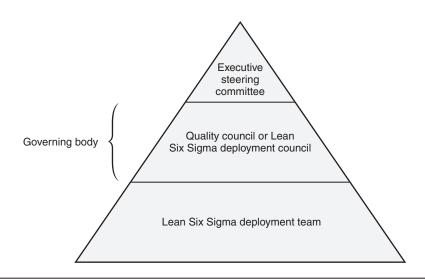


Figure 3.1 A common governance structure within a single organizational unit.

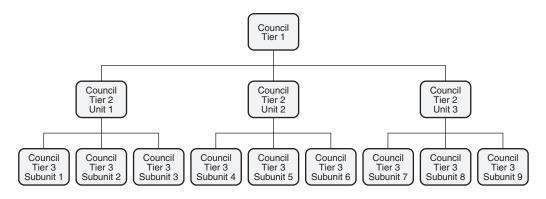


Figure 3.2 Nested governance structures within a complex organizational unit.

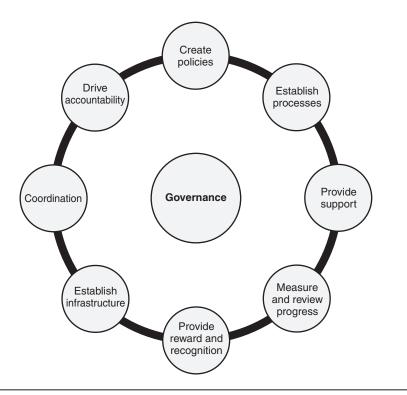


Figure 3.3 The many roles of Lean Six Sigma governance.

• *Establish processes.* An example of this role might be the "project selection process" that identifies who selects the projects, what happens after they're selected and by whom, and so on. While the quality council may not be the generator of the actual process document, it will have final authority over its approval and issuance.

- *Provide support.* Since the quality council has visibility into the overall deployment, it is in a position to work within the total organizational structure to bring resources to bear and reallocate resources among projects.
- *Measure and review progress.* The quality council regularly receives progress reports on projects' activities, attends project portfolio reviews, determines the successfulness of the deployment, and makes adjustments to the deployment strategy as appropriate.
- *Provide reward and recognition.* The quality council provides reward and recognition in a unified and appropriate manner per its policy. Many organizations fail to set such a policy, thus creating great inequities in delivering reward and recognition, or failing to deliver any at all.
- *Establish infrastructure*. Infrastructure is often overlooked by many organizations. If not overlooked, it is overtly avoided since infrastructure creates costs. Examples of infrastructure might be the creation of formal job descriptions for all employees of the Lean Six Sigma deployment team or the purchase of statistical analysis software. In some organizations, the infrastructure is considered the team members themselves. What does this mean? It means that an organization desiring to deploy Lean Six Sigma might hire a Lean Six Sigma deployment leader. However, it chooses not to provide staff to the leader (that is, the people infrastructure necessary for the deployment is deliberately omitted). Organizations doing this are usually testing Lean Six Sigma, and hence leadership is not fully committed. Figure 3.4 depicts a typical Lean Six Sigma deployment

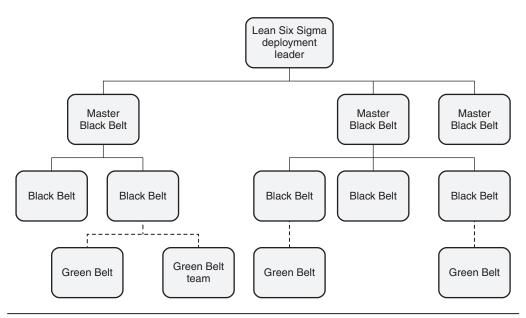


Figure 3.4 A Lean Six Sigma deployment team structure.

team structure. Notice that Green Belts and Green Belt teams are indirect reports into the Lean Six Sigma deployment organization.

- *Coordination.* The role of coordination for a quality council may come in different forms. If the organization is small or simple in structure, the quality council may be required to coordinate projects across departments or even geographies. However, for complex organizations, this coordination role can be significantly compounded since there are many quality councils that it must work with laterally—those that report to it, and those to which it reports. See Figure 3.2.
- *Drive accountability.* Because quality councils, typically in Lean Six Sigma deployments, comprise member representatives across the breadth of the organization, they are in a strong position to drive accountability, a key component in any successful deployment. Some councils choose to make use of the *responsibility, accountability, consultation, inform* (RACI) matrix shown in Table 3.1 (see Price and Works, "Balancing Roles and Responsibilities in Lean Six Sigma"):
 - *Responsibility.* Individuals who actively participate in an activity.
 - Accountability. The individual ultimately responsible for results.
 - *Consultation.* Individuals who must be consulted before a decision is made.
 - *Inform.* Individuals who must be informed by a decision because they are affected. These individuals do not need to take part in the decision-making process.

| Task Player | Set policy | Identify projects | Select projects | Work project | Achieve results | Maintain gain | Coach and mentor |
|------------------|---------------|----------------------|--------------------|-----------------|--------------------|------------------|------------------------|
| Executive | А | | А | | Ι | | |
| Sponsor | | А | R | | | | |
| Process owner | | С | Ι | | | А | |
| MBB | Ι | С | | С | R | | А |
| BB | Ι | | Ι | R | R | | А |
| GB | | | Ι | | А | | |
| Council | R | С | Ι | | С | | |

 Table 3.1
 Example of a RACI matrix.

ASSESSMENT

Describe the following key deployment element: Assessment (maturity models and organizational readiness). (Apply)

Body of Knowledge I.C.2

When we refer to "assessment" in the context of a Lean Six Sigma deployment, we typically think in terms of two distinct concepts. These are:

- *Maturity models.* How do we know whether the organization is moving toward transformation? There are two levels of maturity models to consider:
 - Organization-based
 - Process-based
- *Organizational readiness.* How do we know whether the organization is ready to launch?

Each of these concepts will be discussed in detail below.

Maturity Models—Organization-Based

At the onset of a Lean Six Sigma deployment, it is vital for both the deployment leader and quality council to understand the journey ahead. Lean Six Sigma deployments are cultural-based, organizational-wide, transformational strategic initiatives. As such, they do not happen in the short term. Consequently, it is important to have insight into how organizations develop and mature with regard to the deployment, for a lack of understanding and insight into the maturation process will lead to frustration and impatience on the part of leadership, and likely, the ultimate withdrawal of support. Therefore, a maturity model would be most beneficial.

Table 3.2 depicts such a maturity model. Although designed for a manufacturing organization, Table 3.2 can be modified to accommodate any service or transaction-based organization as well. Notice the focus in Table 3.2 appears to be on quality, in general, and no suggestion for the time in each phase has been indicated. However, the characteristics of each phase have been well defined.

Table 3.3 depicts another maturity model. However, this one is customized to a Lean Six Sigma deployment and is known as the Six Sigma Maturity Model (Raje 2007). Again, five phases are present and each one has now been time-based. Furthermore, another dimension has been added to the model. This dimension, known as the "axes of launch" identifies key characteristics of the Lean Six Sigma deployment, each of which is expected to mature throughout the phases. Note, it is possible, and highly likely, that one or more key characteristics may be in different phases. This is particularly true during early deployment.

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| Table 3.2 An organizationa | An organizational maturity model for a manufacturing organization. | acturing organization. | | |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Level 1 | Level 2 | Level 3 | Level 4 | Level 5 |
| Dysfunctional system | Awakening system | Developing system | Maturing system | World-class system |
| Economy-of-scale focus with long runs preferred. Time-consuming changeovers are the norm. The customer's "voice" is rarely heard, and then only at the top. | Quality Steering Committee has been formed; quality systems are assessed, quality initiatives are planned. A "customer focus" is a goal. | Tested practices are deployed to all major areas of the factory. Customer involvement is sought. | Seeks out and learns about best practices. Adapts improved practices for all areas. Customers, suppliers, and employees are integrated into the systems. | Retaining satisfied customers is key. Plant uses single-piece flow with cellular techniques. Improved throughput achieved through reduction of bottlenecks. |
| Rigid plant layout; nonintegrated systems, erratic workflow prevalent. Buffer stock everywhere. All jobs "rush." "Firefighting" norm. | Applicable lean management practices have been identified. Training is being conducted. | Flexible production layouts and cells are introduced. Cleanliness and neatness of individual work areas is stressed. | Production system allows short runs, greater product mix, speedy introduction of new products, shorter cycle times. | Plant layout is agile and clean. Workers self-inspecting their work. Lean manufacturing tools and techniques liberally applied. |
| Machinery run at maximum speed without regard for its life or performance quality. Workplace is unorganized and unclean. | A small project is under way to implement and test improved quality management practices. | Pull-type production system under test in one area. Employee qualification system is in place. | Operating information is provided immediately with computerized displays. Errors are prevented with mistake- proofing devices. | Preventive maintenance ensures availability, optimizes quality, efficiency, and life cycle cost. |
| No teamwork. "Fiefdoms" fiercely guarded from encroachment by other functions. No linkage between any overall strategy and production scheduling. | Bottlenecks and non- value-added functions in process flow are being examined. An equipment maintenance program is under development. | Cross-functional teams promote adherence to standards and ensure continuous improvement. | Teams, some self- managed, aid adherence to high standards, the focus on customers, and continual improvement. | Management is personally and visibly involved in continual improvement. Quality of information and decision making at all levels is exemplary. |

Source: Adapted from Westcott (2006).

| Table 3.2 Continued. | | | | |
|-----------------------------------------------------------------------------|---------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|------------------------------------------------------------------------------------|
| Level 1 | Level 2 | Level 3 | Level 4 | Level 5 |
| Dysfunctional system | Awakening system | Developing system | Maturing system | World-class system |
| Management by command. Poor workforce commitment and involvement. | A cross-functional team is being initiated to work on cycle-time reduction. | Systems are implemented to provide data for performance measurement, improvement. | An effective strategic planning process is instituted. | All employees are highly motivated, involved, and empowered. |
| Communication is one-way (downward) with little or no feedback loops. | Weekly production review meetings are held, chaired by the VP-manufacturing. | | Overall strategy is linked to production planning and process improvement. | Supplier relations based on collaborative communication and partnerships. |
| Adversarial supplier relationships, focus on price. | A supplier qualification approach is under study. | Supplier certification program in place. | Plant benchmarked by others in industry. | Plant benchmarked by others outside industry. |
| Customers frequently get poor quality and delivery. | Overall performance remains below industry norm. | Overall performance is about equal to industry norms. | Performance is above industry norm. | Performance is world- class. |
| | | | | |

| Model. |
|------------------|
| Maturity |
| x Sigma |
| he Instantis Six |
| Table 3.3 Th |

Part I.C.2

| lable 5.5 | lable 3.3 The Instantis Six Sigma Maturity Model. | urity Model. | | | |
|-----------------------|---------------------------------------------------|-----------------------------------------|---------------------------------------------------------|---------------------------------------------------|----------------------------------------------|
| | Level 1 | Level 2 | Level 3 | Level 4 | Level 5 |
| | 3–9 months | 6–18 months | 12–36 months | 24–48+ months | |
| Axes of Launch | Launch | Early success | Scale replication | Institutionalization | Culture transformation |
| Culture change | | | | | Embedded in culture, Up/down value chain |
| Beyond DMAIC | | | I | DFSS, Lean | Infused in IT, product development |
| Strategy marketing | | | Strategy maps created to align projects | Projects launched to impact strategic goals | Full, closed-loop goals and projects |
| Software | I | Statistical tools for belts | Project tracking, financial reporting | Best practices, lessons learned | Strategy and portfolio aligned |
| Reporting | Anecdotal, qualitative | Aggregate, average, mostly anecdotal | Aggregate, average, predictions | Organization-wide results | Multi-year history, projections |
| Financial impact | Ad hoc, no projects completed | Cost reduction, some impact | Consistent measure in place | Validation of results, strict control in place | Financial impact linked to financial systems |
| Project selection | Burning platform | Low-hanging fruit | Successful replication of projects | Formal idea-generation/ evaluation process | Formalized evaluation |
| People | Few believers, mostly skeptics | More believers, but mostly skeptics | New believers generate momentum | Six Sigma career path, repatriated Black Belts | The way we work |
| Training | Initial Black Belts, champions, executives | External, few waves completed | Many waves across organization, external training | Large scale, mostly external, some internal | Very large scale, internal, e-learning |
| Leadership support | Very few visionaries | Leadership vision validated | Leadership buy-in | Expected to support | Ingrained |

 Support
 Valuated

 Adapted from Prasad Raje. 2007. Maturity Model Describes Stages of Six Sigma Evolution, Second Edition. www.isixsigma.com.

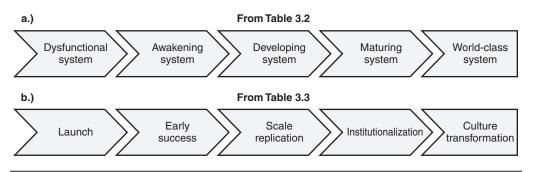


Figure 3.5 A comparison of the phases of two maturity models.

Both models are description-based. That is, they rely on the adequacy of descriptions to assess the level of organizational maturity rather than on a numerical scale. Also, each model relies on five different phases as compared in Figure 3.5. Notice the model given in Figure 3.5. There is nothing special about having five phases other than that the developers of these models felt that five phases adequately described the transformation process. Also, notice that the five phases of each model can not be compared directly to one another.

Figure 3.5a spotlights the organization as a system and provides a broader focus, that is, from dysfunctional to world-class. The full spectrum of performance is included. In contrast, Figure 3.5b focuses solely on the Lean Six Sigma deployment aspect. Unlike Figure 3.5a, the Six Sigma Maturity Model does not appear to assume that the organization is starting out in a dysfunctional state. In fact, state of performance of the organization at time of launch is unknown.

Maturity Models-Process-Based

A wide variety of process-based maturity models have been available for some time. Some are focused specifically on departments such as Information Technology while others have evolved to include more of the broader aspects of organizations. Regardless, most of these models have their roots in the area of software development. Common process-based models include:

- ITIL (Information Technology Infrastructure Library)
- ISO/IEC 15504 (Software Process Improvement and Process Capability Determination)
- CMMI (Capability Maturity Model Integration)

Most of the above models, like the organizational models, are level-based. For example, the CMMI model, which has grown significantly in popularity, is structured into five levels:

- Initial (ad hoc)
- Managed (repeatable)
- Defined (defined and measured)

- Quantitative managed (managed)
- Optimizing (self-optimizing)

(Note: Different authors use different terms for the five levels. Alternate terms are shown in parentheses.)

Although categorized here as process-based, Rao (2010) states that the CMMI "can be used to guide process improvement across a project, a division, or an entire organization." In this regard, we might want to consider it among our organization-based models above.

Several models, in addition to being level-based, are based on a numerical score (for example, 1–5) within a level, and usually result in a certification level being achieved. These types of models should be explored if exact numerical scores are required to measure progress.

Srinivasan and Murthy ("Process Maturity Model Can Help Give a Business an Edge") offer another model, which is delineated by six levels whereby each level includes the characteristics of all levels below it:

- Level 0—Person-dependent practices
- Level 1—Documented process
- Level 2—Partial deployment
- Level 3—Full deployment
- Level 4—Measured and automated
- Level 5—Continuously improving

It was mentioned briefly above that the CMMI has been gaining in popularity. In fact, many authors have begun to write about the integration of CMMI and Lean Six Sigma or, more specifically, how the CMMI can be used to support a Lean Six Sigma deployment. For more details, the reader is encouraged to check out Rao (2010), Sarma ("Six Sigma Encourages Improvement within CMMI Framework"), K. Williams ("CMMI or Six Sigma: Does It Matter Which Comes First?"), and Book ("Roadmap for Integrating ITIL, CMMI, and Six Sigma").

Srinivasan and Murthy are very specific in their definition of a "mature process." They define it as ". . . complete in its usefulness, automated, reliable in information, and continuously improving." Also, they are quick to note that even "fully established businesses can contain processes at different levels."

Organizational Readiness

The *organizational readiness assessment*, as the name implies, is typically performed prior to a strategic deployment. Such an assessment will provide meaningful insight into an organization's readiness to undertake that deployment on many different dimensions. Also, it will provide key input into the process for developing or adjusting the strategic deployment goals discussed in Chapter 2.

Cazar ("Six Sigma Deployment Planning & Readiness Assessment") suggests two aspects of an assessment model, *cultural* and *operational*, and has identified a series of questions for each aspect that needs to be addressed. Table 3.4 identifies those questions. Notice the dichotomy in the figure whereby the cultural

| Cultural Assessment | Operations Assessment | |
|--------------------------------------------|------------------------------------------------------------------------|--|
| | - | |
| How do senior leaders cascade information? | How is success measured? | |
| How are important decisions made? | Are the right things measured? | |
| Who makes the decisions? | How often are these things measured? | |
| How fast are decisions made? | Are there a few metrics that all employees | |
| How are successes and failures recognized? | understand and use? | |
| How does the organization handle failures? | Are decisions based on assumptions or data? | |
| Does everyone understand the mission, | Who owns each critical process? | |
| vision, and strategy? | Who owns the data? | |
| Is everyone aware of critical customer, | Where are the data? | |
| revenue, and expense issues? | Are the data on spreadsheets, laptops, or in a | |
| How are corporate goals established? | data warehouse? | |
| How clear are the goals? | Have the data been validated? | |
| Are the goals measureable? | Are reports written in simple terms and free from fuzzy language? | |
| | Are updated process maps available for most of the critical processes? | |
| | Do executives know what a process map is? | |

 Table 3.4
 The Cazar readiness assessment model.

Adapted from the "Six Sigma Deployment Planning & Readiness Assessment." www.isixsigma.com.

questions address soft issues while the operational questions address hard issues such as measurement, data, and process maps.

Cascella and Graeser ("Importance of Assessing Readiness to Implement Strategy") offer still another perspective on what constitutes a meaningful organizational assessment. In their paper, they cite what they call ten *deployment dimensions*:

- Clarity of strategy definition
- Leadership visibility and support
- Process orientation
- Strategic alignment
- · Process management and accountability
- Cause-and-effect thinking
- Meaningful business measurement
- · Fact-based skills and competencies
- Participative and integrated culture
- Customer orientation and focus

Though the authors do not detail the criteria for achieving each of these dimensions, their paper cites a *strategy deployment readiness assessment* diagnostic tool available at their website.

In addition to the above assessment models, Kubiak (2004) suggests an assessment based on the cultural characteristics depicted in Table 3.5. These characteristics were selected because experience has shown that if these traits are not addressed sufficiently prior to deployment, there is a high probability the deployment will fail.

Although other dimensions are important, only the cultural dimension has been expounded on since it is usually the most difficult to overcome. (Kubiak [2004] provides more details on three other dimensions including: people, leadership, and infrastructure.) Notice that the endpoints of the cultural dimension have been defined. Most organizations fall somewhere in between on each characteristic, though where they fall will likely differ greatly.

Unfortunately, none of the authors discussed present any methodology for determining a quantitative score or otherwise arriving at a predetermined assessment level. (Note: Fortunately, Table 3.5 lends itself to the use of *behaviorally anchored rating scales* [BARS] whereby each line in the table can represent the extremes of the scale that can be plotted for the specific characteristic. A researcher can sample each level of an organization to determine a baseline and progress against that baseline.) Further, some of the models addressed may not meet a particular organization's needs, thus requiring the development of a custom organizational assessment tool. Fortunately, when viewed across many assessment models, a common theme appears. That is, the following should be considered key dimensions of any assessment:

- Culture
- Infrastructure
- Leadership
- People

| Regressive organization | Progressive organization |
|-------------------------------------|------------------------------|
| Metric-averse | Data-driven |
| Process-averse | Disciplined |
| No true north | Aligned |
| Punishment for failure | Risk-takers |
| Soulless and demonic | Value-based |
| No sense of urgency | Speedy and agile |
| Who is to blame? | Accountable |
| Who are the leaders? | Involved/visible leaders |
| Disengaged and uninvolved workforce | Engaged and active workforce |

 Table 3.5
 Critical cultural characteristics for an organizational assessment.

- Processes
- Technology

Characteristics or traits within each of these dimensions can be developed and tailored to fit the needs of any organization type. If so desired, a scoring scheme can be added to provide a numerical score for determining ongoing progress.

RESOURCE PLANNING

Describe the following key deployment element: Resource Planning (identify candidates and costs/benefits). (Apply)

Body of Knowledge I.C.3

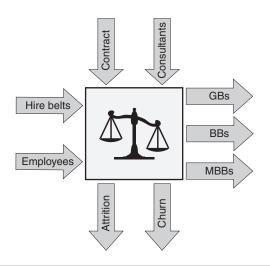
Resource planning associated with a Lean Six Sigma deployment may seem simple and straightforward on the surface. However, below the surface it is fraught with many complicated considerations that must be explicitly addressed because of the widespread organizational impact. These key considerations are best phrased in the form of questions that must be addressed, resolved, and communicated no later than the launch of the deployment. Otherwise, the consequence can be a serious loss of credibility. Some of these questions include the following.

Candidate-Related Questions

• What are the criteria for selecting candidates? Becoming a "belt" is not an employee entitlement. Employees become belts because the organization has embarked on a strategic initiative that it believes is beneficial. Consequently, it must select candidates to become belts (or higher-level belts) judiciously with carefully crafted criteria to maximize each employee's chance of success. This applies to Green Belts, Black Belts, Master Black Belts, or whatever other color scheme of belts the organization uses. The selection to be trained as a belt should not be a foregone conclusion or a political decision. Not every employee is up for the training challenge, though many organizations tend not to believe this. When unqualified candidates are selected, inevitability they will either drop out during the training, or the training will be "dumbed down" to fit the audience of candidates.

• Where will the candidates come from? Quite simply, candidates can come either from within the organization or outside of it, as shown in Figure 3.6. A brief review of this figure illustrates that organizations may:

 Take existing employees and train them with either internal or external resources. Due to qualifications and existing skill levels, some employees may have to start at Green Belt and work their way up, if they so desire. Others who have a strong technical or mathematical background may be able to enter directly into Black Belt or, in some cases, Master Black Belt training.





- Elect to hire belt-trained and/or certified individuals. These may be Green, Black, or Master Black Belts. Although most organizations don't hire Green Belts with the expectation of training them, mature organizations eventually establish Green Belt certification as a condition of hiring.
- Hire contract "employees" who are already Black Belts or Master Black Belts, or hire consultants. However, the latter is typically more expensive.

Another key aspect of Figure 3.6 should be pointed out. That is, organizations lose belts either through churn or attrition. *Churn* is typically defined as movement within the organization, while *attrition* refers to loss to outside the organization.

During resource planning, all elements of Figure 3.6 must be considered and planned for carefully.

• *How long will the candidates/belts be away from their jobs?* Candidates and their direct management will want to know what the length of the commitment will be. Candidates want to know with regard to how the commitment to supporting the Lean Six Sigma deployment initiative affects their careers since they consider it a risk. (Incidentally, any employee willing to step forward to support the organization's new deployment strategy should never be penalized for taking the risk, though many are.) Management will want to know since they must plan manpower and budgets, and must meet objectives.

Many organizations require 18–36 months of commitment to serving the Lean Six Sigma deployment initiative. However, if your organization has certification that is required, it will be necessary to clarify whether this time frame is from the time training begins, training ends, or post certification.

• *Will the candidates be assigned full- or part-time?* This remains one of the most contentious debates the deployment organization faces. The Lean Six Sigma deployment team wants candidates assigned full-time while other departments want candidates to work part-time (that is, their time is split between Lean Six Sigma and their home department).

Many organizations that have gone the part-time path have quickly found that part-time candidates yield little to no results. Part-time candidates find their loyalty split between bosses: one boss to which they report on a solid-line basis and one to which they report on a dotted-line basis. The initial part-time split of say "50/50" becomes "60/40," then "70/30," and so on. Inevitably, the solid-line reporting basis wins and Lean Six Sigma loses.

• *Will the candidates' positions be backfilled?* As mentioned previously, managers of candidates must meet their objectives and commitments. If their staff is reduced, they may be required to make significant changes within their departments. If they are permitted to backfill candidate positions, candidates must clearly understand their career path options when their commitment to the Lean Six Sigma deployment ends. This discussion is addressed below.

• *Will candidates cover all organizational geographies*? This question addresses the fundamental issue regarding how candidates and/or belts align to the organization's geographies. This might become a highly controversial issue in some organizations depending on how budgets are funded. For example, if a facility in Charlotte, North Carolina, has a project but no belt coverage, who provides the Lean Six Sigma service and who pays for it?

• *How will the situation be handled if a candidate drops out?* Expect this to happen, and plan for it. Some organizations have experienced a combined churn and attrition rate of over fifty percent. Candidates are offered or selected for new jobs during training, leave the organization, or simply can't cut it.

• *To whom do the candidates/belts report?* This question was partially addressed before when we discussed the full- or part-time candidate issue. It also connects to the assessment of candidates/belts' performance below. Address this question early, and clearly communicate the decision to both the candidates/belts and their respective management. Leave no room for doubt.

• Also, let's extend this concept to include contract employees and consultants as well. This is particularly important since, in some organizations, departments who pay for outside services are not necessarily the department to which those services report. Again, clarity is essential.

• What is the career path for belts? The answer to this question is essential. Many organizations have chosen the path such that individuals who become Black Belts or Master Black Belts serve the Lean Six Sigma deployment initiative for a specified period. Previously, it was mentioned that 18–36 months is common. Black Belts may be required to commit to a shorter period than Master Black Belts due to the level of investment required for the higher belts. After completing their commitment, belts are usually repatriated back into the rest of the organization. This often takes the form of going back to their original home department or being promoted to a higher, more responsible position in the hopes that they will take their knowledge and skills and drive them into the organization from a position of designated authority.

In addition to the above, some progressive organizations have recognized the need to maintain a strong technical skill base. Consequently, they have created a career path up through the Lean Six Sigma deployment team. Additional positions

within the teams have been created for lower, intermediate positions such as data analysis or data collector, and so on. Occasionally, these new positions have required their own certifications, including the *quality improvement analyst* (CQIA) or the *quality process analyst* (CQPA).

• *Have the expectations of belts been established?* Within any new position, expectations need to be clearly defined. Employees should never discover their expectations during a performance review. Therefore, full-time positions such as the Black Belt or Master Black Belt will require stand-alone job descriptions. When these are required, contact the human resource department.

Many organizations overlook the job requirements of a Green Belt. Specifically, they train individuals and eventually they certify them as Green Belts. Unfortunately, organizations frequently stop right here. More often than not, Green Belt candidates are required to complete one or two projects to achieve their certification. Thereafter, organizations find little use for them again. Skills, once learned, fall into disuse and become lost, along with the costly investment in them.

• *How will the performance of a candidate/belt be assessed?* Candidates and belts must understand how their performance will be assessed. For example, will it be based on the number of projects completed or the dollars saved? Whatever the criteria may be, they must be specific and communicated at the outset of deployment.

• Are candidates expected to achieve certification? Unenlightened management often contests certifications on the premise that it doesn't really accomplish anything for the organization. On the contrary, it provides the organization with a level of confidence in the technical competency of the individual practicing Lean Six Sigma. Of course, this statement is based on ASQ certifications, where rigor and a high degree of expertise and integrity are brought to bear on the development of the bodies of knowledge, examinations, and all related processes. All that is required is a simple, but tactfully phrased, retort to the certification naysayers, "Would you prefer to see non–board certified physicians?"

Assuming the decision has been made that certification is a requirement, there is another aspect of certification that must be addressed. That is, should certification be developed and administered internally or externally? Prior to the ASQ or some other third-party certification, early adopters of Six Sigma had no choice but to develop their own certification process. Unfortunately, internal certification is not without significant drawbacks. For example, it is subject to political pressures, particularly when a key measurement of deployment becomes the number of certifications per time period. This is especially true when candidates report to the managers of their home departments, and those managers are held accountable for their employees achieving certification.

Costs/Benefits-Related Questions

• Will the Lean Six Sigma deployment organization be self-sustaining or underwritten? A self-sustaining department is one that must promote itself to the remainder of the organization with the hope that the organization will fund its services. An *underwritten department* receives a budget and is expected to meet its objectives within the constraints of that budget. In order to answer this question, one must understand the culture of the organization. If the culture is immature and filled with disbelievers (as most are at the outset), the Lean Six Sigma deployment organization will receive few requests for its services, perhaps so few that it is unable to survive.

Let's place this in context. Lean Six Sigma is a strategic initiative. Does it make sense at the outset to jeopardize a strategic initiative by making the deploying organization self-sustaining? Although a few well-known organizations have been successful in this regard, most have not. Subsequently, the initiative either failed or the deployment organization was underwritten.

However, as organizations mature, and Lean Six Sigma is driven into the DNA of the organization, it may be reasonable to revisit the idea of a self-sustaining deployment organization.

• *Will belts be hired or developed from within?* The advantage of developing belts from within the organization is that they are familiar with the culture and workings of the organization. Networks have been created and processes are understood. However, it takes time to train and develop these employees. The return on investment may take time. Further, internal candidates and belts may be biased or blind toward processes they have been exposed to for extended lengths of time. In contrast, outside hires must develop new networks and are regarded as outsiders, and may or may not understand the organization's processes. Outside hires often bring a fresh set of eyes when improving processes designed to expose long-standing assumptions and can apply the 5 whys with less fear of colleague resentment. Outside hires can be productive immediately, thus generating cost savings quickly.

Many organizations have found that some mix of both internal development and external hires is best. Of course, this depends on the strategic deployment goals set, and how quickly the organization wishes to recoup its investment.

• Will training be purchased or developed from within? This is the traditional "make versus buy" problem. To develop the training from within, the organization must already possess the talent. Though this may be the lower-cost alternative, internal development of material is more time-consuming. However, purchasing training may require a large up-front capital expenditure that may not be suitable for small to medium organizations. Actually, a third alternative exists. That is, bring in a consultant to train. The consultant may charge a fixed daily rate plus expenses, a fixed rate per attendee up to a maximum number of attendees per class (also known as a *wave*), or any other possible combination. The key is to secure a complete quote over a specified time frame, determine how long the quote is in force, and do a cost-benefit analysis on a variety of combinations of the options discussed in this bullet that are suitable for your organization and consistent with your strategic deployment goals.

• *Have cost savings categories been clearly defined and are they mutually exclusive?* Since Six Sigma became widely known in the late 1990s, organizations have defined many different categories of cost savings. Some of these include: hard dollar bottom-line cost savings, cost avoidance, revenue growth, capital savings, and so on. Regardless of how you categorize your organization's savings, ensure that they are well defined, unambiguous, and mutually exclusive. There should be no instance where a particular cost savings can fall into more than one category. When this happens, credibility suffers.

• Is there a minimum expectation of cost savings per year or project associated with a specific belt level? One of the questions discussed in the "candidate-related questions" set above related to assessing the performance of candidates and belts. Though this may appear to be duplicated, the viewpoint is different. We ask this question to help plan and predict cost savings for the appropriate fiscal period. Ultimately, if the Lean Six Sigma deployment team does not provide this service, senior management will ask for it.

• *How are cost savings validated and by whom?* Credibility is undermined if cost savings can not be validated. Put another way, they should not be arguable. Consequently, it is beneficial to have a department or authority with the ultimate say and ruling over cost savings. Many organizations consider this authority to be the finance department. The Lean Six Sigma deployment team, in conjunction with the finance department, can develop validation processes. This only adds to the credibility.

• Who provides key data inputs used to facilitate cost savings calculations? Stabilizing key data input into financial calculations is often overlooked. For example, when computing labor savings, we might be faced with several questions:

- Do we use actual or weighted average pay for a given job family?
- Do we use the actual or fringed hourly wages?
- Should we use an average month of 30 days, an actual month, or a fiscal month?

It should be readily apparent that many additional questions can be added to this, and it is likely these will not all be known in advance. Therefore, it is imperative that a department or authority be accountable for addressing questions relating to financial calculations and for establishing key data inputs such as those described above. As before, this is often the finance department.

• What is the time frame associated with the cost savings? Cost savings must be claimed. The question is, "Over what time frame should they be claimed?" Typically, organizations will annualize their cost savings for a particular project. This makes sense to most organizations because annualizing their savings provides a clear and clean way to both understand and communicate the progress the Lean Six Sigma deployment is making. In some instances, savings may be a one-time event. For example, the result of the process makes it unnecessary to spend budgeted capital dollars on a piece of equipment. Not spending the capital dollars would be a one-time event. There is no right or wrong time frame. Choose whatever makes sense for your organization.

• *How does a unit's cost savings impact its budget?* This question will create quite a fury in most organizations. So stand at a safe distance when this issue is discussed. Generally, there are several possible choices to make:

 No impact. Savings are collected in a general ledger account and senior management can reallocate the funds as they deem appropriate.

- The budget of the department impacted by the project cost savings is decremented by the cost savings beginning immediately upon project implementation or beginning with the next fiscal year or budget cycle.
- The budget of the department remains unchanged at the beginning of the next fiscal year or budget cycle. Some organizations may view this as a true savings since the budget was maintained and not increased.

Organizations choosing the second and third bullets need to be particularly confident in their cost savings calculations as well as their ability to sustain the gains obtained through the project.

• *Will suppliers be trained?* This question is aimed at how far back into the value chain the organization should train. Certainly, it is advantageous for any organization to have suppliers who have adopted Lean Six Sigma. However, organizations early in their deployment may want to reconsider pushing their supplier to adopt Lean Six Sigma or offering to train them. Inevitably, the suppliers will require support, and the organization will not be in a position to provide.

• Are the cost savings real or have they just been reallocated? This can be recognized as "squeezing the water-filled balloon." Squeezing the balloon only shifts the water from one place to another. It doesn't lessen the amount of water in the balloon. The authority charged with validating cost savings should pay particular attention for this possibility.

• Will certification be developed and handled internally or externally? Although we addressed internal and external certifications previously, we did not discuss them from the costs-benefits viewpoint. Internal certifications are costly both to develop and maintain. In addition, denial of a certification could result in the candidate bringing a lawsuit against the organization. External certifications, such as the ASQ certifications, remove the organization from making a pass/fail decision, eliminate any potential for political maneuvering, maintain an arms-length distance from the certification process, and are very low-cost to maintain.

A summary of the candidates and costs/benefits questions is provided in Table 3.6.

As you continue with your resource planning, give considerable thought to the above questions. If you don't, be assured the rest of the organization will.

Consider preparing a Frequently Asked Questions (FAQs) document with answers to these questions. Pilot them and refine your list.

RESOURCE DEVELOPMENT

Describe the following key deployment element: Resource Development (train and coach). (Apply)

Body of Knowledge I.C.4

| Candidates | Costs/benefits |
|---------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------|
| What are the criteria for selecting candidates? | Will the Six Sigma deployment organization be self-sustaining or underwritten? |
| Where will the candidates come from? | |
| How long will the candidates/belts be away from their jobs? | Will belts be hired or developed from within? |
| | Will training be purchased or developed from within? Have cost savings categories been clearly defined and are they mutually exclusive? |
| Will the candidates be assigned full- or | |
| part-time? | |
| Will the candidates' positions be backfilled? | |
| Will candidates cover all organizational geographies? | Is there a minimum expectation of cost savings per year or project associated with a specific belt level? |
| How will the situation be handled if a candidate drops out? | How are cost savings validated and by whom? |
| To whom will the candidates/belts report? | Who provides key data inputs used to facilitate cost savings calculations? |
| What will be the career path for belts? | |
| Have the expectations of belts been established? | What is the time frame associated with the cost savings? |
| How will the performance of candidates/ belts be assessed? | How does a unit's cost savings impact its budget? |
| Are candidates expected to achieve certification? | Will suppliers be trained? |
| | Are the cost savings real or have they just be reallocated? |
| | Will certification be developed and handled internally or externally? |

Table 3.6 Summary of key deployment questions related to candidates and costs/benefits.

Resources must be developed consistent with the organization's needs. Needs drive training. This means:

- Organizations shouldn't make blanket statements such as, "Everyone who is a salaried professional will be Green Belt certified by (fill in some future point in time)." Many well-known organizations have made such statements publicly. These same organizations are guilty of training and certifying thousands of Green Belts only to have them never participate on another project.
- Not everyone who wants to be a Green Belt, Black Belt, or Master Black Belt will become one. Qualifications count!

Resources can be developed and maintained in three primary ways:

- Training
- Coaching
- Continuing education

Each of these ways will now be discussed in detail.

Training

As mentioned in the previous section, training may be delivered through internal or external resources. Table 3.7 identifies key characteristics associated with internal and external training. No attempt has been made to classify these characteristics as advantages or disadvantages since each organization may perceive them differently.

One aspect of training is not identified in Table 3.7 because it applies regardless of whether training is conducted by internal or external resources. That is, "Who pays for the training?" This is a simple question, but requires a complicated answer: "It depends." Some organizations will have the Lean Six Sigma deployment organization pay external costs, who, in turn, may or may not charge back (or prorate) candidate costs to their home departments. You may want to look to your finance department for help in this area. Another aspect is that of software. Generally, specialized statistical software is used and must be accommodated for in the budget. Usually such software runs on a common computer laptop and doesn't require any special type of additional hardware consideration.

Take another opportunity to review Table 3.7 as it is important to understand how each of these characteristics enters into your specific deployment plan. Only then will you be able to determine the most effective cost–benefit trade-off. Such a trade-off might include a mix of both internal and external resources.

Coaching

Coaching is a cost element some organizations like to omit during deployment, particularly if it is seen as an external cost and not viewed as value-added by management.

In fact, coaching adds significant value to the deployment. It helps expedite the assimilation and understanding of knowledge gained in the classroom by transferring theory into application. Furthermore, the mere presence of a coach

| Internal | External | | |
|-------------------------------------------------------------------------------------------|-------------------------------------------------------------------|--|--|
| Less expensive | Can be very expensive | | |
| Takes time to develop course material | A variety of courses can be delivered almost on a moment's notice | | |
| Requires that a certain number of high-level belts be on board to develop the material | Does not require high-level belts to be on board | | |
| Future instructors can pair with current instructors for training | The development of internal instructors is unclear | | |
| Consistency of material content likely to degrade with large, decentralized organizations | Material content consistent | | |
| Deployment paced by the rate instructors are developed | Instructors usually available almost on a moment's notice | | |

 Table 3.7
 Key characteristics associated with internal versus external training.

who meets regularly with a candidate serves to accelerate projects to successful completions.

As with training, coaching may be conducted by internal or external resources. Although there is great disparity among Master Black Belts, a typical recommendation for coaching during belt training is approximately one hour per week between classes. Some Master Black Belts vary this recommendation according to the belt level.

Internal resources generally do not incur additional costs to the organization, but they do commit and constrain resources. As you plan for coaching, consider who will do the actual coaching and the associated time and cost of each. Common coaching schemes in use are:

- Certified Black Belt coaches Green Belt candidates.
- Master Black Belt candidates coach Green Belt candidates.
- Master Black Belt candidates coach Black Belt candidates. This approach is used to help Master Black Belt candidates develop strong coaching skills, which will be necessary for coaching executives, champions, and sponsors.
- Certified Master Black Belts coach Master Black Belt candidates, Black Belt candidates, and Green Belt candidates.

External coaching resources can represent a sizeable cost to the organization. However, they are additional resources, thus freeing Black Belts and Master Black Belts within the organization for project work.

Regardless of whether your organization chooses to conduct coaching with internal, external, or a mix of resources, a cost–benefit analysis is required to make an informed decision.

Continuing Education

As with most knowledge and skills, the old adage of "use it or lose it" is at play. Lean Six Sigma is no different. It is exactly for this reason organizations should consider Lean Six Sigma using continuing education courses. This type of education might be reflected as a scheduled set of short courses over a period of time that reintroduces the belt to the concepts already learned during the initial training cycle. It keeps the belt familiar with tools and techniques, and updates to the course can be made to keep everyone up-to-date. Continuing education is typically offered internally after the course material has been developed or purchased.

EXECUTION

Describe the following key deployment element: Execution (deliver on project results). (Apply)

Body of Knowledge I.C.5

This is where the proverbial rubber meets the road. Candidates are trained, projects are conducted, results are achieved, and the gains are sustained. When this happens, the deployment is successful. Remember the old adage, "Plan your work and work your plan." There is no shortcut. Shortcuts often lead to failure and often represent a circumvention of established processes. Sometimes this circumvention is directed or intentional; other times it is the result of happenstance or ignorance of the processes.

Unfortunately, things go awry. Candidates may be trained improperly or insufficiently. Projects may be poorly selected. Potential gains may have been overestimated, thus the results achieved are disappointing. And sometimes the gains are not sustained. This may be due to a poor hand-off back to the process owner, an incomplete control plan, or any number of reasons. Collectively, the execution has failed. When this occurs, it becomes imperative to understand the failure modes in the deployment process. Hopefully, the Lean Six Sigma deployment organization has been using its own tools and has been measuring the various deployment processes, as these measurements will be needed to understand and improve both the deployment process and measurement system. See the next section (I.C.6.).

Of course, the above represents what sounds like a doomsday scenario. It doesn't have to take place as long as effective processes are put in place. Furthermore, these processes must be established with a set of checks and balances. For example, routine reviews by the Lean Six Sigma deployment leader, quality council, and executive committee, and other techniques such as intervention. Intervention is an important check and balance role played by the Master Black Belt and is addressed in Chapter 5.

MEASURE AND IMPROVE THE SYSTEM

Describe the following key deployment element: Measure and Improve the System (drive improvement into the systems, multiphase planning). (Apply)

Body of Knowledge I.C.6

A Lean Six Sigma deployment is not and should not be considered a deterministic activity. As we have seen, the strategic deployment comprises many different processes over a potentially long time frame. Many dynamic forces act on these processes; yet, they must work in harmony to ensure a successful outcome. Of course, this is suspiciously similar to improving a process any belt might face. In fact, the question must be asked, "Why not use Lean Six Sigma tools to improve the Lean Six Sigma deployment?" Of course, that is what we will do.

Since the Lean Six Sigma deployment team has been judiciously measuring both the in-process and end-of-process results for the deployment processes, it has gained significant insight into the efficiency and effectiveness of each process. Because of this insight, the team now has the ability to adjust its planning process based on new knowledge gained, fix any low-hanging fruit, or, if necessary, establish projects to improve the efficiency and effectiveness previously mentioned.

Consider the deployment plan for the moment. The plan is just that, a plan. A plan is an estimate of how work will proceed and is based on assumptions known at the time. Plans require adjustments as knowledge is gained.

Occasionally, it is necessary to dissect the planning time frame into smaller time elements or phases to recognize or even celebrate significant events or accomplishments and to demonstrate progress. In situations like this, *multiphase planning* is beneficial. Recall Figure 3.5b. This figure depicts an example of the phases of a maturity model that began with the launch of a deployment. If the deployment was planned without recognizing the five phases shown, the organization would simply be unaware of its progress along the transformational spectrum. Multiphase planning also allows organizations to "rest" and "catch its breath" between phases. This is particularly important if an organization is undergoing significant change, regardless of the type or source. Organizations have a limited amount of appetite for change. Too much and they become gorged. Then nothing moves. It happens. Recognize it. And plan for it.

Multiphase planning is also an effective tool when dealing with large "super projects," sometimes known as *megaprojects*. In general, these projects are so large that they can't be managed effectively. Therefore, they are decomposed into smaller, more manageable projects that can be completed in shorter times with fewer resources. When this occurs, it is common for the smaller projects to develop predecessor–successor relationships to each other. This concept is illustrated in the activity network diagram (AND) shown in Figure 3.7. Though the projects appear to be neatly aligned in time buckets, this is for the convenience of having just five phases and for illustrative purposes only. If resources are still constrained, additional phases may be added. However, recognize that this will extend the overall completion time of the megaproject.

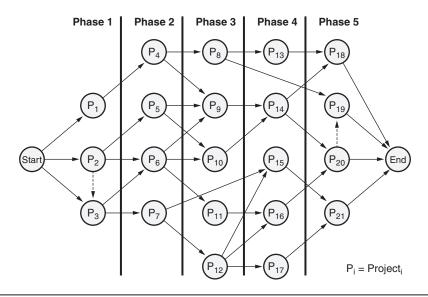


Figure 3.7 Decomposing a megaproject into multiple phases.

Chapter 4 Six Sigma Methodologies

In this chapter, we will address an expanded view of DMAIC, DFSS (including its component methodologies), and Lean; how these key methodologies relate to one another; and the emerging concept of business process management. In addition, we'll observe how the tools align with each phase of *define-measure-analyze-improve-control* (DMAIC).

Many times, the discussion of Six Sigma, DFSS, Lean, or even project management with senior management results in stares and a certain level of disinterest. Often this is because they do not understand how these methodologies are separated or used. Consider this: how many times within your organization have all projects been considered Lean Six Sigma projects regardless of which methodology is appropriate?

Figure 4.1 illustrates how each methodology is chosen. Notice that project management has been included. It has been included, in particular, because many organizations have both project management and Lean Six Sigma organizations that seem to be in perpetual conflict with one another. Keep this figure in mind as you read the remainder of this chapter.

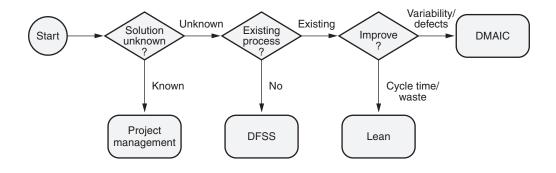


Figure 4.1 Selecting the proper project methodology.

DMAIC

Demonstrate an advanced understanding of the following methodology, including their associated tools and techniques: DMAIC. (Apply)

Body of Knowledge I.D.1

As all Lean Six Sigma professionals know, DMAIC is the de facto methodology for improving processes using Lean Six Sigma. The reason for this is quite simple. The methodology is:

- Easy to understand
- Logical
- Complete

Further, it is sufficiently general to encompass virtually all improvement situations whether they are business or personal in nature.

When belts first learn about DMAIC, they, for the most part, see it depicted in a linear manner as shown in Figure 4.2. Unfortunately, this is an illusion that leads inexperienced belts to plow ahead from phase to phase, steamrolling weak champions/sponsors, particularly if the belts lack the support of a Master Black Belt coach. When this occurs, projects either fail or results are not sustained.

Figure 4.3 depicts DMAIC in the real world. As we all know, tollgates occur after each phase. Diligent application of the proper tools and, above all, significant preparation are required to pass them. Passing them is not a given. Notice that up to and including the *analyze* phase, the failure of a tollgate could set the project back one or two phases. This usually occurs when knowledge is gained in a phase contrary to expectations. In this case, the project team may be forced to retreat all the way back to *define* in order to restate the problem. Beyond the analyze phase, a failed tollgate usually results in correcting or performing additional work within the phase. However, at any phase, a project may be terminated. Generally, this will happen when projects are no longer synchronized with strategy, champions/ sponsors have lost interest or transferred, or the potential savings are less than expected.

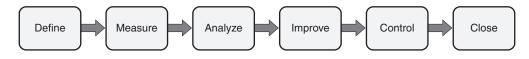
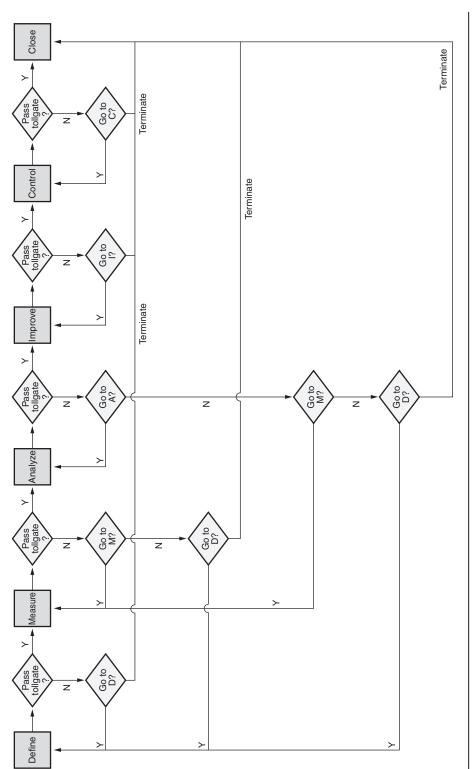


Figure 4.2 DMAIC in a dream world.



Chapter 4: Six Sigma Methodologies

EXAMPLE 4.1

I have just received my annual performance appraisal, which was based on a 360-degree review process. I am not happy with the result since on a scale of 1–5 (5 being high), I averaged a 2. However, rather than complain, I decide to take the results to heart and do something about them. After careful consideration, I plan to use the DMAIC approach as a means of driving personal improvement:

- *Define*. I am unhappy with my personal performance and want to move my average performance from a 2 to a 3 by the next annual performance appraisal.
- *Measure*. The 360-degree performance appraisal is very detailed and allows me to decompose my average result into various components that I can associate with particular behaviors. Each of these component parts has, in turn, an average result determined for them as well.
- Analyze. Since I am able to analyze the results of each of the component parts, I create a Pareto chart to determine which components have the greatest influence on my overall average score. Consequently, I am able to determine the top five behaviors. This is an important consideration since I must balance my actual productive work time against my personal improvement time and work within the time allotted to my annual training plan.
- *Improve*. With the top five drivers of behavior identified, I set about to develop and execute a plan. In this case, my plan consisted primarily of taking company-sponsored training and education courses.
- Control. As I take the courses, I diligently practice what I have learned, and decide that it might be beneficial to keep a journal that details any particular interactions that might influence my performance ratings. This allows me to keep what I have learned in the foreground and to practice it continuously. In addition, I decide to use intermediate 360-degree performance appraisals that "don't count" as I am the only one who will see the feedback. As long as the feedback on the top five that I am focusing on is improving, I am on track. If not, I will need to revisit my plan and make appropriate changes. Furthermore, I am concerned that behaviors that were "in control" have not moved in the wrong direction. If any have, they will be included in the plan moving forward.

Although this is a very simplified example, it demonstrates how useful and easily applied DMAIC is, even in common or routine situations.

Tools of the Trade

Tables 4.1 through 4.5 have been compiled to reflect the current thinking across contemporary authors about which tools should be applied in which phases of DMAIC. It should be readily apparent that these tables extend well beyond the alignment of tools by phase outlined in the ASQ bodies of knowledge for Green Belt, Black Belt, and Master Black Belt. However, all of the tools are covered in the corresponding three handbooks: *The Certified Six Sigma Green Belt Handbook, The Certified Six Sigma Black Belt Handbook*, and *The Certified Six Sigma Master Black Belt Handbook*. As the reader and a practitioner of Lean Six Sigma, you may or may

| 5 whys | Data collection plan | Prioritization matrix |
|--------------------------------|-------------------------------|--------------------------|
| Activity network diagrams | Failure mode and effects | Process decision program |
| Advanced quality planning | analysis (FMEA) | chart |
| Affinity diagrams | Flowchart/process mapping | Project charter |
| Auditing | (as is) | Project management |
| Benchmarking | Focus groups | Project scope |
| Brainstorming | Force field analysis | Project tracking |
| 0 | Gantt chart | Quality function |
| Cause-and-effect diagrams | Interrelationship digraphs | deployment (QFD) |
| Check sheets | Kano model | Run charts |
| Communication plan | Matrix diagrams | Sampling |
| Control charts | Meeting minutes | Stakeholder analysis |
| Critical-to-quality (CTQ) tree | Multi-voting | Supplier-input-process- |
| Customer feedback | Nominal group technique | output-customer (SIPOC) |
| Customer identification | Pareto charts | Tollgate review |
| Customer interviews | Project evaluation and review | Tree diagrams |
| Data collection | technique (PERT) | Y=f(X) |

 Table 4.1
 Common tools used in *define* phase.

| Tuble 1.2 Common tools used in medsure phase. | | | |
|-----------------------------------------------|-----------------------------|----------------------------------------------------|--|
| Basic statistics | Histograms | Project tracking | |
| Brainstorming | Hypothesis testing | Regression | |
| Cause-and-effect diagrams | Measurement systems | Run charts | |
| Check sheets | analysis (MSA) | Scatter diagrams | |
| Circle diagrams | Meeting minutes | Spaghetti diagrams | |
| Correlation | Operational definitions | Statistical process control | |
| Data collection | Pareto charts | (SPC) | |
| Data collection plan | Probability | Supplier–input–process– output–customer (SIPOC) | |
| Failure mode and effects | Process capability analysis | | |
| analysis (FMEA) | Process flow metrics | Taguchi loss function | |
| Flowcharts | Process maps | Tollgate review | |
| Gage R&R | Process sigma | Value stream maps | |
| Graphical methods | Project management | | |

Table 4.2Common tools used in *measure* phase.

| Affinity diagrams | Histograms | Qualitative analysis | |
|---------------------------------|-----------------------------|-------------------------|--|
| ANOVA | Hypothesis testing | Regression | |
| Basic statistics | Interrelationship digraphs | Reliability modeling | |
| Brainstorming | Linear programming | Root cause analysis | |
| Cause-and-effect diagrams | Linear regression | Run charts | |
| Components of variation | Logistic regression | Scatter diagrams | |
| Design of experiments (DOE) | Meeting minutes | Shop audits | |
| Exponentially weighted | Multi-vari studies | Simulation | |
| moving average charts | Multiple regression | Supplier–input–process– | |
| <i>Failure mode and effects</i> | Multivariate tools | output–customer (SIPOC) | |
| analysis (FMEA) | Nonparametric tests | Tollgate review | |
| Force field analysis | Preventive maintenance | Tree diagrams | |
| Gap analysis | | Waste analysis | |
| General linear models (GLMs) | Process capability analysis | 5 | |
| Geometric dimension and | Project management | Y=f(X) | |
| tolerancing (GD&T) | Project tracking | | |

| Table 4.3 | Common | tools | used i | in <i>anal</i> v | <i>ze</i> phase. |
|-----------|-----------|-------|--------|------------------|------------------|
| rabie no | 001111011 | | | and they | ac prideet |

Table 4.4Common tools used in *improve* phase.

| Activity network diagrams | Measurement systems | Project management |
|------------------------------|-------------------------------|--------------------------|
| Analysis of variance (ANOVA) | analysis (MSA) | Project tracking |
| Brainstorming | Meeting minutes | Reliability analysis |
| Control charts | Mixture experiments | Response surface |
| D-optimal designs | Multi-vari studies | methodology (RSM) |
| Design of experiments (DOE) | Multi-voting | Risk analysis |
| Evolutionary operations | Nominal group technique | Simulation |
| (EVOP) | Pareto charts | Statistical tolerancing |
| Failure mode and effects | Project evaluation and review | Taguchi designs |
| analysis (FMEA) | technique (PERT) | Taguchi robustness |
| Fault tree analysis (FTA) | Pilot | concepts |
| Flowchart/process mapping | Prioritization matrix | Tollgate review |
| (to be) | Process capability analysis | Value stream maps |
| Gantt charts | Process sigma | Work breakdown structure |
| Histograms | Project management | Y=f(X) |
| Hypothesis testing | , , | |

| | in control phase. | |
|----------------------|------------------------------------------------|-----------------------------|
| 5S | Lessons learned | Standard operating |
| Basic statistics | Measurement systems | procedures (SOPs) |
| Communication plan | analysis (MSA) | Standard work |
| Continuing process | Meeting minutes | Statistical process control |
| measurements | Mistake-proofing/ | (SPC) |
| Control charts | poka-yoke Pre-control Project management | Tollgate review |
| Control plan | | Total productive |
| Data collection | | maintenance |
| | Project tracking | Training plan deployment |
| Data collection plan | Run charts | Visual factory |
| Kaizen | Six Sigma storyboard | Work instructions |
| Kanban | | |

| Table 4.5 | Common | tools | used | in control | phase. |
|-----------|--------|-------|------|------------|---------|
| iubic 1.5 | common | 10015 | uscu | micontrot | priuse. |

not agree with what the noted authors in the field of Lean Six Sigma suggest for how the tools should be aligned, based on your personal experiences. Nevertheless, a review of these tables indicates that certain tools have extensive use across the phases. For example, failure mode and effects analysis (FMEA) appears in four phases, thus indicating its importance to the DMAIC methodology and perhaps even the degree of emphasis that should be placed on it during training. For the reader's convenience, tools with applications in multiple phases have been italicized. Another key aspect of these tables should be noted. It is obvious that many of the tools in these tables can be seen to fall under the general classification of other tools listed in the same table. However, they are listed separately to allow visibility into the emphasis the various authors have placed on these tools. For example, statistical process control (SPC) is identified as a general classification of tools while control charts is identified as a basic set of tools. Recall that the control chart is considered one of the seven basic quality tools while SPC encompasses much broader concepts.

DFSS

Demonstrate an advanced understanding of the following methodology, including their associated tools and techniques: DFSS. (Apply)

Body of Knowledge I.D.2

While DMAIC may be traditionally viewed as the foundation for Lean Six Sigma, its application is primarily limited to improving existing processes; it does little to

address the design of new products or processes. This concept is illustrated in the flowchart given in Figure 4.1.

Fortunately, several additional structured methodologies exist. Several of the common methodologies are depicted in Figure 4.4. Each of these methodologies has its usefulness as long as the nuances of each are fully understood.

Table 4.6 has been created to provide a loose comparison between the DMAIC and DFSS methodologies. These comparisons are not exact, but do provide a sense of how the methodologies align. This table may be beneficial for readers who may be required to use multiple methodologies or find themselves in need of selecting one suitable for a particular project.

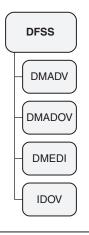


Figure 4.4 The DFSS family of methodologies.

| Table 4.6 Comparing DMAIC and DFSS methodologies. | | | | |
|-----------------------------------------------------------|----------|----------|-----------|-------------|
| | DFSS | | | |
| DMAIC | DMADV | DMADOV | DMEDI | IDOV |
| Define | Define | Define | Define | Identify |
| Measure | Measure | Measure | Measure | |
| Analyze | Analyze | Analyze | Explore* | Design |
| Improve | Design | Design | Develop | |
| | | Optimize | | Optimize |
| | Verify** | Verify** | Implement | Validate*** |
| Control | | | | |

*Loosely aligned

** Often confused with "Validate"

*** Often confused with "Verify"

Table 4.7 identifies some of the commonly used tools in DFSS, including the DFSS methodologies shown in Figure 4.4. However, this table also lists other tools that are useful regardless of the DFSS methodology chosen. Examples include axiomatic design and TRIZ.

DMADV

DMADV is a well-recognized Design for Six Sigma (DFSS) methodology and an acronym for *define, measure, analyze, design,* and *verify*. Note that the ASQ Black Belt Body of Knowledge replaces "verify" with "validate." The difference is likely because, although different, "verify" and "validate" are often used synonymously.

The DMA portion of DMADV has been covered in *The Certified Six Sigma Black Belt Handbook,* so we'll concentrate on the remaining DV portion:

- *Design.* Quite simply, this means carrying out the process of designing a product or process. Many organizations have well-established policies and procedures for their respective design processes. One valuable Lean Six Sigma technique that supports the design process is quality function deployment (QFD), the details of which were discussed in Chapter 15. Additional tools useful in this phase include pilot runs, simulations, prototypes, and models (addressed in Chapter 33).
- *Verify.* This phase is directed at ensuring that the design output meets the design requirements and specifications and is performed on the final product or process. Basically, *verification* speaks to the design meeting customer requirements and ensures that the design yields the correct product or process. By contrast, *validation* speaks to the effectiveness of the design process itself and is intended to ensure that it is capable of meeting the requirements of the final product or process.

Both verification and validation are necessary functions in any design process. As such, this suggests that DMADV might be more appropriately named DMADVV.

| Table 4.7 Common tools used in D155. | | | |
|--------------------------------------|----------------------------------------------------------------------------|--|--|
| FMEA | Quality function | | |
| IDOV | deployment (QFD) | | |
| Porter's five forces | Robust product design | | |
| analysis | Statistical tolerancing | | |
| Portfolio architecting | Systematic design | | |
| Process simulation | Tolerance design | | |
| Pugh matrix | TRIZ | | |
| | | | |
| | FMEA IDOV Porter's five forces analysis Portfolio architecting | | |

| Table 4.7 Common tools used in DFS |
|--------------------------------------------|
|--------------------------------------------|

DMADOV

The basic difference between DMADV and DMADOV is that "O" for "optimize" has been added. This sounds like a trivial observation, but many organizations' design processes do not include this refinement action. Oftentimes they produce only a minimally workable product or process. DMADOV forces attention on the need to optimize the design. Additional tools useful in this phase include *design of experiments* (DOE), *response surface methodology* (RSM), and *evolution-ary operations* (EVOP). These methods help the design team establish and refine design parameters.

DMEDI

The *define–measure–explore–develop–implement* (DMEDI) methodology is appropriate where the limitations of DMAIC have been reached but customer requirements or expectations have not been met, or a quantum leap in performance is required. Unlike DMAIC, DMEDI is not suitable for kaizen events and it is generally more resource-intensive, with projects taking considerably longer.

Let's explore each phase of DMEDI:

- Define. This phase is the same as in DMAIC.
- Measure. Unlike measure in DMAIC, this phase must emphasize the voice of the customer and the gathering of critical customer requirements since no baseline data are available. The quality function deployment (QFD) tool is valuable during this phase.
- *Explore.* This phase focuses on creating a high-level conceptual design of a new process that meets the critical customer requirements identified in the measure phase. Ideally, multiple designs are developed that provide options to be considered.
- *Develop.* The options developed in the explore phase are considered, and the "optimal" one is chosen based on its ability to meet customer requirements. Detailed designs are developed.
- *Implement*. During this phase, the new process may be simulated to verify its ability to meet customer requirements. Also, a pilot is conducted, controls are established and put in place, and the new process is transferred back to the process owner.

IDOV

Unlike most of our other DFSS methodologies, IDOV contains only four phases and stands for *identify-design-optimize-validate*. Woodford ("Design for Six Sigma—IDOV Methodology") states that IDOV phases parallel the original MAIC phases. However, a review of his descriptions does not confirm this. Table 4.6 has been modified to conform to the descriptions of each phase he provides.

Let's explore each phase of IDOV:

- *Identify*. This phase links the design to the voice of the customer via the customer, product, and technical requirements, establishes the business case, identifies critical-to-quality (CTQ) variables, and conducts a competitive analysis. Quality function deployment will be a particularly useful tool during this phase.
- *Design.* This phase focuses on identifying functional requirements, deploying CTQs, developing multiple design concepts, and selecting the best-fit concept.
- *Optimize.* This phase focuses on developing a detailed design for the best-fit design chosen in the previous phase, predicting design performance, and optimizing the design. Additional work in this phase includes error-proofing, statistical tolerancing, prototype development, and optimization of cost.
- *Validate (verify).* During this phase, the prototype is tested and validated, and reliability, risks, and performance are assessed.

Munro et al. (2008) describe each phase of IDOV in more detail on pages 42–43 of *The Certified Six Sigma Green Belt Handbook*.

LEAN

Demonstrate an advanced understanding of the following methodology, including their associated tools and techniques: Lean. (Apply)

Body of Knowledge I.D.3

As with Six Sigma, the tools of Lean are not new, only rediscovered. Table 4.8 lists many of the common tools used by Lean practitioners. Most of these tools are covered in the three ASQ handbooks previously mentioned.

While Six Sigma focuses on reducing process variation and enhancing process control, Lean—also known as lean manufacturing—drives out waste (nonvalue-added) and promotes work standardization and flow. These methodologies are complementary, not contradictory as some organizations and practitioners of each believe and defend.

In recent years, the demarcation between Six Sigma and Lean has blurred. We are hearing about terms such as "Lean Six Sigma" with greater frequency because process improvement requires aspects of both approaches to attain positive results. It is for this reason that Six Sigma practitioners need to be well versed in the tools of Lean.

Many successful implementations have begun with the lean approach, making the workplace as efficient and effective as possible, reducing the (now) eight

| Table 4.8 Common tools used in Lean. | | | | |
|----------------------------------------|-----------------------------------------|-------------------------------------------|--|--|
| 5S | Line balancing | Spaghetti diagram/chart | | |
| Autonomation | Linear programming | Standard work | | |
| Batch/lot size reduction | Load leveling/level | Standardization | | |
| Brainstorming | scheduling | Supermarkets | | |
| Buffer stock/inventory | Non-value-added activities analysis | Takt time | | |
| Bullwhip effect | Overall equipment | Theory of constraints | | |
| Changeover time | 1 1 | | | |
| Continuous flow | Poka-yoke | Total productive | | |
| manufacturing (CFM) | Process efficiency | maintenance (TPM) | | |
| Cycle time | Product family | Transport time | | |
| Demand analysis/ management | Pull systems | Value stream mapping (VSM) | | |
| Facility layout | Queue time | Visual control/ | | |
| First in, first out (FIFO) | Runner | management | | |
| Just-in-time | Safety stock/inventory | Waste/waste reduction/ 7 or 8 wastes | | |
| Kaizen | Simplification | | | |
| Kanban | Single-minute exchange of die (SMED) | Work cell design Work-in-process (WIP) | | |
| Lead time | Single-piece flow | inventory | | |

| Table 4.8 Common tools used in Lean |
|---------------------------------------------|
|---------------------------------------------|

wastes, and using value stream maps to improve understanding and throughput. When process problems remain, the more technical Six Sigma statistical tools may be applied. Furthermore, both methodologies require strong management support to make them the standard way of doing business.

Some organizations have responded to this dichotomy of approaches by forming a Lean Six Sigma problem-solving team with specialists in the various aspects of each discipline but with each member cognizant of others' fields. Task forces from this team are formed and reshaped depending on the problem at hand.

Unfortunately, other organizations have decided to adopt one approach to the exclusion of the other. Perhaps this is due to a strong influence by misinformed senior management and/or possibly both weak skills and leadership on the part of the deployment leader. Regardless, Master Black Belts must be evangelists and passionate about Lean Six Sigma. They must be able to carry forth the word and communicate the symbiosis that exists between Lean and Six Sigma, or any other methodology for that matter, and how those relationships can be exploited to the benefit of the organization. They will be viewed as the expert's expert. They must be out front on the firing line, ready to convert the naysayers. See Kubiak (2004) *What Does It Take to Be a Master Black Belt?*

BUSINESS SYSTEMS AND PROCESS MANAGEMENT

Demonstrate an advanced understanding of the following methodology, including their associated tools and techniques: Business Systems and Process Management. (Apply)

Body of Knowledge I.D.4

Kubiak and Benbow (2009) state that, "A business system is designed to implement a process or, more commonly, a set of processes. Business systems make certain that process inputs are in the right place at the right time so that each step of the process has the resources it needs. Perhaps most importantly, a business system must have as its goal the continual improvement of its processes, products, and services. To this end, the business system is responsible for collecting and analyzing data from the process and other sources that will help in the continual incremental improvement of process outputs." Additional details regarding business systems can be found in Chapter 1 of *The Certified Six Sigma Black Belt Handbook*.

Munro et al. (2008) defines *process management* as "the pertinent techniques and tools applied to a process to implement and improve process effectiveness, hold the gains, and ensure process integrity in fulfilling customer requirements."

When the notions of "business systems" and "process management" are combined, the emerging concept is that of *business process management* (BPM). Business process management has been given many definitions since it surfaced in the early 2000s:

- Some describe it simply as another name for process management.
- vom Brocke and Rosemann (2010) define BPM as a "... holistic management approach that promotes business effectiveness and efficiency while striving for innovation, flexibility, and integration with technology. Business process management attempts to improve processes continuously."
- Zairi and Sinclair (1995) define BPM as "a structured approach to analyze and continually improve fundamental activities such as manufacturing, marketing, communications, and other major elements of a company's operations."
- Elzinga et al. (1995) calls BPM "a systematic, structured approach to analyze, improve, control, and manage processes with the aim of improving the quality of products and services."
- The BPM Enterprise website states, "The phrase *business process management* (BPM) refers to a set of activities an organization implements to optimize its processes. Business process management also encompasses software tools designed to assist firms in achieving process optimization."

While all of the above definitions tend to differ somewhat, we can extract the following key elements:

- Process-oriented
- Systematic or structured
- Focused on analyzing, improving, and controlling processes (notice the strong linkage to DMAIC)
- Software/technology-based
- Organization-wide
- Customer-focused
- Continuous improvement

As there are many definitions for BPM, so there are many life cycle models. Figure 4.5 illustrates three specific models commonly referred to in the literature, while Table 4.9 provides the corresponding explanations for each phase of the life cycles. In addition, a relevant portion of the Association of Business Process Management Professionals BPM body of knowledge has been included for comparison purposes.

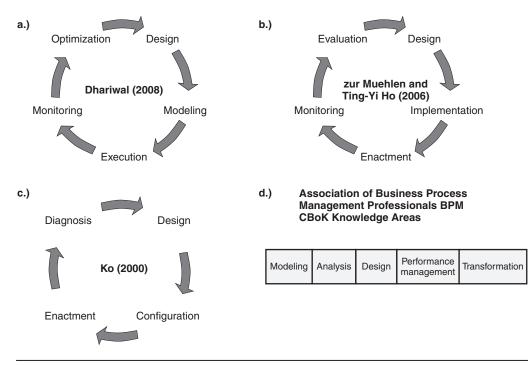


Figure 4.5 Business process management life cycles.

| Credited to | Phase | Explanation | | |
|--------------------------------------|----------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|--|
| Dhariwal (2008) | Design | Identify existing processes; identify the design of "to be" processes; focus on process flow, SOPs, service level agreements, hand-over mechanisms, escalations, and alerts and notifications. | | |
| | Modeling | Takes the theoretical design, introduces differing combinations of variables, and conducts "what if analysis" on the processes. | | |
| | Execution | Automate processes either through development or acquisition. Complex processes might require human intervention. | | |
| | Monitoring | Tracks both the state and the performance of the processes. | | |
| | Optimizing | Input from either or both the modeling and monitoring phases is used as feedback to improve the design. | | |
| zur Muehlen and Ting-Yi Ho (2006) | Design | Identify the processes to be analyzed, designed, redesigned, and/or automated; influencing variables are identified; processes are specified and mapped formally using modeling techniques. | | |
| | Implementation | Processes are transferred into the operational environments. These processes can be either manual or automated. | | |
| | Enactment | Processes are executed in real time. | | |
| | Monitoring | Processes are observed in real time. Measures are taken. | | |
| | Evaluation | Feedback obtained from the monitoring phase can be used to drive improvement back into the design of the processes. | | |
| Ko (2000) | Design | Processes are modeled electronically. | | |
| | Configuration | The underlying system infrastructure of the BPM systems is configured to the roles of the individuals involved in the processes and the associated organizational charts. | | |
| | Enactment | The processes that were modeled electronically are deployed or made operational. | | |
| | Diagnosis | With appropriate analysis tools, the processes are analyzed, bottlenecks removed, and otherwise improved. | | |

Table 4.9BPM life cycle definitions.

A brief search of *business process management* on the Internet will yield numerous results and several undeniable conclusions:

- Many more definitions than provided here are proffered.
- Many more life cycle models than provided here are proffered.
- Several organizations are working independently to promote BPM and offer bodies of knowledge. A few offer professional certifications.
- Technology solutions abound.
- BPM concepts have not unified over the past decade.

Ko (2009) notes that, "To my best knowledge, there is no agreed academic or industrial classification or taxonomy of the different types of business processes. From a higher-level viewpoint, there are two main perspectives of business processes: the level perspective and the core competency perspective."

Further, he defines three levels of management activities for the level perspective:

- *Strategic planning*. This level is considered the process by which the objectives of the organization are determined, changed, and resourced. It also includes the policies that govern, use, and disposition resources.
- *Management control.* This level is considered the process by which managers assure that resources are used both efficiently and effectively to achieve the organization's goals.
- *Operational control.* This level is considered the process by which tasks are executed both efficiently and effectively.

He goes on to provide the levels of the core competency perspective:

- *Core business processes.* These constitute revenue-generating processes. Examples include production and service departments.
- *Management business processes.* These include processes that address efficiency, governance, and corporate compliance. Examples include requests and notifications.
- *Support business processes.* These constitute non-revenue-generating processes. Examples include human resources and information technology departments.

Harmon (2004) offers a means of evaluating an organization's business process maturity based loosely on the capability maturity model (CMM) developed by the Software Engineering Institute (SEI). Recall, the CMM defines five progressive levels: initial, repeatable, defined, managed, and optimizing. Further, Harmon provides sufficiently detailed descriptions at each level to allow an organization to independently evaluate its status.

Figure 4.6 illustrates how business processes cross many functional departments. Notice the intersections between the business processes and functional

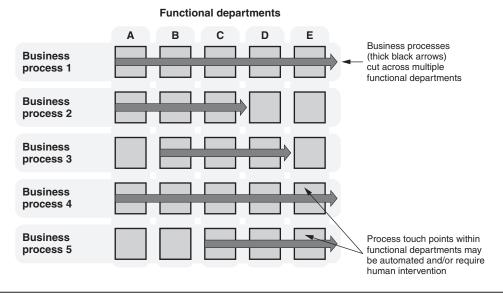


Figure 4.6 Integrating Lean Six Sigma and BPM.

departments. This is where the proverbial rubber meets the road. In our case, Lean Six Sigma meets BPM. The processes at these intersections may be automated or manual, cross-functional or intradepartmental. Regardless of the process type, Lean Six Sigma has a role. Some might argue a responsibility to step in and improve the processes because allowing ill-defined or otherwise poorperforming processes to be automated would be disastrous, as many organizations have learned.

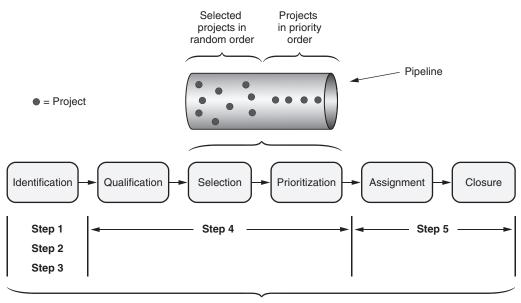
Chapter 5

Opportunities for Improvement

The concepts of project identification, selection, prioritization, and so on, have been sprinkled throughout this body of knowledge. Rather than revisit them each time, definitions will be provided once in this subsection and referenced when necessary. Also, this subsection will contain all the appropriate figures and tables to provide continuity of presentation and to enhance the readability of the material.

Figure 5.1 identifies the processes associated with a project from its creation to its completion. Six key important processes have been identified:

• *Project identification*. Project identification means the delineation of clear and specific, but potential, improvement opportunities vital to the organization. Project identification will be discussed later in this chapter.



Greiner's five-step process

Figure 5.1 Managing the project flow from creation to completion.

- *Project qualification.* Project qualification means the translation of an identified improvement opportunity into Lean Six Sigma charter format for recognizing its completeness and the ability to assess its potential as a viable project. Project qualification will be discussed later in this chapter. (Note: Some organizations tend to combine the qualification and identification processes.)
- *Project selection*. Project selection means a qualified project has met some organizational threshold and is considered suitable for execution later. In short, it has been approved. Project selection is discussed in Chapter 6.
- *Project prioritization*. Project prioritization means that a selected project will be independently assessed against specific criteria in order to establish a rank score. Rank scores of other projects will be compared and a final ranking of projects will be established. This final ranking will set the precedence and order for project assignment. Project prioritization is discussed in Chapter 6. (Note: Some organizations tend to combine the selection and prioritization processes.)
- *Project assignment.* Project assignment means that a prioritized project and a resource (that is, Black Belt or Black Belt candidate) are available to be matched. If the matching is successful, the resource is assigned to the project and the project gets under way. Project assignment is discussed in Chapter 21.
- *Project closure*. Project closure means that a project has been terminated for one of two reasons: successful completion or no longer viable. Project closure is discussed in Chapter 14.

It is important to note that some organizations will combine some of the above processes. However, for the purpose of this discussion, all processes will be addressed separately to ensure no aspect of a process is lost during combination.

PROJECT IDENTIFICATION

Facilitate working sessions to identify new project opportunities that can be prioritized. (Apply)

Body of Knowledge I.E.1

Numerous sources of information exist that provide ideas for identifying potential Lean Six Sigma projects. Perhaps Figure 5.2 will serve as a memory jogger. In particular, notice that "action plans" has been identified as an input. Action plans are discussed at length in Chapter 12 where their relationship to the strategic plan is highlighted. This linkage is important because it ensures that the Lean Six Sigma

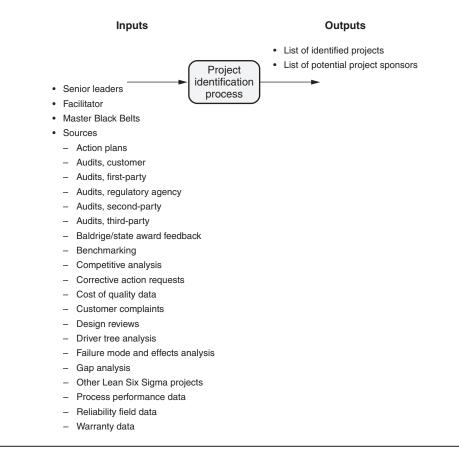


Figure 5.2 The project identification process.

projects are aligned to support the organization's goals and objectives through its strategies.

Also notice in Figure 5.2 that the inputs rely heavily on data and analysis or observable findings. Additionally, once the initiative has been up and running, many organizations find that other Lean Six Sigma projects contribute input as well.

Furthermore, Figure 5.2 identifies several key participants. These include senior leaders, a facilitator, and Master Black Belts. Master Black Belts have the technical expertise and experience that allows them to conceptualize and transform a project concept into a well-written charter for a Lean Six Sigma project. The facilitator may or may not be required depending on the skill level of the Master Black Belts needed to perform this function. Regardless of who performs this function, this role must be played when dealing with senior leaders. Interestingly, senior leaders have been identified as a key player in the project identification process. Involving them early in this process is crucial and allows them to demonstrate their commitment, engagement, and visibility to the organization. Further, if conducted as a project identification workshop, it provides a valuable and accelerated education and learning experience for them.

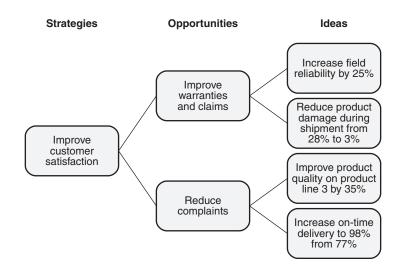


Figure 5.3 Example of a value driver analysis.

As noted previously, the role of the facilitator is critical. This individual must be tactful and diplomatic, yet forceful. Harnessing and channeling the energy of the senior leaders will be difficult, but essential for success. Techniques such as brainstorming, six thinking hats, and nominal group technique are useful for generating ideas. Additional tools and techniques are discussed later in this chapter.

The output of this process will be a succinct list of identified projects and an associated or initial list of potential project sponsors. Project sponsors are foundational to the concept of Lean Six Sigma projects. No project should exist without a sponsor.

One type of analysis some organizations have found useful for generating project ideas is *value driver analysis*. This type of analysis makes use of the tree diagram and links the strategic plans to opportunities to project ideas. Figure 5.3 illustrates this concept. In this figure, we start with the end in mind and work from right to left. Notice that this approach has the familiarity of a cause-and-effect diagram where the effect is the strategy and the cause is the idea.

PROJECT QUALIFICATION

Determine the elements of a well-defined project (i.e., business case), the process for approving these projects, and tools used in project definition (QFD, process maps, value stream maps, FMEA, CTx (critical to ... customer, ... design, ... quality), etc. (Apply)

Body of Knowledge I.E.2

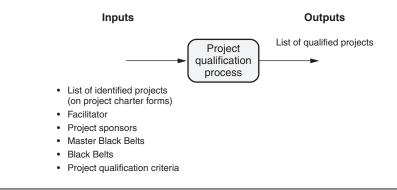


Figure 5.4 The project qualification process.

Once project opportunities have been identified, they must be qualified. This means translating them from a concept or idea into a well-written project charter. Figure 5.4 illustrates the inputs and outputs for the qualification process at a high level.

While the translation of an identified project into a project charter may seem like a trivial activity done simply for project qualification purposes, this activity is quite worthwhile. It normalizes the playing field by ensuring that all projects under consideration by the evaluation team have, at least, the same minimal information (that is, as defined by the project charter) presented in the same format.

Successful input to a project qualification workshop includes: the list of identified projects in the form of well-written charters, project sponsors, Master Black Belts, Black Belts, facilitator, and any other criteria the organization might want to stipulate. Such criteria might include charter limits (for example, scope, duration, type of savings), the problem must be systematic (that is, driven by common cause), and so on.

The Project Charter

The project charter, though a simple document, requires significant preparation time. There are typically six key elements of a meaningful charter document:

- *Business case.* This identifies the dollars to be saved and establishes how the project aligns to the organization's strategies.
- *Problem statement.* This identifies what is working, what is not working, or where the pain point lies. This critical statement is often glossed over.
- *Goal statement.* This identifies the project's objectives and targets and how the success of the project will be measured. More than one measure (that is, primary measure) may be used. Baseline performance must be established. Frequently, primary measures are accompanied by secondary or countermeasures—measures to ensure that when primary measures are improved, other measures (that is, secondary measures) do not suffer in the process. Remember, goals

should be SMART (that is, *specific, measurable, achievable, relevant,* and *timely*).

- *Project scope.* This specifies the boundaries of the project. Examples of common boundaries include:
 - *Process.* Processes receive input from other processes and feed output to other processes. Tightly bound the process or subprocess associated with the project.
 - *Demographics*. This would consider factors such as employee categorizations, gender, age, education, job level, and so on.
 - *Relationships.* This would include such entities as suppliers, customers, contract personnel, and so on.
 - *Organizational*. Which business units, divisions, sites, or departments are included?
 - Systems. Which manual or computerized systems are included?
 - *Geographical.* Which country or site is included? Note: In this example, a site was used in the context of both an organizational and geographical scope.
 - Customer. Which segmentation or category will be considered?
 - Combinations. Any combination of the aforementioned.

In addition to defining what is in scope, it is often useful to define what is out of scope. Although one would seem to define the other, experience has shown that what is out of scope is frequently overlooked or not understood fully unless explicitly stated.

Unwieldy scopes are one of the most frequent reasons cited for the demise of projects. When the scope is too large to be completed within the project plan time frame, or additional resources are not available, there may be a reluctance to go back and re-scope the project. When it is too small, projected savings may be overstated. Remember, the project charter should be considered a living document and adjusted as knowledge occurs and learning takes place.

- *Project plan.* The greater the amount of detail that can be provided the better. However, an estimated completion date is required. Project durations must be determined so that, from a portfolio perspective, projects can be managed in terms of cost, schedule, and resources.
- *Project team.* The project team members should be identified along with their expected time contributions to the project. Project team members are knowledgeable and valuable resources who will likely be in demand for other projects as well. Knowing the total demand requirements placed on team members is essential to ensure they are not overloaded and that their "regular" jobs do not suffer from their contributions to projects.

Furthermore, it is prudent to have charters replete with the appropriate signatures of authorization, not the least of which are the sponsor and financial representatives. The inclusion of the financial approval lends credibility and support to the proposed financial gains. This aspect is so important that many organizations establish financial approval as a requirement for a project to be considered for qualification.

The project sponsor has the ultimate accountability for developing the project charter. He or she will likely seek counsel from a Master Black Belt or Black Belt. However, in the end, senior leaders will look to the project sponsor for the success or failure of the project. It is for this reason that they need to be a major contributor to the project qualification process.

Figure 5.5 illustrates an alternate form of a project charter. Note how this form emphasizes baseline and goal performance and provides the opportunity to link the project directly to customer satisfaction.

Project Charter

Project Authorization

| Organization: | Champion: | Process Owner: |
|----------------------------|---------------------|--------------------------|
| Project: | | Project #: |
| Project | | |
| Problem Statement: | | |
| | | |
| Project Objective: | | |
| | | |
| Estimated Defect Level: | Initial Goal: | Estimated Benefits: |
| Approval Date: | Champion Signature: | Process Owner Signature: |
| Estimated Completion Date: | Project Leader: | Financial Analyst: |

Project Team

| Name | Role | Comments | Phone |
|------|------|----------|-------|
| | | | |

Figure 5.5 Example of a project charter document.

Source: Courtesy of Minitab Quality Companion 3 software, Minitab Inc.

Project Definition and Scoping

Metrics (unit of measure):

Critical to Satisfaction (linkage to customer):

Defect Definition (include opportunity):

Scope of Project:

Goals and Benefits

| | Defe | ct Levels/Goals | : | | |
|-----------------|-----------------|-----------------|--------------|--------------|-----------------------------------------------------|
| | Date | DPMO(LT) | Zbench(ST) | Cpk | |
| Baseline | | 0 | 0.00 | 0.00 | |
| Goal | | 0 | 0.00 | 0.00 | |
| Stretch Goal | | 0 | 0.00 | 0.00 | |
| A Important inf | | \$0 | | | |
| Soft Savings | | \$0 | | | |
| Implementati | on Costs | \$0 | | | |
| Based on how | many months: | 12 | | | |
| | wed and revi | sed as needed | after the er | | eline DPMO, and Zbench asure phase when you have |
| Measure phase | completed o | n: | | | |
| Were goals | revised after | r completion of | Measure ph | ase? | |
| 🔲 Were financ | cial benefits r | evised after co | mpletion of | Measure ph | ase? |
| Approved by Fi | nance Repres | sentative: | Da | ate of Finan | ce Approval: |
| | | | | | |

Figure 5.5 *Continued.*

Project Qualification Tools

Chapter 4 addressed many of the common tools used in each phase of DMAIC. Table 4.1 specifically listed these tools. Though not complete, the list is sufficiently comprehensive and can be easily expanded. For example, the critical-to-quality (CTQ) tree tool shown in the table could be generalized as "CTx."

The "define" phase provides many tools that help shape and crystallize a Lean Six Sigma project. When these tools are properly applied, projects can be easily qualified. Those that are eventually selected, prioritized, and finally assigned will have a higher rate of success because the appropriate level of attention has been placed on defining the project.

STAKEHOLDER ENGAGEMENT

Describe how to engage stakeholders. (Apply)

Body of Knowledge I.E.3

Tague (2005) defines a *stakeholder* as "anyone with an interest or right in an issue, or anyone who can affect or be affected by an action or change. Stakeholders may be individuals, groups, internal or external to the organization." To help identify stakeholders related to a project, she recommends asking the following set of eight questions:

- Who might receive benefits?
- Who might experience negative effects?
- Who might have to change behavior?
- Who has goals that align with these goals?
- Who has goals that conflict with these goals?
- · Who has responsibility for action or decision?
- Who has resources or skills that are important to this issue?
- Who has expectations for this issue or action?

A form similar to the one depicted in Table 5.1 can be used to capture relevant information about the stakeholders. Let's review the form:

• *Stakeholder.* The list provided above is an excellent starting point for identifying stakeholders. There are two types of stakeholders: primary and secondary. *Primary stakeholders* are directly affected by the project. *Secondary stakeholders* are involved in implementing, funding, monitoring, and so on, and are considered intermediaries. Consider identifying each stakeholder as primary or secondary. An alternate

| Stakeholder | Project impact | Level of influence | Level of importance | Current attitude | Action plan |
|-------------|-------------------|-----------------------|------------------------|---------------------|-------------|
| | | | | | |
| | | | | | |
| | | | | | |

Table 5.1 A simple form for completing a stakeholder analysis.

stakeholder analysis form is shown in Figure 5.6. Notice that this form allows for the categorization of stakeholders.

- *Project impact.* Clarify the impact of the project on the stakeholder. This establishes the relationship between the stakeholder and the project.
- *Level of influence*. Rate the level of influence the stakeholder has on the project. Consider a scale from 1 to 5, or low, medium, and high. Low and high works well, too.
- *Level of importance.* Rate the level of importance each stakeholder has on the project. Consider a scale from 1 to 5, or low, medium, and high. Low and high works well, too.
- *Current attitude.* Rate the attitude the stakeholder has toward the project. Consider a scale from 1 to 5, or low, medium, and high. Low and high works well, too.
- *Action plan.* Outline different strategies for dealing with each stakeholder. The strategies should focus on reducing opposition and increasing support.

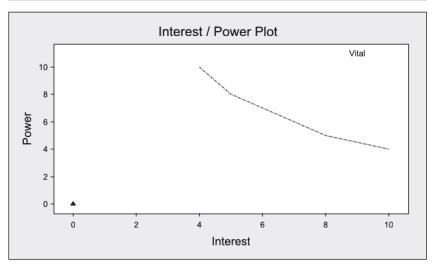
Once Table 5.1 has been completed, the rating from the level of influence and importance columns can be transferred to Table 5.2. Table 5.2 provides the added benefit of classifying stakeholders:

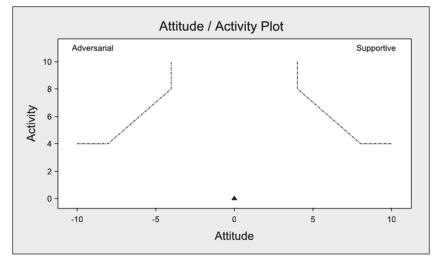
- *High influence/high importance.* Collaborate with these stakeholders.
- *High influence/low importance.* Work with these stakeholders to involve them and increase their level of interest. These stakeholders are capable of sabotaging plans and escalating problems and are considered high risk.
- *Low influence/high importance.* Protect and defend these stakeholders. Work to give them a voice and help increase their level of influence.
- *Low influence/low importance.* Monitor these stakeholders, but don't spend resources on them.

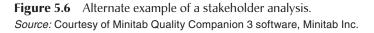
| Stakeholder Analysis | |
|------------------------------------------------------|------------------------|
| Prepared By: | Date: |
| | |
| Project: | |
| Project | |
| Description: | |
| Use to organize a quality improvement project into p | hases that you define. |

How to Use this Table

| Stakeholder Categories | Relevant Stakeholders | Code | Attitude | Activity | Attitude Rating | Power | Interest | Power Rating |
|---------------------------|--------------------------|------|----------|----------|--------------------|-------|----------|-----------------|
| | | | 0 🗸 | 0 🔻 | 0.00 | 0 🔻 | • 0 | 0.00 |







| | Influence | | | | | |
|------------|-----------|-------------|------------------------|--|--|--|
| | | High Low | | | | |
| Importance | High | Collaborate | Protect and defend | | | |
| | Low | High risk | Do not spend resources | | | |

| Table 5.2 | Example of an | influence-im | portance stak | eholder table. |
|------------|---------------|---------------|----------------|----------------|
| I UDIC J.L | LAumpic of un | minucinee min | Joi tunee Stuk | choluci tubit |

Source: Adapted from Tague (2005).

| Table 5.3 | Example of an impact-cooperation stakeholder tab | ole. |
|-----------|--------------------------------------------------|------|
| | | |

| | Potential to impact | | | | |
|------------------------------|---------------------|----------------|------------|--|--|
| | | High | Low | | |
| Potential for cooperation | High | Mixed blessing | Supportive | | |
| cooperation | Low | Nonsupportive | Marginal | | |

Source: Adapted from Andersen (2007).

Whenever possible, involve stakeholders with high influence; otherwise, neutralize their influence.

Andersen (2007) suggests another means of classifying stakeholders. His method provides additional insight and is depicted in Table 5.3. In this table, stakeholders are classified as to their potential impact on the organization versus their potential for cooperation with the organization. Again, this requires categorizing each stakeholder as either low or high. Once complete, stakeholders can be charted and viewed as:

- *High impact/high cooperation*. These stakeholders are considered a "mixed blessing" and should be handled through cooperation.
- *High impact/low cooperation.* These individuals are considered "nonsupportive." Minimize dependency on this type of stakeholder through a defensive strategy.
- *Low impact/high cooperation.* These stakeholders are considered "supportive" and should be involved in relevant discussions and decisions.
- Low impact/low cooperation. These stakeholders are considered "marginal" and should be monitored. Like Tague's low influence/low importance, don't spend resources on them.

Master Black Belts may find that a significant portion of their time is dedicated to engaging project stakeholders. As Covey (1989) would say in his fifth habit, "Seek first to understand, then to be understood." The stakeholder analysis is a highly

structured method dedicated to this purpose. However, if done properly, it can be a very sensitive document. If left open to public consumption within the organization, it can be damaging, hurtful, or perceived as offensive. Use this tool effectively, but use it with discretion.

INTERVENTION TECHNIQUES

Describe techniques for intervening across levels to prevent potential project failures. (Apply)

Body of Knowledge I.E.4

Thoughts of "intervention" may bring about images of impending doom, demise, and disaster. However, recognize that some elements of the concept of intervention are inherent within the Lean Six Sigma implementation structure. Consider the following:

- *Tollgates*. Tollgates are the bonds that hold the DMAIC framework together. They serve as meaningful checkpoints, facilitate communications, and ensure sponsor participation. Tollgates focus projects on moving forward.
- *Mentoring*. Mentoring provides the opportunity for the Master Black Belt and Black Belt to address the progress of the project, proper application of tools, and strategies for moving forward, and to make mid-course corrections as necessary.
- *Sponsors*. Sponsors provide the linkage to higher levels of the organization and, to some extent, provide visibility across the organization as well. As such, they are expected to remove roadblocks, anticipate potential problems, and be proactive with regard to their project.
- *Project teams.* Project teams are meeting and communicating on an ongoing basis, as discussed in Chapter 14. Open communication should exist such that team members make each other, including the team leader, aware of current and potential problems.
- *Project reports.* Team members provide the team leader with status reports as defined in the project plans (see Chapter 14). This formal communication provides ongoing awareness of current and potential project-related problems.

In addition to the above, many organizations instigate an integrated and routine set of reviews across all active projects by higher levels of management. If a large number of projects are under way, half of the projects might be conducted one month and the other half the next month and rotated on this cycle thereafter. This permits management to view all of the projects over a relatively short time frame. Projects not performing to plan may receive additional support and resources to rectify them.

CREATIVITY AND INNOVATION TOOLS

Use these tools to develop concept alternatives. (Apply)

Body of Knowledge I.E.5

Frequently, generating ideas about opportunities for improvement projects requires a bit of creative thinking. Merrill (2008) helps us understand the difference between creativity and innovation when he states, "Innovation involves the successful implementation of creative ideas. *Creativity* is about coming up with ideas while *innovation* is about creating ideas and putting those ideas into practice."

Jonson (2005) defines *ideation* as "the creative process of generating, developing, and communicating new ideas, where an idea is understood as a basic element of thought that can be visual, concrete, or abstract." Greiner (1997) suggests a simple five-step process for ideation:

- 1. *Create an opportunity statement.* The opportunity statement is broken into two components: the goal and the schedule. The goal is usually stated in the form of a statement such as, "I will create and implement an idea to . . ." The schedule relates to the completion of the remainder of the five-step process.
- 2. *Collect the raw material*. Recall the inputs shown in Figure 5.2. Many sources of raw material are listed.
- 3. *Hold an idea workout.* This may be conducted as a workshop away from work. It is most important that an idea-prone atmosphere be created. This would include the use of humor, playfulness, and, of course, the elimination of judgment. Rank and organizational levels should be avoided or neutralized.
- 4. *Evaluate the ideas.* The quality toolbox has numerous tools to help and advise us in this area, some of which include the seven management and planning tools. Numerous others exist and are nicely consolidated and described in the following list:
 - Bjørn Andersen. 2007. *Business Process Improvement Toolbox,* Second Edition.
 - T. M. Kubiak and Donald W. Benbow. 2009. *The Certified Six Sigma Black Belt Handbook*, Second Edition

- Jack B. ReVelle. 2004. *Quality Essentials: A Reference Guide from A to Z.*
- Nancy R. Tague. 2005. *The Quality Toolbox,* Second Edition.
- 5. *Implement the ideas.* Good ideas should be implemented in accordance with other priorities, resource availability, and funding.

Greiner's five-step process has been mapped against the pipeline shown in Figure 5.1 to provide a perspective as to how it relates to the pipeline management process. Remember, the emphasis in this section is on identifying improvement opportunities. Three of Greiner's five steps focus on this area.

Further, Table 5.4 maps a set of twenty tools recommended by Greiner against her five-step process. Nineteen of these tools apply to steps 1, 2, and 3 of her process and align with the "identification" process shown in Figure 5.1.

Some organizations have created "idea banks." These banks serve to capture and share suggestions for improvement. Also, they have the effect of sparking ideas in others. Essentially, ideas beget ideas.

Straker (Creating Minds) compiles a comprehensive list for creating ideas on his website and indicates whether they are suitable for individual or group use. The following list is referenced verbatim (with permission) from the website:

- *Absence thinking*. Think about what is not there.
- *Art streaming*. Keep creating until you get through the blocks.
- Assumption busting. Surfacing and challenging unconscious assumptions.
- *Attribute listing.* Listing attributes of objects and then challenging them.
- Braindrawing. Good for reticent groups.
- Brainmapping. Combining brainwriting and mind mapping.
- Brainstorming. The classic creative method for groups.
- Brainwriting. Group doodling for nonverbal stimulation.
- *Breakdown*. Careful decomposition to explore the whole system.
- *Challenge*. Challenge any part of the problem.
- *Chunking*. Go up and then down elsewhere.
- *Crawford slip method*. Getting ideas from a large audience.
- *A day in the life of* Building creative tension from contextualized situations.
- Delphi method. Explore ideas or gain consensus with remote group.
- *Doodling*. Let your subconscious do the drawing.
- *Essence*. Looking elsewhere whilst retaining essential qualities.
- *Forced conflict*. Using conflict to stimulate the subconscious.

| Technique | Step 1: Create an opportunity statement | Step 2: Collect the raw material | Step 3: Hold an idea workout | Step 4: Evaluate the ideas | Step 5: Implement the ideas |
|-------------------------------------------|--------------------------------------------------|-------------------------------------------|---------------------------------------|-------------------------------------|--------------------------------------|
| Benchmarking | ~ | ~ | | | |
| Brainstorming | | | ~ | | |
| Brainwriting | | | ~ | ~ | |
| Catalyst library | ~ | ~ | ~ | ~ | ~ |
| Check out the competition | | v | | | |
| Curiosity file | | ✓ | ~ | | |
| Devil's advocate | ~ | ✓ | ~ | | |
| Diagramming | ~ | | ~ | | ~ |
| Disgruntled employees and customers | | ~ | | | |
| Engage the whole brain | | | ~ | | |
| The "F" word | | | ~ | ~ | ~ |
| Idea-inspiring places | | | ~ | | |
| If Einstein were here | | | ~ | ~ | |
| Just do it | | | | ~ | ~ |
| Manipulate materials | | | ~ | | |
| Metaphorical thinking | | | ~ | | |
| Mind mapping | ~ | | ~ | | |
| Pay attention | ~ | ~ | ~ | ~ | ~ |
| Reporter's questions | ~ | | | ~ | |
| Why ask why? | ~ | | | | |

| Table 5.4 | Idea creation to | ools and techniq | ues and the crea | tion process. |
|-----------|------------------|------------------|------------------|---------------|
| IUDIC J.I | | Jois and teening | ues and the cree | nuon process |

Source: Adapted from Greiner (1997).

- *Guided imagery*. Letting your subconscious give you a message.
- How-how diagram. Break down problem by asking "how."
- *How to*. Frame statements as "how to" to trigger focused thinking.
- *Incubation*. Letting the subconscious do the work.

- The Kipling method (5W1H). Ask simple questions for great answers.
- Lateral thinking. Thinking sideways to create new ideas.
- Lotus blossom. Unfold the flower of extended ideas.
- Mind mapping. Hierarchical breakdown and exploration.
- Modeling. For the artist in everyone.
- Morphological analysis. Forcing combinations of attribute values.
- *Nominal group technique.* Getting ideas with minimal personal interaction.
- *Pause*. Think more deeply for a minute.
- Post-up. Brainstorming with Post-It Notes.
- *Provocation*. Shake up the session by going off-piste. [That is, off the beaten path.]
- *PSI*. Problem + Stimulus = Idea!
- *Random words.* Using a random word as a stimulus.
- Remembrance. Remembering solutions not yet discovered.
- *Reversal.* Looking at the problem backwards.
- Reverse brainstorming. Seek first to prevent your problem from happening.
- *Right-braining*. Combine incomplete doodles around the problem.
- *Role-play.* Become other people. Let them solve the problem.
- *Rubber-ducking.* Get someone else to listen to your talk.
- SCAMPER. Using action verbs as stimuli.
- *Six thinking hats.* Think comfortably in different ways about the problem.
- Storyboarding. Creating a visual story to explore or explain.
- Take a break. When creativity is fading.
- *Talk streaming*. Just talk and talk and talk until you unblock.
- TRIZ contradiction analysis. Use methods already used in many patents.
- Unfolding. Gradually unfolding the real problem from the outside.
- *Value engineering*. Deep analysis to understand and innovate in areas of key value.
- *Visioning*. Creating a motivating view of the future.
- Wishing. State ideas as wishes to expand thinking.
- Write streaming. Write and write and write until you unblock.

Details about when and how to use each of the tools described are provided at the Creating Minds website (creatingminds.org).

This section addressed a wide variety of tools to generate ideas for identifying improvement opportunities. At this point, it may be noted that the classification of what tools may be used to generate ideas is somewhat subjective. Regardless, as Master Black Belts, it is incumbent on you to be intimately familiar with these tools and to know when and how to apply them in the appropriate situations. As always, some tools are more effective than others, and some organizations are more receptive to the use of some tools than others. Understanding the culture of your organization will help you select the right tools.

Chapter 6

Risk Analysis of Projects and the Pipeline

E ach time a project is undertaken, it is accompanied by risk. Many organizations fail to adequately consider this aspect of a project until it surfaces. However, risk can be identified in advance and steps taken to manage it properly. This chapter will discuss the concept of risk and the tools available for successfully managing it.

RISK MANAGEMENT

Use risk management and analysis tools to analyze organizational elements, to appraise portfolios and critical projects, and to identify potential problem areas. (Evaluate)

Body of Knowledge I.F.1

Risk is the probability of not achieving a project goal or schedule, budget, or resource target because something did or didn't occur. Risk may be negative in the form of threats—or positive—in the form of opportunities. Every project incurs some degree of risk.

The Components of Risk Management

The major components of any risk management program include the following key areas as defined by the Project Management Institute:

• *Risk management planning*. According to Turk (2008), "The Risk Management Plan presents the strategy and ground rules, defines the stakeholders, sets the objectives of the program, defines the process and organization structure, and presents roles and responsibilities. It may contain the templates for documentation associated with the program. . . . The plan should also present requirements for prioritizing and closing the risks."

• *Risk identification.* Identify as many risk factors as possible. Initially, the team will be a good source for identifying risks. However, if a failure mode and effects analysis (FMEA) has been conducted, it is an excellent starting point.

What is a risk factor? A risk factor must describe the risk event clearly and concisely in a manner similar to the way the "potential failure mode" or "potential cause" columns in FMEA documents are completed. Table 6.1 provides a useful starting point for identifying various types of risk. Often it is useful to categorize risks in a meaningful way. For example, risk might be categorized as: schedule, budget, technical, quality, personnel, and so on. Categorization can bring additional insight into the risk management process, as it is possible that a risk strategy that applies to a particular risk event could apply to more than one event in the same category.

• *Risk assessment.* This is usually taken as a combination of the probability of the risk occurring and the impact of the risk. Taken together, they are considered the *severity* of the risk. The risk assessment may be conducted in matrix form with one axis labeled *probability* and other labeled *impact*. Each axis is divided into low, medium, and high categories, yielding nine boxes. This concept is illustrated

| Legal action | Noncompliance with regulatory requirements |
|------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------|
| Environmental violations | Customer errors |
| Customer payment delinquencies and nonpayment | Supplier errors |
| Raw material defects | Subcontractor nonconformance |
| Errors and omissions | Financial investments (unexpected or unacceptable yield) |
| Failed projects or inadequate return on investment from projects | Product liability |
| Employee wrongdoing | Sabotage |
| Accidents | Catastrophic loss |
| Civil unrest or terrorist attack | Damage from military action or political upheaval |
| Vandalism | Product obsolescence |
| Inadequate or omitted controls (over processes, finances, employees, suppliers, subcontractors, and so on) | Inattention to danger signals from controls |
| Illegal or unethical behavior on the part of management | Disqualification for certifications, licenses, permits |
| Unwanted buyout/takeover of organization | Unexpected death, disability, or departure of key personnel |

Table 6.1 Potential types and forms of risk that could affect an organization.

Source: Adapted from Westcott (2006).

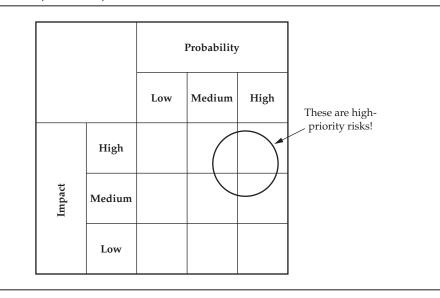


Table 6.2 Example of a simple risk assessment matrix.

in Table 6.2. Each risk is applied to the matrix. Higher risk requires more monitoring and action, and consequently has a higher priority. An alternate project risk assessment is shown in Figure 6.1. Note that this document has been tailored for a Lean Six Sigma project.

• *Risk quantification (risk calculations).* Risk quantification adds a bit more expected value theory to the assessment of risk. When analyzing risk, we often deal with the concept of expected profit. Expected profit is defined as follows:

Expected profit =
$$\sum_{i=1}^{n} (Profit_i \times Probability_i)$$

where:

 $Profit_i = Profit$ for the *i*th outcome

Probability_{*i*} = Probability of the *i*th outcome

n = Total number of outcomes

• *Risk response planning.* Generally, there are four ways to manage or respond to risk. They are: avoidance, transfer, monitoring, and mitigation. Avoidance refers to the practice of eliminating the risk factor. For example, falling back on a tried-and-true software package rather than introduce a new, flashier, but relatively unproven version. Transfer refers to moving the risk to another party or individual and allowing them to assume the risk responsibility. For example, contract a construction job to an outside contractor rather than have it completed by inhouse personnel. Another example, though simple, would be buying insurance. Monitoring refers to continuing to observe low risks where the cost of mitigation

| Project Risk Assessment | | |
|-------------------------|-------|--|
| Prepared By: | Date: | |
| Project: | | |
| Project Description: | | |
| | | |

<u>Weight</u>

Suggest using a 1 to 9 scale, where 9 signifies a high-risk factor and 1 signifies a low risk factor.

<u>Criteria</u>

Click D to add new criteria or remove existing criteria.

<u>Answer</u>

Choose from the following:

- Yes, if criteria do not present any risk for this project (multiplies weight by 1)
- Probably (multiplies weight by 3)
- Maybe (multiplies weight by 4)
- Probably Not (multiplies weight by 5)
- No, if criteria present risk for this project (multiplies weight by 10)

Interpreting the scores

High-risk scores correspond to projects that have a relatively high degree of risk associated with their execution and could be potentially difficult to close. Pay particular attention to the **Scope** score because poor scoping is often the main cause for project failure.

| Scope / | Definition | of Project |
|---------|------------|------------|
|---------|------------|------------|

| Weight | Evaluation Criteria | Answer | Risk Score |
|--------|---------------------------------------------------------------------------------------|------------|------------|
| 1 | The project has a clearly identified customer | (choose) 🔻 | * |
| 1 | The project has a clearly definable defect | (choose) 🔹 | * |
| 1 | The defect can be effectively and accurately measured | (choose) 🔹 | * |
| 1 | The defect metric can be defined at the opportunity level | (choose) 🔹 | * |
| 1 | Costs can readily be associated with the defect | (choose) 🔻 | * |
| 1 | Defects are of an ongoing nature | (choose) 🔻 | * |
| 1 | The process to be improved has reasonably high output volume | (choose) 🔹 | * |
| 1 | The project can be completed in a timely manner | (choose) 🔹 | * |
| 1 | The project has only one defect (one DPMO) | (choose) 🔹 | * |
| 1 | The project will involve only one product | (choose) 🔹 | * |
| 1 | Project improvements will not depend on modifying operations outside of department | (choose) 🔻 | * |
| 1 | Project improvements will not depend on modifying operations outside of organization | (choose) 🔻 | * |
| 1 | The project is limited to only one geographic location | (choose) 🔻 | * |
| 1 | The project will have visible management support | (choose) 🔹 | * |
| 1 | The inputs can be readily modified (e.g., no regulatory requirements) | (choose) 🔹 | * |
| | Scope/Definition of Project Score | | 0 |

Figure 6.1 Example of a project risk assessment document.

Source: Courtesy of Minitab Quality Companion 3 software, Minitab Inc.

Deployment / Leadership

| Weight | Evaluation Criteria | Answer | Risk Score |
|--------|--------------------------------------------------------------------------------|------------|------------|
| 1 | The team leader will be a full time Black Belt | (choose) 🔻 | * |
| 1 | The project will be assigned a Six Sigma Champion (willing to remove barriers) | (choose) 🔻 | * |
| 1 | The project will have a clearly defined Process Owner | (choose) 🔻 | * |
| 1 | The project will have an assigned financial representative | (choose) 🔻 | * |
| 1 | The Black Belt will be allowed proper team resources | (choose) 🔻 | * |
| 1 | Funding will be available (as appropriate to the benefits) | (choose) 🔻 | * |
| | Deployment/Leadership Score | | 0 |

Team Leader (Black / Green Belt)

| Weight | Evaluation Criteria | Answer | Risk Score | |
|--------|---------------------------------------------------------------------|------------|------------|--|
| 1 | The BB/GB will have received proper Six Sigma Training | (choose) 🔻 | 36 | |
| 1 | The BB/GB will have access to a Master Black Belt | (choose) 🔻 | 342 | |
| 1 | The BB/GB will have sufficient technical background for the project | (choose) 🔻 | * | |
| 1 | The BB/GB will have access to subject matter experts as needed | (choose) 🔻 | * | |
| 1 | The BB/GB will be fully empowered by management | (choose) 🔻 | * | |
| | Team Leader Score | | | |

Project Closure / Benefits

| Evaluation Criteria | Answer | Risk Score |
|------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| The project has a clear end point | (choose) 🔻 | * |
| The project will have a clearly identified hand off (to process owner) | (choose) 🔻 | * |
| The benefits (payback) will start at or before project hand off | (choose) 🔻 | * |
| The payback will be expressed in hard savings | (choose) 🔻 | * |
| Project Closure/Benefits Score | | 0 |
| Totals | | |
| | The project has a clear end point The project will have a clearly identified hand off (to process owner) The benefits (payback) will start at or before project hand off The payback will be expressed in hard savings | The project has a clear end point (choose) The project will have a clearly identified hand off (to process owner) (choose) The benefits (payback) will start at or before project hand off (choose) The payback will be expressed in hard savings (choose) Project Closure/Benefits Score |



or avoidance is too high. The risk is monitored until the severity level is no longer acceptable. Mitigation refers to minimizing the impact of the risk. For example, splitting a contract across multiple suppliers rather than one supplier. Although this may introduce other forms of additional risks, it limits the impact of the failure of a single supplier.

• *Risk monitoring and control.* The above four ways address managing risk before events occur. Once a risk event occurs, some action must be taken. This action is described by a contingency plan. Let's consider an organization with multiple suppliers of the same part. If a supplier should fail, the contingency plan might be to immediately move the work to the remaining suppliers, if their capacity or schedule permits. If their capacity or schedule does not permit this, perhaps the work is moved to a backup supplier who is not already performing the work.

EXAMPLE 6.1

A proposed Lean Six Sigma project is aimed at improving quality to attract one or two new customers. The project will cost \$3M. Previous experience indicates that the probability of getting customer A is between 60% and 70%, and the probability of getting customer B is between 10% and 20%. The probability of getting both A and B is between 5% and 10%. One way to analyze this problem is to make two tables, one for the worst case and the other for the best case, as shown in Table 6.3.

Assuming the data are correct, the project will improve profit of the enterprise by between \$1M and \$2.5M.

| | | Worst case | | | Best case | | |
|-------------------|--------------|-------------|------------------------|---------------|-------------|------------------------|--|
| Outcome | Profit (\$M) | Probability | Profit× Probability | Profit (\$M) | Probability | Profit× Probability | |
| A only | 2.00 | 0.60 | 1.20 | 2.00 | 0.70 | 1.40 | |
| B only | 2.00 | 0.10 | 0.20 | 2.00 | 0.20 | 0.40 | |
| A and B | 7.00 | 0.05 | 0.35 | 7.00 | 0.10 | 0.70 | |
| None | (3.00) | 0.25 | (0.75) | (3.00) | 0.00 | 0.00 | |
| Expected profit = | | 1.00 | Expe | cted profit = | 2.50 | | |

Table 6.3 Example of quantifying risk.

Source: Adapted from Kubiak and Benbow (2009).

Consider the impact of the March 2011 Japanese earthquake on paint supplies for several large automobile makers. A lack of a contingency plan affected production.

Risk management is not a static activity. It must be monitored and controlled on an ongoing basis throughout the project. Swift action must be taken when risk levels become unacceptable. Furthermore, it is important to recognize that new risks may be entering the project at any given time.

Risk Analysis Tools

There are three commonly used tools to analyze risk. All of them have been addressed in *The Certified Six Sigma Black Belt Handbook* and will only be identified here. They include the following:

- Failure mode and effects analysis (FMEA) (Note: This tool was mentioned earlier in this section).
- Process decision program chart (PDPC) (Note: This tool is one of the seven management and planning tools).
- Risk quantification (Note: This tool was discussed earlier in this section).

Portfolio Analysis of Projects

Westcott (2006) notes with regard to portfolio analysis that, "Usually the comparative value of the proposed projects to the organization is a major consideration in the acceptance of projects. This comparative value relates to the financial impacts of ongoing projects to the potential impact on resources of the proposed projects." He goes on to state that the L-shaped prioritization matrix that compares projects against weighted criteria might be a suitable tool for managing a portfolio of projects.

Tague (2005) identifies three criteria-setting methods (that is, analytical criteria method, consensus criteria method, and the combination interrelationship digraph/matrix method) to be used with the L-shaped prioritization matrix, and provides clear guidelines as to when each method should be used.

Unfortunately, the L-shaped matrix tends to be a pairwise comparison approach. It works well with a small number of projects. However, when the number of project begins to grow, the pairwise approach quickly becomes unwieldy. An alternate approach that might be considered is a modified form of the causeand-effect matrix. While retaining some of the characteristics of the L-shaped prioritization matrix, this approach permits each project to be assessed against specific criteria rather than compared to other projects.

Consider the matrix outlined in Table 6.4 as a template for conducting a portfolio analysis. The process for developing the matrix is:

- All projects proposed, in process, or a combination thereof would be listed in the first column.
- Relevant criteria for evaluating the project are placed along the top. Generally, this is limited to seven. Useful criteria might include: strategic fit, some measure of financial performance, estimated time to complete, customer impact, crosses lines of business, urgency of the project, project duration, availability of subject matter experts, and so on. Multiple measures of project financial performance might be required (as noted in Chapter 9) as a single measure may be misconstrued.
- Each criterion is weighted. Normally, one hundred percent is spread across the criteria in increments of five percent.
- An ordinal rating scale, similar to one used in an FMEA, is created and is used to populate the light gray area in Table 6.4. Such a scale might be: 1, 3, 5, 7, and 9, with 1 being the lowest and 9 the highest. Words should describe the numeric values in a consistent and meaningful way.
- Cross-multiply the weight for each criterion by the corresponding rating for a specific project and add across criterion columns to determine the rank score for the project. This is simply:

$$\operatorname{Rank}_{j} = \sum_{i=1}^{n} (\operatorname{Weight}_{i}) (\operatorname{Rating}_{i})$$

| Project | <i>C</i> ₁ | <i>C</i> ₂ | C_n | Rank |
|---------|-----------------------|-----------------------|--------------------|------------------|
| | W_1 | <i>W</i> ₂ | W _n | Weight× Score |
| 1 | | | | |
| 2 | | | | |
| ÷ | | | | |
| m | | | | |

Table 6.4 Example of a simple portfolio analysis matrix.

| Project | Strategic fit | Customer impact | | ROI | Rank |
|---------|------------------|--------------------|-----|------|------------------|
| | 0.50 | 0.30 | ••• | 0.20 | Weight× Score |
| 1 | 3 | 7 | | 5 | 4.6 |
| 2 | 7 | 7 | | 9 | (7.4) |
| : | : | : | | : | : |
| m | 9 | 3 | | 5 | 6.4 |
| | | | | | |

Top priority

where

 $W_i = Weight_i$

 $C_i = Criterion_i$

This method requires consensus be developed in three areas: development of the evaluation criteria, allocation of weights, and creation of the rating scale. Involving the individuals who will ultimately evaluate and rank the projects in reaching consensus on these items will increase buy-in and support of the projects. Initially, establishing a consensus will take considerable time. Use of a trained facilitator may be beneficial. However, once established and applied to several projects, the evaluation team tends to move through subsequent projects quickly.

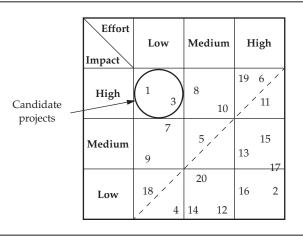
The rank scoring approach just discussed and depicted in Table 6.4 can be refined further. Consider Table 6.5. Additional criteria were added to this table and subtotals computed in the same manner as before:

Total impact_j = $\sum_{i=1}^{k} (Weight_i)(Rating_i)$ Total effort_j = $\sum_{i=k+1}^{n} (Weight_i)(Rating_i)$ Rank_i = Total impact_i + Total effort_i

| | | | | 1) | | , 1 | | | | |
|---|---------|-------|-----------------------|-------|--------|------------|-----------|-------|--------|----------------|
| Р | Project | C_1 | <i>C</i> ₂ | C_k | Total | C_{k+1} | C_{k+2} | C_n | Total | Rank |
| | | W_1 | <i>W</i> ₂ | W_k | Impact | W_{k+1} | W_{k+2} | W_n | Effort | Weight × Score |
| | 1 | | | | | | | | | |
| | 2 | | | | | | | | | |
| | ÷ | | | | | | | | | |
| | m | | | | | | | | | |

Table 6.5 Example of a project selection and/or prioritization matrix.

| Table 6.6 | Charting impact v | ersus effort. |
|-----------|-------------------|---------------|
| 14010 010 | end ang impace i | croac chert |



The subtotals computed were "Total impact" and "Total effort." In this case, a portion of the criteria was devoted to determining the total impact of the project on the organization. This might be construed as a benefit to the organization. Another portion of the criteria was devoted to determining the total effort required to execute the project. This dichotomy proves useful, and can be graphed easily for each project as illustrated in Table 6.6. Those projects that require the least amount of effort yet provide the greatest impact (that is, yield the most benefit) are likely to be selected as projects.

In addition to the above, Mawby (2007) suggests linear, integer, and nonlinear programming methods for project portfolio selection. These methods, though interesting, will not be addressed here. The reader interested in learning about these methods should consult Mawby (2007).

Ultimately, the successful use of a portfolio analysis tool will allow an organization to select, prioritize, postpone, and eliminate projects. When used in conjunction with project reviews, additional support can be brought to bear on those projects that are worthwhile, but experiencing difficulties.

PIPELINE CREATION

Create, manage, and prioritize a pipeline of potential projects for consideration. (Create)

Body of Knowledge I.F.2

In Chapter 5, the first two processes (identification and qualification) of Figure 5.1 were discussed. This subsection will focus on project selection and prioritization.

Project Selection

To select a project means that it has met the criteria of being a Lean Six Sigma project (that is, it has been qualified) and now must be judged to determine whether it is sufficiently worthy such that the organization is willing to invest time and resources to achieve the expected benefits of the project.

In the previous subsection, we discussed several approaches for rank-scoring projects in the context of analyzing a portfolio of projects. These approaches are equally suitable for the selection process as well. A high-level view of the selection process is represented in Figure 6.2. Notice that some of the inputs are familiar, such as the facilitator and Master Black Belts. Additionally, project sponsors and senior leaders should participate in this critical process, as both are primary stakeholders. Further, the list of qualified projects is included. Remember, this was an output from the previous process. At this point, all projects should be in top-notch form, signed by sponsors and financial representatives, and ready for senior leadership review. Finally, project selection criteria are included in the event they are different from the qualification criteria.

Once projects are selected, they enter the project pipeline. Again, we refer to Figure 5.1. Notice that the projects in the pipeline over the selection process are in random order. Those projects have been selected, but not prioritized. They are essentially sitting in the proverbial hopper.

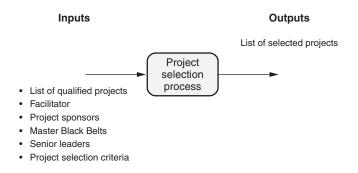


Figure 6.2 The project selection process.

Project Prioritization

To prioritize a project means the project has been judged and deemed sufficiently worthy such that the organization is willing to invest time and resources, and the organization is now ready to determine the order in which the projects will be executed. This is an important, but fine, distinction because projects that have been selected may be re-prioritized many times as new projects are identified, qualified, and selected to enter the pipeline.

In Figure 5.1 we saw that selected projects in the pipeline were in random order. Once they are prioritized, they are in rank order and ready for assignment.

Priorities will remain in rank order until they change. Organizations work in a dynamic environment and are subject to both internal and external influences. Projects may be identified, qualified, and selected on an ongoing basis. Incoming projects may yield significantly better projected results than projects already prioritized and in the pipeline. This dynamic nature of the pipeline must be recognized and addressed accordingly.

A high-level view of the prioritization process is portrayed in Figure 6.3. As before, senior leaders are key participants in this process. They hold the purse strings and are in the best position to determine how project priorities relate to overall organization-wide priorities. Notice that the prioritization criteria are also included as an input in the event they are different from the selection criteria. Again, familiar inputs from the previous process are present as well.

In one of the previous subsections, we discussed several approaches for rank-scoring projects in the context of analyzing a portfolio of projects and selecting projects. These approaches are equally suitable for the prioritization process as well.

Frankly, separation of the qualification, selection, and prioritization processes can be unwieldy. Some organizations have elected to combine two or more of these processes. However, there is danger in doing so. Each aspect of the process given in Figure 5.1 requires specific inputs and generates specific outputs. Shortcutting this can undermine the pipeline management process and hinder its overall effectiveness. Process inputs and outputs can be truncated or forgotten; senior leaders or projects sponsors may be cut from meetings; and there is always the strong temptation to "get it done." Rigorous adherence to the process will ensure it is done properly and consistently and will achieve its intended objectives.

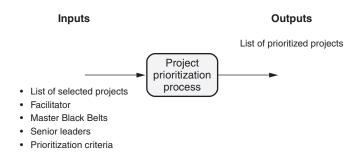


Figure 6.3 The project prioritization process.

PIPELINE MANAGEMENT

Create a selection process that provides a portfolio of active six sigma opportunities that are clearly aligned and prioritized to meet/exceed strategic goals. (Create)

Body of Knowledge I.F.3

Earlier in this chapter and in Chapter 5, a process was identified for managing the entire pipeline process. Methods were suggested for uncovering project ideas as well as many sources of data, some of which were linked to the organization's action plans. Consensus-based techniques for qualifying, selecting, and prioritizing projects were presented. Likewise, these were linked to the organization's strategies, as in the value driver analysis tool shown in Figure 5.3.

The pipeline management process depicted in Figure 5.1 provides an organization with a rigorous and systematic methodology for managing a portfolio of Lean Six Sigma projects and opportunities that will necessarily be aligned with the organization's strategic goals. This alignment is assured when a direct line of sight is established between the sources of data and ideas and the strategic plan, as demonstrated in earlier chapters.

One addition to the pipeline management process many organizations have found useful is the establishment of a knowledge management system that focuses on the capture and distribution of project activities and results. This facilitates the sharing of both failures and successes. Also, it permits the replication of project results, wherever applicable.

Managing and keeping track of project progression throughout the entire process from creation to closure does not have to be complicated or expensive. Generally, an Excel spreadsheet is sufficient. However, it is prudent to ensure that Lean Six Sigma processes are developed with data collection in mind. This is necessary not just for project management purposes, but for improving the overall management system of the Lean Six Sigma deployment process as discussed in section I.C.6.

Chapter 7 Organizational Design

This chapter will address the overall concept of organizational design. When discussing organizational design, it is necessary to address the supporting concepts of systems thinking, organizational culture, and organizational maturity. Once these concepts are understood, we have a formal basis for discussing organizational change techniques.

SYSTEMS THINKING

Apply systems thinking to anticipate the effect that components of a system can have on other subsystems and adjacent systems. Analyze the impact of actions taken in one area of the organization and how those actions can affect other areas or the customer, and use appropriate tools to prevent unintended consequences. (Analyze)

Body of Knowledge I.G.1

What Is Systems Thinking?

"Systems thinking" was founded in 1956 by MIT professor Jay Forrester and has its foundation in the field of system dynamics. Essentially *systems thinking* is a problem-solving approach that expands the view of something under analysis to determine how it interacts with other elements of the system of which it is a part. Note that this is in direct contrast to the traditional approach to analysis (that is, decompose a whole down to smaller and smaller parts and study them in isolation).

Skyttner (2006) identified several characteristics critical to systems thinking. These include:

- *Interdependence of objects and their attributes.* A system must comprise interdependent elements. Elements are never independent.
- *Holism.* Properties emerge at the system level that are not detectable by analysis at the element level.

- *Goal seeking.* The purpose of the system (that is, interaction of the elements) must have a goal or final state.
- *Inputs and outputs (including the transformation thereof).* The inputs and outputs in a closed system are determined and constant while an open system permits the entry of additional inputs and outputs. Transformation is the process by which goals or the final state is achieved.
- *Entropy*. This represents the amount of disorder, chaos, or randomness present in the system.
- *Regulation.* This is the method by which feedback is provided. Also, it permits the system to operate in a predictable manner.
- *Hierarchy.* Complex systems are made up of smaller systems and so on down to the elemental level.
- *Differentiation*. There are specialized elements of the system that perform specialized functions.

A brief perusal of these characteristics quickly reveals that systems thinking is applicable to organizational design. Consider any organization you are familiar with and you will readily see how the above eight characteristics apply. Granted, goals may not be fully aligned, hierarchies may be redundant, feedback may be ignored, and entropy may reign. Nonetheless, organizational design will benefit from the application of systems thinking.

EXAMPLE 7.1

A large multinational organization decided to start its implementation of Lean Six Sigma. An organizational structure was developed with a position of vice president of Lean Six Sigma reporting directly to the CEO. A lengthy search was conducted, the vice president hired, and funding issued to develop a self-contained department. A mandate is given that the department will include curriculum development and delivery and a specified number of Master Black Belts who will be hired from outside the organization, and some who will be developed from inside the organization.

Also part of the CEO's staff is the vice president of human resources, who is responsibile for organizational learning, and the vice president of quality, who has been responsible for driving quality improvement.

Soon after the vice president of Lean Six Sigma began to develop the department, she met with severe resistance from her two peers. Why? The CEO had not consulted her peers. Furthermore, it had not been determined how the new Lean Six Sigma department would integrate into the existing organization or how responsibilities and accountabilites would change with regard to the organizational learning and quality departments. Critical questions were left unanswered and conflict quickly arose. Essentially, a new element (Lean Six Sigma department) was dropped into an existing system (large multinational organization) with no thought given to the consequences.

Systems Thinking Diagrams

In its purest form, systems thinking uses an established set of rules, diagramming structures, and the like for depicting the relationships between the various entities, parts, or elements within a system. Some of these rules and structures can be quite complex. As such, they will not be addressed here. The interested reader is encouraged to seek out resources that specialize in this content. Senge (1994) is one author who provides a significantly detailed discussion of this topic.

However, an example of a simpler form of systems thinking diagrams will be addressed. This approach is less complex and reminiscent of the circle diagram technique addressed in Chapter 18 of *The Certified Six Sigma Black Belt Handbook*.

Law of Unintended Consequences

When systems thinking is not applied, unintended consequences are inevitable. When applied diligently, unintended consequences can be minimized.

EXAMPLE 7.2

An organization is beginning its implementation of Lean Six Sigma. At the outset, the Lean Six Sigma leader brought the newly established team together to determine how each of its key elements would interact. During the first draft, the team identified the five activities or entities illustrated in Figure 7.1. This figure depicts what the team believes to be the flow of information and communication. At this point in the development of the department, it is not known who will be responsible for the activities or roles other than Master Black Belt. The team's thinking behind each of the five activities or entities is as follows:

- *Master Black Belt*. The Master Black Belt is key to the functioning of the department. This individual will provide and receive input to curriculum development, conduct training, and be central to project and candidate selection.
- *Curriculum development*. This entity will interact with the Master Black Belt and will develop the actual curriculum and training material that will be delivered to the candidates.
- *Training*. This entity will use the training material and provide feedback to curriculum development on the effectiveness and deliverability of the training material.
- *Candidate selection*. Candidates will be selected from outside the Lean Six Sigma department (not shown here) and their suitability as a candidate will be evaluated by the Master Black Belt. Candidates will be sent to training.
- *Project selecton*. The Master Black Belt will be highly involved in the project selection process as well as determining how projects and candidates should best be paired.

This example, although simplified, is meant to illustrate the interaction between the system components and recognize that this subsystem (Lean Six Sigma department) is part of a greater system (total organization). How so? Recall from candidate selection above, candidates enter into the subsystem from outside the department. Essentially, they are drawn from the remainder of the organization.

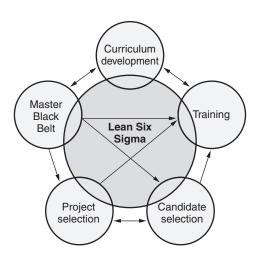


Figure 7.1 A simplified systems thinking relationship diagram.

The *law of unintended consequences*, named by sociologist Robert K. Merton, is meant to describe outcomes not intended by purposeful actions. "Unintended consequences" is sometimes known as *unanticipated consequences*, *unforeseen consequences*, or even *side effects*. Causes of unintended consequences include stupidity, ignorance, error, self-deception, incentives, or simply failing to account for the human element. Unintended consequences can be grouped as follows, though the difference between the last two bullets may be considered a matter of degree:

- *Unexpected benefits (positive).* The sinking of ships in shallow waters during war for defense purposes created artificial reefs
- *Unexpected drawbacks (negative)*. Prohibition during the 1920s in the United States was intended to reduce consumption of alcohol. Instead, it created a black market and allowed organized crime to grow and prosper.
- *Perverse results.* Rent control was enacted to make housing more accessible. Instead, it created a housing shortage. Consequently, landlords circumvented rent control through bribery.

EXAMPLE 7.3

Asian carp is a collection of bighead and silver carp and is considered an invasive species. Catfish farms imported Asian carp decades ago as a means of controlling algae in their ponds. Eventually, the fish escaped and were found in the lower reaches of the Mississippi River in the early 1970s. Currently, they are approaching the Great Lakes. The concern is that these carp are considered eating machines. They eat plankton, and would be major competitors for the lowest layers of the food chain. This could disrupt the entire ecosystem of the Great Lakes by eliminating the lowest layers. As a result, trout and other native species could starve out.

EXAMPLE 7.4

Conventional wisdom suggests that if insects are damaging a crop, apply an insecticide. This works in the short term. In the long term, the opposite is true. The insects eliminated by the insecticide were controlling another population of insects who, in turn, could cause crop damage. Consequently, the controlled insect population grows and crop damage returns.

EXAMPLE 7.5

Cogon grass is a hardy weed native to Asia. It was brought to the United States through its extensive use as packing material in shipping crates. Today, it has spread throughout the Southeastern United States.

ORGANIZATIONAL MATURITY AND CULTURE

Describe the implications these factors can have on Six Sigma implementation, including potential barriers. (Understand)

Body of Knowledge I.G.2

Is Your Culture Mature? Is Your Organization Ready?

Recall our discussion in Chapter 3 regarding cultural maturity models and organizational readiness assessments. These instruments are crucial in understanding and defining the present state of your culture with regard to your cultural objectives as well as providing great insight into your ability to succeed with the implementation of Lean Six Sigma. They can be designed to accommodate the needs of any organization and to assess all aspects of culture maturity and readiness. Also, they can be stratified according to business units, divisions, sites, and departments to help identify potential implementation starting points. Furthermore, the forces opposing cultural change that are identified in Figure 11.2 can be assessed. Once surfaced and acknowledged, techniques discussed later in this chapter can be applied to mitigate these forces and move the culture toward the desired objectives. Cultural objectives are addressed in Chapter 11.

If your organization's maturity level is low or your readiness assessment indicates your organization is not prepared to accept the launch of a strategic initiative such as Lean Six Sigma, then the organization must step back and prepare itself. Likely, this may mean delaying the implementation of Lean Six Sigma until maturity and readiness can be improved. To do otherwise is foolhardy. Unfortunately, all too often organizations choose the path of foolhardiness. Many organizations plow forward into defeat and lay the blame on Lean Six Sigma when this occurs. However, all is not lost. An alternative is to divide your organization into elements of varying degrees of readiness and place them on tailored implementation schedules. Implementation schedules would have to be expanded beyond normal Lean Six Sigma activities. Cultural maturity considerations would have to be included as well. Of course, this adds to the complexity and will likely require the use of skills beyond Lean Six Sigma.

Potential Barriers to a Successful Implementation

There are numerous barriers to a successful implementation. More often than not, they can be grouped into the following categories identified in Chapter 3:

- Culture
- Infrastructure
- Leadership
- People
- Processes
- Technology

Culture. Figure 11.2, as mentioned previously, identifies several of the forces that oppose cultural change. It should be clear from this list that it is not complete or mutually exclusive from the other categories above. Consider this list a starting point as you develop your own cultural assessment instrument.

Infrastructure. This may be seen as the "pieces of the puzzle" as well as the impact of the organizational structure. From the "pieces of the puzzle" viewpoint, one must ask the question, "Do I have everything I need to do the job?" For example, because of a recent performance appraisal, an employee's goals and objectives for the coming year require her to take specific personal development courses. However, her manager's budget does not have funding for training, nor does the organization have a training department. This creates a conflict for her that must be resolved swiftly. Also, from the organizational structure viewpoint, there is a significant gap in the structure of the organization, particularly if in-house training was intended.

Note that many organizations use a structure long after the need for that structure has passed. They maintain the structure because of inertia, and never update it to reflect the current business needs or operational environment. Highly centralized organizational structures tend to have more of this inertia than decentralized ones. Centralized structures tend to develop cultures that resist change, making it difficult to achieve improvements. By contrast, there are those organizations that change their structure quite frequently. These organizations are so dynamic and fluid that responsibilities and accountabilities for goals and objectives become lost in the change. Employees find it difficult to build relationships and take work seriously because "this too shall pass."

Furthermore, the division of labor in an organization, both horizontally and vertically, creates several roadblocks, for reasons outlined in Table 7.1. The solutions suggested represent only a limited number. Remember, today's possible solutions may be limited by today's culture. Mature the culture and the size of the solution set and the future may open vastly.

Leadership. The theories of motivation, leadership styles, communication styles, and influencing without authority are well known, and discussed in Chapter 11. Leaders who are unfamiliar with these theories or execute them badly will have a devastating effect on the organization in terms of its culture and its ability to mature.

| Roadblock | Possible solutions | | |
|-----------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|--|
| Lack of cross-functional collaboration | Ensure that improvement initiative and projects, performance measures, and rewards are designed such that collaboration must occur across functional boundaries to achieve specific objectives. Develop internal customer–supplier agreements and couple this with joint progress rewards. | | |
| Lack of authority | Identify an individual, possibly a champion or sponsor, who has authority over an entire area impacted by a change. Ensure that this individual has a direct link to higher levels of the organization in the event they need additional support outside their area of authority. | | |
| Inward focus | Consider having employees visit the customer sites or locations where the final product or service is in the hands of the end user. This would also include trade shows, conferences, and workshops where new sources of information and ideas might exist. | | |
| Internal competition for resources or rewards | Ensure that strategic, tactical, and operational plans are clearly prioritized. Conduct budget planning both horizontally and vertically throughout the organization and link it to the operational plans. | | |
| Lack of understanding | Ensure that communication is consistent throughout the organiza- tion and relevant to specific units and departments. Use different media to reinforce message and communicate in a timely manner. | | |
| Slow decision making | Identify authority levels for typical situations and on an exception basis for particularly difficult situations across organizational boundaries. | | |

 Table 7.1
 Common organizational roadblocks and possible solutions.

People.

"If you don't like change, you're going to like irrelevancy even less."

-General Eric Shinseki

"If it weren't for the people, Lean Six Sigma implementations would go just fine." However, that isn't the case. People are *part* of the implementation, and we must learn to deal with the issues they create. Some of these issues include:

- *Fear.* This is a broad category, but it could include fear of being measured, following processes, departure from the norm, job loss, the unknown, and so on—all creating resistance.
- *Impatience*. This could easily fall under leadership, but it is equally applicable to people in general. People seek instant gratification. They want quick feedback and they want to see instantaneous process improvement.
- *Structure*. The methodologies of Six Sigma (for example, DMAIC, DFSS and its derivatives) are highly structured. While there is flexibility in the use of the tools within each phase, we simply do not skip phases, tollgate reviews, and the like. The structure consumes time, yet it is what gives us success. Unfortunately, many individuals are opposed to the use of such structure and want to shortcut it whenever possible. Shortcuts are a recipe for disaster.
- *Rigor.* Generally, people abhor the rigor of processes put in place through Lean Six Sigma. They seek flexibility, and Lean Six Sigma seeks standardization.

Although this list is rather abbreviated, you can easily expand it by listening to the complaints of employees in any organization undergoing an implementation.

Processes. An organization's processes may be long institutionalized, dictated by regulatory agencies, customers, or even higher levels of the organization. In some cases, the rationale behind why or how a process operates may have been lost over time, or the reason for it is no longer valid. Consequently, processes may be obsolete, contradictory, redundant, or completely ineffectual.

That being said, people become comfortable with existing processes and will resist changing them for reasons discussed previously. In organizations where processes are automated, resistance to change can be almost insurmountable.

Technology. As great as technology can be, it can be *improvement*'s worst enemy. Technology such as SAP and Oracle software packages can be expensive. Often, an organization must adapt its way of doing business to the methodology integrated into these packages. However, such a methodology may not be optimal or even suitable for the organization. Sometimes, the organization requests custom changes to the software. Frequently, this is done without consulting the Lean Six Sigma implementation team. However, once adopted, the expense of changing or improving the process is prohibitive. In all likelihood, hope for improvement is lost.

ORGANIZATIONAL CULTURAL CHANGE TECHNIQUES

Describe techniques for changing an organizational culture, such as rewards and recognition, team competitiveness, communications of program successes, and appropriate cascading of goals throughout the organization. (Apply)

Body of Knowledge I.G.3

Planning for Change

Only when an organization determines and acknowledges the current state of its culture can it begin to plan for cultural change. Cultural maturity and readiness assessments can establish a baseline, but senior leadership must be accepting (that is, acknowledging) of that baseline. In many instances, senior leadership will be in denial of the results. This is most often the case when they are far less than expected. When this occurs, cultural change will be immediately impeded.

A generic plan for cultural change will likely include:

- *Rewards and recognition.* An integrated system of rewards and recognitions should be established. This system should focus on achieving results through proper behaviors. One such critical behavior might be teamwork.
- *Team competitions.* Competition builds relationships and the spirit to succeed. Also, it provides the opportunity to engage multiple individuals in a single improvement activity.
- *Communications plan.* This plan would ensure a two-way flow of communication (that is, from employees to leadership and from leadership to employees). The plan should communicate information relevant to particular categories and groups of employees in a timely manner using multiple media as necessary. Messages should be clear, consistent with, and reinforcing of other messages. Time frames of events should be evident, and who is affected by what event or action and when should be clear. Everyone in a leadership role should be on the same page. Employees should understand why the change is required and how they are personally affected by the change. Tout successes.
- *Goals and objectives.* These should be aligned horizontally and vertically, and budgeted and resourced properly to permit success. Responsibility and accountability should be evident. Authority levels should be identified.
- *Define the scope.* Clarify the scope of the change. How big is it? Where does it start and end? Is there an implementation sequence? If so, what is the logic behind it? What is out of scope and why?

• *Throttle change.* Change is inevitable. Some say it is the only constant. Some leaders say change is good, thus, more change must be better. Employees are people, and people are highly adaptable. However, they have their limits. They can only absorb a limited amount of change in a limited amount of time. How much is that? It all depends on the person. Too much of a single type of change or too many different types of change and an individual can shut down. Consider an employee in a department caught in the transition of a large system that is central to her daily job. Life is tough. Now assume that same individual deals with four large systems daily, all undergoing transitions simultaneously. Life may now have transitioned from tough to unbearable. The organization may now lose a knowledgeable and hard-to-replace employee due to what we might call "runaway change." When we say we must manage change, we mean more than get it done. Change may have to be scheduled, delayed, or integrated with other change activities so that change occurs in a "manageable size."

In addition to the above elements of a cultural plan for change, Westcott (2006) summarized several useful considerations and errors to avoid when managing change. These are shown in Table 7.2.

| Useful considerations | Common errors | | | |
|-----------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------|--|--|--|
| Create an awareness of the need for change | Not sufficiently emphasizing the urgency and allowing people to be complacent | | | |
| Organize a project with sufficient authority to guide the process | Those guiding process do not have sufficient power | | | |
| Define the vision and strategies for achieving it | Lacking a clear and compelling vision or not communicating it strongly and/or frequently enough | | | |
| Communicate the vision and demonstrate personal commitment to it | Failing to manage the forces that resist change | | | |
| Remove obstacles that prevent others from acting on the vision | Not ensuring some early successes that encourage others | | | |
| Go for early and visible wins | Celebrating victory prematurely | | | |
| Build on success by rewarding supporters and involving more people | Not changing other organizational systems and cultural elements that are required for long-term continuation of change | | | |
| Institutionalize the new methods by aligning other systems with them | | | | |

Table 7.2 Key guidelines regarding change.

Source: Adapted from Westcott (2006).

Who Leads Change?

Good question! The answer, of course, depends on whom you ask. Senior executives often try to delegate the responsibility for leading change. As such, and as Master Black Belts, you will likely be called on to "lead change." Hence, you will serve in the role of a change agent. According to Hutton (1994), *change agents* are "individuals who play a specific role in the planning and implementation of the change management process. They may be members of the organization or may be outsiders."

However, the definition of "change agent" doesn't really speak of leading change. Master Black Belts have the skills and in-depth knowledge to develop the plans for change, perhaps as a collective team, and advise senior leadership on what actions to take. Therefore, the Master Black Belt's role in change management is primarily one of support. (Note: This does not preclude the Master Black Belt from executing actions from change management plans.) This approach allows senior leadership to remain committed, engaged, involved, and visible to the organization. If senior leadership should waver on any of these, their behavior becomes transparent and they can no longer serve as role models to the organization. Consequently, change will not occur or be sustained.

Tools of Change

There are several tools available to help drive change. Several have been addressed previously. They include: organizational cultural and readiness assessments, communications plans, systems thinking, rewards and recognitions processes, and an integrated, top-down, organization-wide system of goals and objectives linked with performance measures and applicable to all employees. In addition, there is a well-known, but little used, quality tool that is quite helpful in driving organizational change. It is known as the force field analysis.

Siebels (2004) defines *force field analysis* as a "technique for analyzing the forces that aid or hinder an organization in reaching an objective." Such an objective could be cultural change.

Figure 7.2 illustrates the general format of a force-field analysis. The desired change is listed at the top in simple terms. Driving forces are listed on the left side of the vertical line. *Driving forces* are those forces that aid in achieving the objective. Similarly, restraining forces are placed on the right side of the line. *Restraining forces* are those forces that hinder or oppose the objective.

EXAMPLE 7.6

Figure 7.3 illustrates an example of losing weight provided by Tague (2005). As you review this example, notice that the driving and restraining forces are not necessarily or intended to be opposites. Also, notice that the relative strength of each force is not stated. This may be considered a drawback of the force field analysis tool.

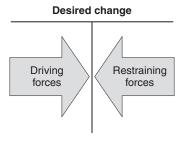
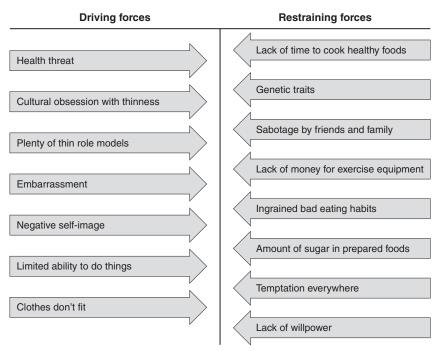


Figure 7.2 A force field analysis diagram.



Losing weight

Figure 7.3 A force field analysis for Example 7.5. *Source:* Adapted from Tague (2005).

The cultural maturity and readiness assessment will contain a wealth of information regarding both the driving and restraining forces. In addition, consider the use of brainstorming sessions. These can be conducted candidly and will likely surface additional and insightful information.

A force field analysis can be done for change at any level and is useful anytime there are two opposing sides to an issue that needs to be considered. Identifying which forces can be changed and which can not will help reduce wasted energy. Focusing energy on removing or reducing forces that are restraining change, or ensuring that the driving forces are maintained or increased, will be more productive.

Tools of Change at the Project Level

Numerous quality improvement tools are at our disposal. Tague (2005) and Andersen (2007) provide an exhaustive list of such tools and techniques. Many of these tools involve the use of teams (that is, engaged individuals), and when systematically applied allow the team to reach a logical conclusion or at least one that can be supported by consensus. The real point to be made is that we have been applying change management tools all along. Perhaps we just haven't recognized them as such.

Chapter 8

Organizational Commitment

Successful Lean Six Sigma deployments require organizational commitment. This chapter will discuss:

- · Techniques and methods to gain commitment
- The organization structure necessary to support that commitment
- · How to communicate with management to maintain commitment
- The role of the Master Black Belt and the various techniques available to effect change and support a successful deployment.

TECHNIQUES TO GAIN COMMITMENT

Describe how to gain commitment from the organization's leadership for the Six Sigma effort. (Understand)

Body of Knowledge I.H.1

On occasion, we find ourselves in a position where a leader's commitment to the Lean Six Sigma implementation is waning or perhaps absent in the first place. Consequently, we find the need to rebuild or instill it. In Chapter 11, we will discuss in detail various leadership styles, communication styles, and influencing without authority, all of which are critical to gaining or regaining commitment from leaders.

In 1989, Stephen Covey introduced his 7 *Habits of Highly Effective People*. His fifth habit, "Seek first to understand, then to be understood," is most appropriate when seeking to gain commitment for it invokes the need for empathetic listening. Empathetic listening is important because it provides insight into the leaders' concern or reluctance to provide commitment.

One often-cited source on gaining commitment is Locke and Latham (1990). They identify seven factors that affect the ability to gain commitment in leaders and employees. These include:

- *Valence. Valence* is the value placed on achievement and reveals the "why" beneath the "what" that is being asked of someone. Employees who value a goal will find a way to get it done in spite of its difficulty or adversity. One mistake leaders make is confusing commitment with *compliance*. Committed employees give their heart and soul while compliant employees just put in their time. One way to help leaders to see the benefits of achieving their goals is through brainstorming. Consider asking them, "What personal benefits do they receive by achieving the goals?"
- *Rewards.* Monetary rewards of substantial value have been shown to influence commitment to goal setting (that is, participation in the Lean Six Sigma implementation). However, if goals are too difficult or impossible to achieve, then such extrinsic motivation will likely act to lower commitment and performance. This is often the situation when people see they are not receiving a reward. One possible solution is to set both short- and long-term goals so that performance can be rewarded for attaining short-term goals that lead to long-term objectives.
- *Involvement.* Commitment to goals can be increased when leaders are involved in the goals-setting process rather than simply being assigned goals (or projects). This will help actively engage them and their employees and go a long way toward increased morale and participation.
- *Authority*. Influential requests by authority figures do not induce or coerce commitment. Commitment is a choice. Those in authority can increase employee commitment by explaining why the goal is important and the benefits of achieving the goal, exerting reasonable pressure for performance, being knowledgeable about the task, and serving as a role model for the behavior they seek.
- *Competition.* Healthy competition among leaders, if not carried to the extreme, can create a high-performance organization. Competition creates a psychological climate that focuses employees' attention on rewards that are contingent on comparisons of their performance against that of peers. Consequently, employees become more focused on their goals and commitments. Also, peer pressure and group norms can influence an individual's behavior and subsequent performance.
- *Publicness.* This is based on the idea that leaders perform better when their goals and performance to goals are public rather than kept anonymous. This is predicated on the idea that one is motivated to preserve one's self-image and public image.
- *Expectancy*. Expectancy theory, developed by Vroom (1964), is based on the idea that people are motivated by their conscious expectations of what will happen when they take certain actions, and are more productive when they believe their expectations resulting from those

actions will be achieved. According to Vroom, an individual will ask three questions:

- If I attempt this behavior, how likely is it that I will succeed? (Expectancy)
- If I am successful, will the outcome be desirable? (Instrumentality)
- How much do I value the outcomes? (Valence)

In its simplest terms, expectancy is the belief that putting in the effort will lead to the performance necessary to receive the rewards. Of course, this assumes there is a positive correlation between effort and performance.

While the Master Black Belt may not be fully in a position to implement all of the above factors outlined by Locke and Latham, he or she should consider working through his or her leadership or higher levels of leadership to ensure such factors are put in place, perhaps on a wider organizational scale.

In addition to the above factors, Kubiak (2004) recommends an additional factor—partnership. Often in Lean Six Sigma implementation, organization leaders feel abandoned. They are expected to do this thing called "Lean Six Sigma." Perhaps they attended a few Lean Six Sigma leadership introduction workshops or were visited by a Master Black Belt and asked to come up with projects or donate potential Black Belt candidates. They may be confused, suspicious, and unclear of where to proceed. In short, they feel abandoned.

Partnering with leaders can go a long way toward gaining their commitment. They will know they are not alone and that you, the Master Black Belt, have "skin in the game." They know that if they fail, you fail. Let them know you will be with them every step of the way. This will provide them with encouragement and confidence to move ahead. Serve as their personal mentor and coach. Provide tactful feedback. Encourage them to publish and celebrate successes. Soon they will champion the cause, and subsequent efforts to gain commitment will become easier.

However, there are those cases where leadership commitment remains based solely on financial returns. Where this is an important consideration, Chapter 6 discusses ROI as it relates to risk analysis and project prioritization while Chapter 9 addresses a variety of measures suitable for selecting projects.

NECESSARY ORGANIZATIONAL STRUCTURE FOR DEPLOYMENT

Develop the inherent organizational structure needed for successful deployment. (Apply)

Body of Knowledge I.H.2

Organizational Design

Numerous opportunities are available to structure the organizational design when first entering into a Lean Six Sigma deployment. Those in use today cover the gamut from simplistic to complex. Some are easy to navigate and some couldn't be navigated with a GPS and flashlight. Whether a structure is right or wrong depends on whether it contributes to or detracts from achieving a successful implementation.

As organizations begin to develop and ready their organization structure for implementation, several key considerations should be addressed during the design. Let's discuss the few critical ones:

• *Centralization versus decentralization*. This addresses both the total organization as well as the Lean Six Sigma department. *Centralization* versus *decentralization* refers to how much decision-making authority has been delegated to lower management levels. This includes business units, divisions, sites, and so on.

In general, at the outset, experience has shown that a centralized organization is more conducive to a smoother implementation. Consider just a few of the reasons:

- Processes are immature or nonexistent.
- Curriculums have not been developed or purchased
- Job descriptions have not been established

Once the topmost organization structure has been put in place and all the necessary infrastructure, processes, and the like developed, they can be deployed throughout the remainder of the organization to business units, divisions, sites, and so on, who are ready to begin their deployment. However, at all times, a reporting relationship (whether solid or dotted) should be retained up to the highest level of the organization. This is necessary to ensure consistency throughout the organization. This concept is depicted in Figure 8.1. At some time in the future, when the organization matures, it may find the need to decentralize. Keep in mind

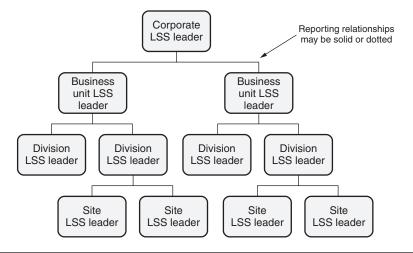


Figure 8.1 Maintaining a cohesive, centralized Lean Six Sigma organization.

that many organizations that have migrated from a centralized to a decentralized organization have found the need to recentralize their Lean Six Sigma implementation after a few years. The most frequent reason given is that the curriculums and processes in the subordinate organizations begin exhibiting significant variation. This is more aptly known as "doing their own thing."

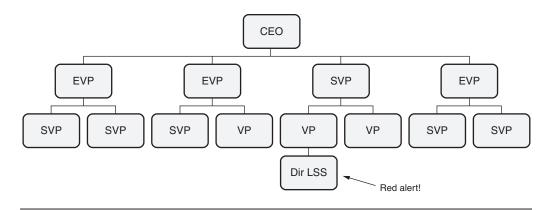
• *Multiple cultures and subcultures.* In Chapter 2, the concept of multiple cultures and subcultures within an organization was introduced. This is highly likely with large, multinational organizations. Furthermore, it is possible to have subcultures within an organization largely due to the mergers and acquisitions craze that has found favor over the last two decades. There is no prescription here on what to do. Each situation is different. Take the situation seriously and address it carefully.

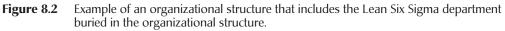
• *Geographical dispersion.* Contemporary organizations can be highly dispersed around the world, connected only through technology. Sites may be many time zones apart, and multiple languages may be involved. (Note: Although English has generally been accepted as the language of business around the world, expect workers at the lowest levels of an organization to be more comfortable with their native tongue.) Consider how support is provided to a site half a world away. How can training materials be translated, and in what language will the training be delivered?

• *Union environment*. Union leadership may influence organization structures and reporting relationships as well as blockage of process improvement initiatives. Operating in a union environment can create additional levels of difficulty. For this reason alone, it would be wise to include someone from the organization on the design team who has experience dealing with the union or unions as the case may be.

• *Customer influence.* Customers who have already started a Lean Six Sigma implementation may apply pressure to establish an organization that parallels their own. Those who haven't might request access to yours. Therefore, you may need to have a customer liaison position inside the department. Similar thinking may be applied to key suppliers as well.

• Organizational placement of the department. The actual placement of the Lean Six Sigma department within the organization is a major consideration that can have long-term consequences on the success of the implementation. If the implementation of Lean Six Sigma is to be seen as a strategic and/or culture-changing initiative, then it must be placed at the highest level of the organization. Similarly, the same applies for all business units, divisions, sites, and so on. Doing otherwise is tantamount to a nonverbal statement that "it really isn't as important as we say it is." Figures 8.2 and 8.3 depict two common examples of organizational structures in play today. Figure 8.2 illustrates the situation where the Lean Six Sigma leader is at director level two levels removed from the CEO and reports to an individual who is not familiar with Lean Six Sigma. Figure 8.3 illustrates the situation where the Lean Six Sigma leader reports to the CEO, but whose title and authority level are grossly inconsistent with others reporting directly to the CEO. This creates an uneven playing field among the CEO's direct reports, making it difficult for the Lean Six Sigma leader's peers to take him or her seriously. Both cases





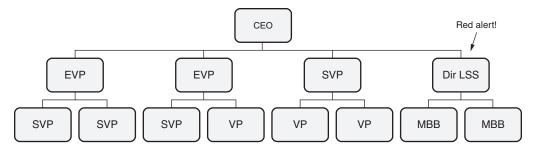


Figure 8.3 Example of an organizational structure that includes the Lean Six Sigma department reporting to the CEO.

suggest "red alert" situations, and indicate that the organization's leadership is just testing the Lean Six Sigma waters.

Traditional Lean Six Sigma Organization

So the question before us now is, "What does a traditional Lean Six Sigma organization look like?" The answer is, "There is no traditional Lean Six Sigma organization structure." They are as varied as there are organizations. As before, what is right is what works. Figure 8.4 illustrates one possible Lean Six Sigma organization structure.

A quick review of Figure 8.4 shows that:

- Master Black Belts are reporting directly to the leader as well as other functional leads, such as Curriculum Development and Training. This structure assumes that both of these functions are specific to Lean Six Sigma and are not a corporate function. If so, they may report in on a dotted-line basis.
- Each Master Black Belt has several Black Belts reporting to them. They will likely be used to run projects at the corporate level and to provide additional implementation support.

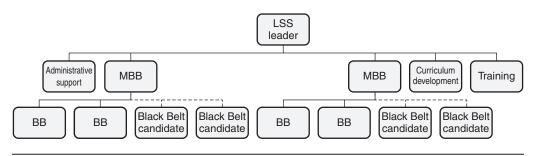


Figure 8.4 One possible Lean Six Sigma department structure.

- Black Belt candidates report to the Master Black Belts. Of course, this is quite controversial and is a key issue that must be determined up front between the Lean Six Sigma department and the functional departments supplying Lean Six Sigma Black Belt candidates. For the purpose of this chart, the candidates report in to the Master Black Belts on a dotted-line basis. Hence, they still report to the functional managers. (Note: Green Belts do not appear on the organizational structure. They reside in their functional departments.)
- Administrative support has been included that reports directly to the Lean Six Sigma leader, but is available to support all members of the department.

COMMUNICATIONS WITH MANAGEMENT

Describe elements of effective communications with management regarding organizational benefits, failures, and lessons learned. (Apply)

Body of Knowledge I.H.3

Elements of Effective Communications

This section will deal only with the elements of effective communications. Other important aspects of communication are addressed in other chapters. The key elements of any effective communication include the following:

• *Clarity of the message.* Is the purpose or central ideal clearly delineated? The audience shouldn't have to guess. Is the message simple or complicated? Is there a single message or are multiple messages to be delivered? If there are multiple messages, is it better to be handled in multiple communications? Is the communication providing

information or seeking approval? Have more questions been raised than answered? If you are requesting action be taken, ensure that those who have actions know it, and when such actions are due and to whom.

- *Knowing your audience.* Is the audience heterogeneous or homogeneous? If heterogeneous, it may be in terms of organizational level, skill, knowledge, interest, and so on. These all have an impact on the design of the communication and the ability of the audience to receive and understand it. Does the audience have any hot or otherwise emotional buttons that might accidentally be pushed? Be aware of these up front. Understand the knowledge base of your audience. Do you need to build a foundation for them before proceeding with the message?
- *Presentation and delivery.* The element depends greatly on the medium used and takes into account many factors such as:
 - Language. Consider the level of language used, acronyms, jargon, slang, active rather than passive voice, and so on. Use of jargon is a frequent cause of audiences losing interest in the subject and tuning out the speaker.
 - *Physical stature and mannerisms.* Such factors in this category would include: stammering, slouching, speaking too fast, not maintaining eye contact, blocking the screen, and so on.
 - Correctness of material. All too often, incorrect spelling, poor formatting, or bad grammar derails a communication and obscures the message. It works to undermine the credibility of the communicator.
 - Organization, structure, and progressive development. Simply, is the communication developing in a logical manner? Has the foundation been established properly and subsequent thoughts built thereupon? Are examples relevant, accurate, and persuasive? Are the conclusions arrived at logically?
- *Medium used.* What type of communication medium will be used? E-mail (personal or blast message), telephone (personal or blast message), face-to-face (one-on-one, small audience, large audience), newsletter, and so on? Select the medium or mediums consistent with the needs of the audience. For example, factory workers will likely have minimal, if any, access to e-mail.
- *Feedback and interaction.* If you are delivering your communication in a face-to-face manner, beware of your audience's reaction and be prepared to adjust your delivery accordingly. Flexibility during the communication activity can mean success or failure. Also, be prepared to assess feedback and interaction from the audience. The audience often participates in the communication process, particularly

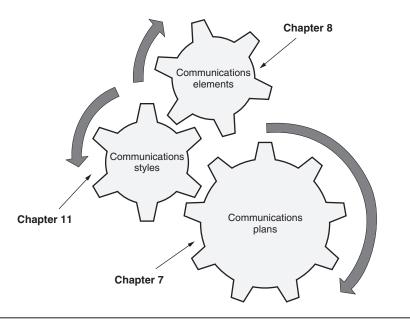


Figure 8.5 The communications trilogy.

during face-to-face communications. Feedback also allows the sender or communicator to determine whether the message was properly sent and understood. If you are delivering a prepared presentation, don't assume the audience will allow you to proceed from start to finish uninterrupted. Expect interruptions and build them into your schedule.

Although this section dealt explicitly with elements of effective communications, it should be evident that these elements are critically integrated with communication styles and communication plans. For this reason, Figure 8.5 was prepared. Note that Figure 8.5 cross-references the chapters in which the other integral parts of communication are addressed.

Communicating with Senior Management

When dealing with the senior management of an organization, additional factors must be taken into consideration when communication takes place. These additional factors include:

- *Creating the picture approach.* Management should understand that what is being communicated or proposed is not being done in isolation, but that all appropriate aspects of the organization were taken into consideration.
- *Identifying the risks and benefits involved.* A thorough analysis in this regard alleviates major concerns management might have.

- *Speak the language of management.* Whenever possible, translate benefits into economic terms. This allows management to more fully understand the impact on the bottom line.
- *Link strategies to goals and objectives.* Demonstrate the linkage. If such a linkage can not be demonstrated clearly, management should question the wisdom of undertaking any proposal.
- *Win support from peers and other senior management.* Don't underestimate the influence of peer pressure in communications, particularly if the peers were privy to the communication or proposal before the delivery.
- *Seek advice from those who have a stake in the outcome.* Individuals who have a stake in the outcome will usually go out of their way to help ensure that your communication is successful. Don't forget about them. You can identify them from a stakeholder analysis.

Keeping these additional factors in mind, along with solid preparation, will greatly improve your communication with management.

Failures and Lessons Learned

Two particularly important aspects of communicating with management deal with conveying failures and lessons learned. When addressing these aspects, keep the following in mind:

- *Failures.* Be factual about the failure. Deal with processes only. Keep people and personalities out of it. If an impact to the bottom line is expected, if a customer is impacted, or if a regulation or law is violated by the failure, let management know. Don't surprise them. Discuss the corrective and preventive actions taken or to be taken. In particular, address what process changes are required and who will be responsible for ensuring the changes are made and when. Advise them that the proper documentation is being updated.
- Lessons learned. Lessons learned can be accumulated throughout any project or activity, but a formal activity of identifying lessons learned typically occurs at the end. Lessons learned may reflect both positive and negative learnings. Like failures, these should be factual and related only to processes. The idea behind identifying lessons learned is to gather and communicate key knowledge and understanding systematically such that future projects and other endeavors benefit. Management will want to be assured this will take place. Furthermore, it is important to communicate that a lessons learned analysis would likely generate actions now when knowledge is fresh.

Communicating failures and lessons learned may, at times, be painful. However, consider using them as a springboard for generating additional improvement activities.

CHANGE MANAGEMENT

Describe the MBB role in change management and apply various techniques to overcome barriers to successful organizational deployment. (Apply)

Body of Knowledge I.H.4

Facilitating or Managing Organizational Change

Change is inevitable. Organizations that continually improve their processes will have a greater probability of success than those who only react to problems. As an organization evolves, there are not only incremental changes but also increasingly major shifts in strategy, technology, and work organizations. Change can occur because of either outside forces or inside forces.

However, fear of change is also a real and valid concern. People are afraid of change because of its potential impact on them. Corporate downsizing causes major disruptions of people's work and personal lives, and continual improvement efforts are sometimes blamed for job losses.

Change management is a process for ensuring that the people affected by change understand the nature of the change and the reasons for it, with the expectation that the new methods of operating will be internalized without creating undue resistance, conflict, and fear. To reduce fear, it's important that the vision of the future be well communicated and that jobs be protected when feasible. Ongoing and open communication during any change process is paramount. Although these will not totally remove fear, they can remove some of the uncertainty of not knowing the direction in which the organization is headed (adapted from Westcott 2006).

Change Agents

In Chapter 7, we discussed the Master Black Belt in the role of the "change agent" and defined them as "individuals who play a specific role in the planning and implementation of the change management process. They may be members of the organization or may be outsiders..." according to Hutton (1994). Collaboration of an internal change agent with an external change agent who has extensive experience in the type of change to be implemented can be a useful strategy.

An internal change agent is a person within the organization usually designated by management to facilitate a particular change effort. Internal change agents possess an understanding of the organization's culture, infrastructure, and the business, and have a stake in seeing change efforts succeed. However, political pressures can prevent them from providing objective feedback when problems arise. Also, they may lack perspective of the big picture or have a stake in preserving certain traditions, which keeps them from seeing opportunities for improvement. A staff or line person, depending on the type and magnitude of the change being implemented, may fill the role of the internal change agent.

An external change agent is a person from outside the organization who has been hired to advise and help facilitate the change process. An external change agent has a greater degree of freedom and should be better able to objectively assess activities and provide honest feedback to senior management without fear of repercussion. Also, organizational members are less likely to have previous experiences with the change agent that might influence their effectiveness, and the agent does not have a stake in preserving long-held organizational traditions. Of course, the danger is that organizations can become so dependent on an external change agent that the change process is adversely affected when the agent leaves. Another disadvantage of external change agents is their lack of familiarity with the specific organizational culture.

External change agents must work diligently to build a relationship with the client organization. This includes becoming familiar with organizational norms, shared beliefs, values, and behaviors, as well as understanding both formal and informal leadership structures. For most organizations, the change agent needs to become acquainted with persons who serve as informal leaders and to whom others turn for new ideas. Building a relationship among informal leaders can be beneficial because other members of the organization will check with them for affirmation that it is beneficial or safe to support the change process (adapted from Westcott 2006).

Deming emphasized the role of external change agents in his view of organizational transformation by stating, "A system can not understand itself. The transformation requires a view from outside."

Techniques and Roles of the Master Black Belt

Change agents may assume several roles and use a variety of techniques, such as (Westcott 2006):

- Coaching top management to:
 - Create an environment in which change can take place with minimum resistance
 - Develop and support an improvement plan
 - Provide the resources to implement the plan
- Supporting and advising management colleagues on how to:
 - Deal with technical issues
 - Cope with intellectual and emotional resistance
 - Measure, monitor, and report progress
 - Handle behavioral issues

- Provide performance feedback, including reinforcement of top management for the decision they made and reinforcing the work of those implementing the change
- Use the change agent as a facilitator when needed
- Managing a specific project or segment of a large project to:
 - Fill in where no other suitable resource person is available
 - Serve as a role model for other project management efforts
- Guiding the development of a network to:
 - Support the implementation of the change
 - Deploy the principles and practices for managing change throughout the organization
- Guiding the assessment of the results of closure of the change, including:
 - Review of lessons learned
 - Evaluating the economic case for the change
 - Documenting the change

As a change agent, the Master Black Belt will serve in varying roles and wear many different hats. He or she will be expected to plan, execute, coach, role model, facilitate, and counsel in addition to being highly competent in the technical aspects of Lean Six Sigma.

Chapter 9

Organizational Finance and Business Performance Metrics

rganizations have come under severe financial scrutiny in recent years due to the Sarbanes–Oxley Act. Consequently, it is important that Master Black Belts have a solid understanding of the various financial measures that may be used to cost-justify projects, and higher-level business performance metrics that may weigh upon project selection.

FINANCIAL MEASURES

Define and use financial measures, including revenue growth, market share, margin, cost of quality (COQ), net present value (NPV), return on investment (ROI), cost-benefit analysis, activity-based cost analysis, and breakeven time performance, etc. (Analyze)

Body of Knowledge I.I.1

When preparing a project proposal or report, it is generally expected to state the project's benefits in financial terms. As such, it is often necessary to conduct a detailed financial analysis. This section will address some of the key tools needed to support such an analysis and will discuss some of the advantages and disadvantages of each. Remember, as with any analysis, it is critical to enumerate all assumptions on which it is based since it may be necessary to revisit them from time to time. Also, whenever possible, work with the organization's financial department. This helps to instill confidence and credibility into the numbers.

Furthermore, some benefits can not be stated financially. For example, improved morale and improved team-building skills. However, these benefits are important as they may be used to break ties between projects with equal financial benefits. Therefore, they should be identified in the report, but recognize that the financial aspects usually dominate the benefit section of the report.

Common Financial Measures

Revenue growth is the projected increase in income that will result from the project. This is calculated as the increase in gross income minus the cost. Revenue growth may be stated in dollars per year or as a percentage per year.

An organization's *market share* of a particular product or service is that percentage of the dollar value that is sold relative to the total dollar value sold by all organizations in a given market. A project's goal may be to increase this percentage. During a market slowdown, market share is often watched closely because an increase in market share, even with a drop in sales, can produce an increase in sales when the slowdown ends.

Conceptually, *margin* refers to the difference between income and cost. However, there are multiple financial concepts of margin, each of which is used to measure the efficiency and effectiveness of an organization's financial capabilities and finesse:

• *Percent profit margin.* This metric provides an overall sense of how efficiently an organization converts sales into profits. It is computed as follows:

Percent profit margin

$$= \left(\frac{\text{Sales of a product or service} - \text{Cost of a product or service}}{\text{Sales of a product or service}}\right) (100)$$

$$= \left(\frac{\text{Net profit}}{\text{Net sales}}\right) (100)$$

• *Percent gross margin.* This metric measures the efficiency of producing sales. It is computed as follows:

Percent gross margin =
$$\left(\frac{\text{Gross profit}}{\text{Net sales}}\right)(100)$$

• *Percent operating margin*. This metric measures how effectively an organization controls costs not related to the production and sales of a product or service. It is computed as:

Percent operating margin =
$$\left(\frac{\text{Operating profit}}{\text{Net sales}}\right)(100)$$

• *Percent net margin*. This metric measures the effectiveness of an organization at generating net profit from sales. It is computed as:

Percent net margin =
$$\left(\frac{\text{Net profit}}{\text{Net sales}}\right)(100)$$

Notice it is computed the same as the percent profit margin. Generally, when we speak of percent profit margin, we are speaking of the percent net margin.

To better understand some of the above equations, we must define some of the component terms such as:

- Net sales = Revenue (from sales)
- Gross profit = Net sales Cost of goods sold
- Cost of goods sold = Cost of raw goods + Cost of labor + Cost of inventory
- Net profit = Net income
 - = Net earnings
 - = Bottom line
 - = Total revenue Total expenses
- Operating profit = Operating income = Earnings = Gross profit Operating expenses

Cost of Quality

The standard categories of quality costs are as follows:

- Appraisal costs. Expenses involved in the inspection process
- *Prevention costs.* Costs of all activities whose purpose is to prevent failures
- Internal failure costs. Costs incurred when a failure occurs in-house
- *External failure costs.* Costs incurred when a failure occurs when the customer owns the product

The total cost of quality is the sum of these four amounts. Cost-of-quality calculations have been successfully used to justify proposed improvements. However, it is essential that representatives from the accounting function be involved to ensure that appropriate conventions are used in such proposals.

It is the bias of most quality professionals that if businesses spent more on prevention, they could reduce the total cost of quality because failure costs and, in many cases, appraisal costs could be reduced.

The traditional way to sketch the relationship between quality costs and the quality level is illustrated in Figure 9.1. It indicates that as the quality level approaches 100%, appraisal and prevention costs approach infinity. The optimum total quality cost level is where the total quality cost curve is minimized. This optimum point is located directly above the intersection of the appraisal/ prevention and failure curves.

Figure 9.2 demonstrates a different path of thought with regard to quality costs. It illustrates that if it were possible to achieve perfect quality (that is, 100% quality level), the total quality cost curve would be minimized, with finite appraisal/prevention and failure costs.

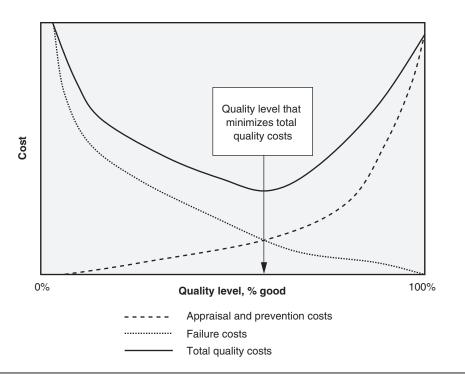


Figure 9.1 Traditional quality cost curves.

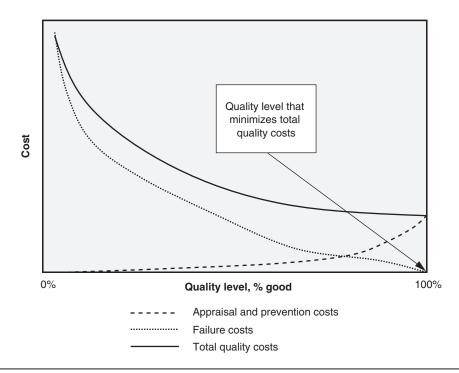


Figure 9.2 Modern quality cost curves.

Net Present Value

Discounted cash flow techniques were developed to take into account the time value of money and to improve the accuracy of cash/capital project evaluations. One such method, net present value (NPV), consists of:

- Finding the present value of each cash flow, discounted at an appropriate percentage rate (determined by the cost of capital to the organization and the projected risk of the flow—the higher the risk, the higher the discount rate required).
- Computing the project's NPV by adding the discounted net cash flows.
- Determining if the project is a candidate for acceptance, when the NPV is positive. If two projects are positive, the one with a higher NPV is a more likely choice.

EXAMPLE 9.1

In four years, \$2500 will be available. What is the NPV of that money, assuming an annual interest rate of 8%?

 $P = 2500(1+0.08)^{-4}$ P = 1837.57

Therefore, \$1837.57 invested at 8% compounded annually will be worth \$2500 after four years.

Suppose a project requires an investment of \$1837.57 and will return \$3000 in four years. Using the NPV approach, the gain is \$500, assuming a compound annual interest rate of 8% per year. The interest rate is usually set by company policy on the basis of the cost of money.

EXAMPLE 9.2

Suppose a project requiring an initial investment of \$4000 pays \$1000 every six months for two years beginning one year from the date of the investment. What are the NPV and the gain?

| Net present value of the first payment | $= NPV_1 = 1000(1 + 0.08)^{-1}$ | = 925.93 |
|-----------------------------------------|-------------------------------------|-----------|
| Net present value of the second payment | $t = NPV_2 = 1000(1 + 0.08)^{-1.5}$ | = 820.97 |
| Net present value of the third payment | $= NPV_3 = 1000(1 + 0.08)^{-2}$ | = 857.34 |
| Net present value of the fourth payment | $= NPV_4 = 1000(1 + 0.08)^{-2.5}$ | = 824.97 |
| Net present value of the fifth payment | $= NPV_5 = 1000(1 + 0.08)^{-3}$ | = 793.83 |
| Total net present value of the income | | = 4293.04 |

Since the initial investment is \$4000, the gain is about \$293.04. Note: This calculation assumes a lump sum payback at the end of four years.

The NPV of an amount to be received in the future is given by the formula

$$P = F(1+i)^{-n}$$

where

P = Net present value

F = Amount to be received n years from now

i = Annual interest rate expressed as a decimal

This formula assumes the money is compounded annually.

Return on Investment

One of the most easily understood financial numbers is the *return on investment* (ROI):

$$ROI = \frac{Income}{Cost}(100\%)$$

where *income* includes money that will be made due to the project dollars saved, and costs avoided, and *cost* is the amount of money required to implement the project.

In basic terms, the ROI metric measures the effectiveness of an organization's ability to use its resources to generate income.

Return on investment is used both as an estimating tool to determine the potential return on an investment contemplated as well as a measure of actual payoff upon project completion. ROI tends to be more useful for evaluating the payoff potential of shorter-term projects inasmuch as it typically ignores the time value of money.

Payback Period

The *payback period* (also known as the *payout period*) is the number of units it will take the results of a project or capital investment to recover the investment from net cash flows. (Units may refer to the amount of product, passage of time, or other meaningful component of measure.) The first formal method developed to evaluate capital budgeting projects, the payback period is easily determined by accumulating the project's net cash flows to locate the point where they reach zero. The number of units associated with this zero point represents the payback period.

The payback period method provides a measure of *project liquidity*, or the speed with which cash invested in the project will be returned. Organizations short of cash may place greater value on projects with a high degree of liquidity. The payback period is often considered an indicator of the relative risk of projects inasmuch as managers generally can forecast near-term events better than those more distant. Overall, projects with relatively rapid returns are generally less risky than longer-term projects.

However, Grant et al. (1976) note that "Except for the special case where funds are so limited that no outlay can be made unless the money can be recovered in an extremely short time, the payout period is never an appropriate way to compare a group of proposed investments." They go on to state, "Sometimes the payout technique is combined with the stipulation that no proposal will be accepted unless it has an extremely short payout period, such as one year or two years. Such a stipulation, if rigidly adhered to, tends to block the approval of projects that would earn excellent returns."

Internal Rate of Return

The *internal rate of return* (IRR) is the discount rate that causes the sum of the NPV of the costs (negative cash flows) plus the sum of the NPV of the savings (positive cash flows) to equal zero. This is often expressed as the rate that causes the difference between the NPV of the costs and the NPV of the savings to equal zero. If the IRR is greater than the minimum required by the organization (for example, cost of capital or alternative investment options), the project is likely to be accepted.

When projects are mutually exclusive, the IRR should be used to make a decision against a single project, not to compare projects. For example, consider two projects A and B. Project A may have a lower IRR than project B, but a higher NPV (shareholder wealth). In this case, project A would be selected over project B, assuming there are no capital constraints.

Mathematically, we define the IRR as:

$$NPV = \sum_{n=0}^{N} \frac{CF_{n}}{(1+r)^{n}} = 0$$

where

N = Number of periods (units)

CF = Cash flow

r = Internal rate of return

NPV = Net present value

Although only the most common form of the IRR equation (that is, single outflow, multiple inflows) was presented, several other variations are available that

EXAMPLE 9.3

Consider the data given in Table 9.1. Using the first two columns, compute the IRR. The Excel built-in IRR function is perfect for this computation. Using it and starting with an initial default guess of 10%, we can compute the IRR to be 4.295%. This result is easily confirmed using columns 3 through 6 of Table 9.1. Notice the

total NPV is (\$0.04) instead of zero. This difference is due to rounding.

| Table 7.1 Internal rate of return calculation for Example 7.5. | | | | | |
|----------------------------------------------------------------|-----------|---------|---------|------------------|-----------|
| Year | Cash flow | IRR | 1+IRR | (1+IRR) <i>"</i> | NPV |
| 0 | (\$5000) | 0.04295 | 1.04295 | 1 | (\$5000) |
| 1 | \$1200 | 0.04295 | 1.04295 | 1.04295 | \$1150.58 |
| 2 | \$1450 | 0.04295 | 1.04295 | 1.087744703 | \$1333.03 |
| 3 | \$1800 | 0.04295 | 1.04295 | 1.134463337 | \$1586.65 |
| 4 | \$1100 | 0.04295 | 1.04295 | 1.183188538 | \$929.69 |
| | | | | Total | (\$0.04) |

| Tahle 91 | Internal rate of return | calculation | for Example 93 |
|----------|-------------------------|-------------|-------------------|
| | | calculation | IOI LAGINDIC 7.5. |

depend on the situation at hand. The reader is encouraged to engage other and more thorough sources on this topic when faced with complex situations.

Cost–Benefit Analysis

Top management must approve funding for projects that require significant financial resources. Therefore, before a proposed project is initiated, a cost-benefit analysis using projected revenues, costs, and net cash flows is performed to determine the project's financial feasibility. This analysis is based on estimates of the project's benefits and costs and the timing thereof. The results of the analysis are a key factor in deciding which projects will be funded. Upon project completion, accounting data are analyzed to verify the project's financial impact on the organization. In other words, management must confirm whether the project added to or detracted from the company's financial well-being.

In today's competitive global environment, resource allocation is complicated by the fact that business needs and opportunities are greater and that improvements are often more difficult to achieve. Many of the easier projects have already been done. Therefore, top management requires that project benefits and costs be evaluated so that projects can be correlated to overall revenues, costs, customer satisfaction, market share, and other criteria. These factors are analyzed to maximize business returns and to limit risks, costs, and exposures. Therefore, quality improvement projects are considered business investments—as are all cash or capital investments—in which benefits must exceed costs. Quality managers are additionally challenged in that many of these benefits and costs are not easily quantifiable, as is normally expected by executives, accountants, and financial officers.

A *cost–benefit* (also known as a *benefit–cost*) *analysis* attempts to evaluate the benefits and costs associated with a project or initiative. Two types of benefits can be evaluated: direct or indirect. Direct benefits are easily measured and include issues such as increased production, higher quality, increased sales, reduced delivery costs, higher reliability, decreased deficiencies, and lower warranty costs. Indirect benefits are more difficult to measure and include such items as improved quality of work life, increased internal customer satisfaction, and

better-trained employees. Similarly, two types of costs can be evaluated. Direct costs include equipment, labor, training, machinery, and salaries. Indirect costs include downtime, opportunity costs, pollution, and displaced workers.

The simple formula for computing a benefit–cost (occasionally referred to a as cost–benefit, which is the reciprocal) ratio is:

$$\frac{\sum NPV \text{ of all benefits anticipated}}{\sum NPV \text{ of all costs anticipated}}$$

If the formula above yields a value greater than one, the project is generally worth pursuing, but it may not be as attractive as other projects.

Typically, a cost–benefit analysis is conducted on the basis of direct costs and direct benefits to:

- Assist in the performance of a what-if analysis
- Compare the benefits, costs, and risks of competing projects
- Allocate resources among multiple projects
- Justify expenditures for equipment, products, or people

Because many indirect benefits and costs are difficult to quantify, and time is needed to identify such costs, some worthwhile quality projects or initiatives could fail to obtain the necessary funding.

Quality professionals are expected to help their organizations profit from quality investments, so no project should receive automatic approval and resource allocation. To be competitive, companies must find ways to increase profitability while providing customers with high levels of satisfaction. The challenge is to find the link between critical business performance measurements, such as profitability, and quality investments, for improvement projects, programs, and initiatives. Although some quality benefits and costs are difficult to measure or are unknowable, the quality manager still must help determine and measure quality's effect on the organization's finances.

EXAMPLE 9.4

The NPV costs to conduct a project are estimated to be \$90,000. The NPV benefits or savings due to the project are estimated at \$855,000. Compute the benefit-to-cost ratio. Using the above formula, we have:

$$\frac{\sum NPV \text{ of all benefits anticipated}}{\sum NPV \text{ of all costs anticipated}} = \frac{\$855,000}{\$90,000} = 9.5$$

This means \$9.50 in benefits for every dollar expended.

Activity-Based Cost Analysis

Four cost allocation methods commonly used by organizations are:

- *Undifferentiated costs.* This method considers those costs that are known or assumed associated with producing the product or rendering the service. There is no categorization by cause or type of product or service.
- *Categorized costs (cost of quality).* Such costs may be aligned to the cost of quality categories: appraisal, prevention, and failure.
- *Traditional cost model.* Costs allocated by a percentage of direct labor dollars, or some other arbitrary factor such as amount of space occupied, number of personnel in a work unit, and so on.
- *Activity-based costing (ABC).* The costs of materials and services are allocated by the activity or process (cost drivers) by which they are consumed.

Since the undifferentiated cost method provides little confidence in actual product costs, we will not address it further. Likewise, the cost of quality method will not be discussed since it was addressed earlier in this chapter. Therefore, we will elaborate on the two remaining methods: traditional cost and the activity-based costing models.

The traditional model for allocating costs to a product is given by:

Product cost = Direct material costs + Direct labor costs + Overhead costs

Traditional cost accounting practice involves computing the sum of all costs from the work unit accounts that have been identified as a part of the direct cost of goods, then adding to the sum the indirect (overhead) expenses (for example, utilities, space allocation dollars, cleaning and maintenance).

This model worked well for allocating overhead costs when, say, a single base such as direct labor accounted for a significantly large proportion of the product cost and overhead a smaller percentage. As technology entered the workforce, this percent allocation reversed, with a much smaller proportion of direct labor and overhead much larger. With such cost structure changes, many organizations have found their product costs now askew.

In some cases, organizations choose a cost base other than direct labor. For example, space allocation is also a common choice. However, it is possible that the causal relationship that existed between direct labor hours and the product cost will be diminished, as space requirements may be less of a cost driver than direct labor hours. This will lessen the credibility of the costing method considerably.

In contrast, *activity-based costing* allocates overhead expenses to the activities based on the proportion of use, rather than proportion of costs. It does this by identifying all cost drivers (for example, machine setups, purchase orders, shipments, maintenance requests) and the number of driver occurrences or events (for example, number of setups). The results of this exercise are costs/activity or service: \$/setup, \$/purchase order, \$/shipment, and \$/maintenance request. Consequently, it is now possible to allocate costs based on use. A product requiring a setup is charged for a setup. Similarly, a product that does not require a setup is not charged for one.

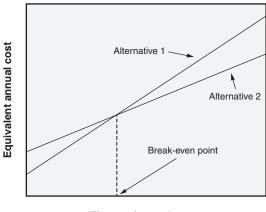
The ABC approach provides a much more accurate depiction of what it really costs to produce each type of product or service. Some organizations that have used ABC have curtailed or discontinued production of some of their products once the real cost to manufacture became known. They hadn't realized the extent to which such products were "eating" their profits.

Break-Even Time Performance

The *break-even point analysis* is a useful approach when there are multiple alternatives each of which may be economical under its own set of conditions. The break-even point is determined by altering one variable while holding all others constant. The variable generally represents some passage of time or number of units processed and can be calculated or determined graphically to identify where the alternatives are equal from an economical viewpoint. This point of equality is known as the *break-even point*.

To determine the break-even point, it is essential that all appropriate costs for each alternative be collected or estimated and normalized to the same time frame (for example, year). Such costs include: initial start-up costs, annual investment costs, maintenance costs, power costs, replacement costs, estimated life times, projected utilizations, and so on. Consider this list a starting point since each breakeven point analysis will be different. These costs will be used to determine the equivalent annual cost (that is, future costs that will need to be annualized). The determination of these costs may be somewhat complicated and involve the use of engineering economy theory. Therefore, you may want to consult a text on engineering economy.

The cost examples just mentioned are used to create cost projection lines similar to those shown in Figure 9.3. The break-even point occurs on the *x*-axis at the



Time, units, and so on

Figure 9.3 A graphical approach to the break-even point.

point of intersection of the lines. From Figure 9.3, we can determine that alternative 2 is more cost-effective to the right of the break-even point while alternative 1 is more cost-effective to the left.

Additionally, the break-even point may be calculated by: establishing the equivalent annual cost equations for each alternative, setting the alternatives equal to each other, and solving for the unknown variable. In actuality, the equivalent annual cost equations for each alternative must be developed just to plot the graphs. Solving for the unknown variable is simply another algebraic step.

BUSINESS PERFORMANCE MEASURES

Describe various business performance measures, including balanced scorecard, key performance indicators (KPIs), and the financial impact of customer loyalty; and describe how they are used for project selection, deployment, and management. (Analyze)

Body of Knowledge I.I.2

Many of the Lean Six Sigma tools deal with analyzing numerical values. The metrics the organization chooses reflects its business philosophy and, largely, determines its success. Unfortunately, many organizations use too many metrics, often lagging in nature, which may drown them in data and cause them to be unable to change with customer needs. This section lists some tools that will help in the selection and use of business performance measures.

Balanced Scorecard

The *balanced scorecard*, a term coined by Robert S. Kaplan and David P. Norton in the early 1990s, separates organizational metrics into four perspectives:

- *Financial perspective.* This provides the organization with insight into how well its goals, objectives, and strategies are contributing to shareholder value and the bottom line. It provides shareholders with a direct line of sight into the health and well-being of the organization. Common metrics used for this perspective are given in the next section.
- *Customer perspective.* This perspective defines an organization's value proposition and measures how effective the organization is in creating value for its customers through its goals, objectives, strategies, and processes. The customer perspective is not easily obtained. Multiple approaches and listening posts are frequently required, which must be compared and reconciled to ensure a correct understanding of the customer's stated and unstated needs, expectations of the organization's products and services, and the strength of its customer relationships.

- *Internal business processes perspective.* Internal business processes includes all organizational processes designed to create and deliver the customer's value proposition. Organizations that perform these processes well generally have high levels of customer satisfaction and strong customer relationships since efficient and effective internal processes can be considered predictors of the metrics included in the customer perspective.
- *Learning and growth perspective.* Generally, this perspective includes the capabilities and skills of an organization and how it is focused and channeled to support the internal processes used to create customer value. The development and effective use of human resources and information systems, as well as the organizational culture, are key aspects of this perspective.

Key Performance Indicators

Key performance indicators (KPIs) are both financial and nonfinancial metrics that reflect an organization's *key business drivers* (KBDs), also known as *critical success factors* (CSFs). Organizations will benefit from the use of KPIs that are defined with the following criteria in mind:

- Quantitative and measurable
- Goal based
- Strategy based
- Time bounded

Another way of defining KPIs is in terms of the SMART acronym:

- Specific. KPIs should be laser focused and process based.
- *M*easurable. KPIs must be quantitative and easily determined.
- Achievable. KPIs may be set with regard to benchmark levels, yet remain obtainable. KPIs set at higher levels adversely impact employee morale and subsequently organizational performance.
- *R*elevant or *R*esults based. KPIs should be linked to the organization's strategies, goals, and objectives.
- *T*ime bounded. KPI levels should reflect a specific period of time; they should never be open-ended. Time-bounded KPIs can be measured. When an appropriate time frame is chosen, KPIs can create a sense of urgency and employee focus.

Some organizations have developed and embedded their KPIs within the framework of the balanced scorecard. Common examples include the following:

- Financial
 - Return on investment

- Return on capital
- Return on equity
- Economic value added
- Customer
 - Customer satisfaction levels
 - Retention rates
 - Referral rates
 - Quality
 - On-time delivery rates
- Internal business processes
 - Defect rates
 - Cycle time
 - Throughput rates
 - Quality
 - On-time delivery rates
- Learning and growth
 - Employee satisfaction
 - Employee turnover rate
 - Absentee rate
 - Percentage of internal promotions

These examples should be considered a starter set and subjected to the criteria identified earlier to ensure their appropriateness. Also, note that the metrics in these perspectives are not necessarily mutually exclusive and may very well overlap. For example, quality and on-time delivery rates appear in both the customer and internal business process perspectives. Customers judge an organization by these metrics and make determinations regarding the continuance of a business relationship with a supplier. On the other hand, these same metrics may be used by a supplier as a predictor of the customer satisfaction levels that might be achieved because of its internal business processes.

Organizational metrics do not end with KPIs. Lower-level metrics down to the individual process level may be required to support KPIs and to provide deep, meaningful, and actionable insight for organizations to effectively conduct and run their businesses.

Financial Impact of Customer Loyalty

Customer loyalty is a term used to describe the behavior of customers—in particular, customers who exhibit a high level of satisfaction, conduct repeat business,

or provide referrals and testimonials. It results from an organization's processes, practices, and efforts designed to deliver its services or products in ways that drive such behavior.

Many organizations recognize that it is far easier and less costly to retain customers than attract new ones. Hence, customer loyalty drives customer retention, and by extension, has a financial impact. However, customer loyalty can be a double-edged sword. Some organizations strive blindly to retain all their customers, including those that are unprofitable. Chapter 15 of *The Certified Six Sigma Black Belt Handbook* discusses the concept of customer segmentation. In one context of segmentation, customers may be classified as profitable or unprofitable. Conceptually, the idea of unprofitable customers is difficult to understand. Moreover, severing ties with this customer segment may be even more difficult, as it may be seen as highly unusual or simply "not done." Enlightened organizations understand this distinction well and act accordingly. Such organizations have gained insight into the cost of maintaining customer relationships through extensive data collection and analysis. This insight has given rise to such terms as "relationship costs" and "relationship revenue."

Organizations must be careful not to confuse loyal customers with tolerant customers. Customers have a zone of tolerance whereby a single bad experience or even a series of bad experiences spread over a sufficiently long time frame may strain the customer–supplier relationship yet keep it intact. However, the cumulative effects of such experiences may push a customer beyond his or her zone of tolerance, resulting in customer defection. At this point, a customer may deem that it is no longer worth maintaining the business relationship. Consider your own banking relationship for the moment. How many times would it take for your bank to provide bad service before you are willing to forsake convenience and take your business elsewhere?

Organizations, especially those in a niche market or those that may be a single/ sole source provider, should have a deep understanding of what drives their customer satisfaction levels and relationships. Limited competition or the cost of changing suppliers or service providers may be all that is keeping customers loyal. Given the right opportunity, these "loyal" customers might readily defect.

Using Metrics for Project Selection, Deployment, and Management

Several methods have been discussed to aid in the selection of projects. These methods are financially based, with each possessing a set of advantages and disadvantages. Typically, an organization will select one method and apply it to all of its projects under consideration. Obviously, the best method is the one that selects projects that will maximize an organization's value. Unfortunately, these methods can lead to different conclusions. Recall the project A versus project B discussion using the IRR and NPV methods earlier in this chapter.

If a project selection method is to lead consistently to making correct capital budgeting decisions, the method must:

- Include cash flows for the entire life of the project
- Address the time value of money

- Ensure projects that are mutually exclusive
- Select projects that maximize the organization's value or wealth.

From the foregoing discussions, we have learned that the payback method fails to account for cash flow and the time value of money. Likewise, the ROI ignores cash flow and the time value of money, thereby rendering this method useful for only rudimentary analysis or smaller, short-term projects. Finally, both the NPV and the IRR provide appropriate financial accept/reject decisions for independent projects, but only the NPV method allows for project selection that maximizes shareholder value under all conditions.

Once a project is selected and placed into the pipeline, it may remain there for an extended period of time before being deployed. During this dormant period, the underlying assumptions on which the financial analysis was based will likely change. In fact, the greater the number of assumptions, the greater the likelihood that one or more assumptions will become invalid. After all, cost estimates are rarely static. Consequently, the financial rationale for selecting a project and placing it into the pipeline may no longer be true at time of assignment.

Therefore, projects residing in the pipeline will need to be validated periodically and just prior to assignment. This validation process may result in eliminating the project from the pipeline, retaining it in the pipeline, or expediting its assignment based on whether the updated financial analysis projects a lower return, similar return, or higher return, respectively.

Once assigned, a project takes time to complete. Just as in the pipeline phase, it will be necessary to revisit the financial justification for the project. Should the updated analysis indicate a lower than acceptable return, the organization must decide whether or not to continue with the project. Generally, if the return is lower than some prespecified threshold, the decision should be straightforward. Unfortunately, some organizations are reluctant to stop a project and redirect its resources as this is viewed as a "mistake" instead of the prudent and judicious execution of management's duty.

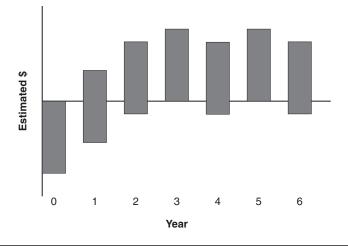
PROJECT CASH FLOW

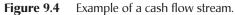
Develop a project cash flow stream. Describe the relation of time to cash flow and difficulties in forecasting cash flow. (Analyze)

Body of Knowledge I.I.3

Figure 9.4 illustrates an example of a project's cash flow stream over a six-year time period. As you review this figure, consider the following general points about cash flow streams:

• The *y*-axis represents estimated dollars. Dollars below the *x*-axis represent cash outflows, while those above are cash inflows.





- As you might expect, nearer-term estimates are typically more accurate than out-year estimates.
- The cash flow ignores the time value of money.
- Cash outflows or inflows may be paid or received at the beginning or end of each year (or per time period). This needs to be specified in advance.

Let's briefly examine Figure 9.4 more specifically:

- *Year 0.* There is a large cash outflow. This is likely start-up or investment cost.
- *Year 1.* The cash outflow has been reduced dramatically, and the cash inflows or returns are being received for the first time.
- *Years 2 through 6.* The project appears to be operational, with stable cash outflows and inflows.
- *Years 2, 4, and 6.* Note the cash outflows in these years. Given they are all the same, these are probably replacement estimates or some otherwise recurring investment costs.

Although the project cash flow stream was depicted graphically in Figure 9.4, it could have easily been shown in tabular form, as illustrated in Table 9.2. Either way, the financial measures discussed in the previous section can be applied.

Based on the foregoing discussions, it should be readily apparent that forecasting cash flow could be a difficult endeavor. The most common complicating factors include: estimating costs and savings in distant future time frames, invalidation of assumptions, and, of course, the inevitable introduction of unforeseen circumstances.

However, don't let this dissuade you from attempting to develop a meaningful cash flow stream projection. Initially, your estimates may be crude, but they

| Year | Cost | Benefit | Net cash flow | Cumulative investment | Cumulative benefits | ROI ratio | Cumulative net cash flow |
|--------|------------|-----------|------------------|--------------------------|------------------------|--------------|--------------------------------|
| 1 | (\$15,000) | \$800 | (\$14,200) | \$15,000 | \$800 | 0.05 | (\$14,200) |
| 2 | (\$12,200) | \$5,600 | (\$6,600) | \$27,200 | \$6,400 | 0.24 | (\$20,800) |
| 3 | (\$4,000) | \$45,000 | \$41,000 | \$31,200 | \$51,400 | 1.65 | \$20,200 |
| 4 | (\$4,000) | \$61,000 | \$57,000 | \$35,200 | \$112,400 | 3.19 | \$77,200 |
| Totals | (\$35,200) | \$112,400 | \$77,200 | | | | |

will become more refined over time as the project progresses and further insight is gained. Always remember to keep your project cash flow stream up-to-date.

SARBANES-OXLEY ACT

Understand the requirements for financial controls dictated by SOX. (Understand)

Body of Knowledge I.I.4

This section will address the Sarbanes–Oxley Act. The intent is to provide:

- A brief background of the Act
- Describe how the Act relates to Lean Six Sigma

This section will not debate the wisdom or the effectiveness of the Act.

Background

The *Sarbanes-Oxley Act of 2002* (SOX), enacted after a series of financial scandals, mandates strict requirements for financial accounting and for top management's responsibility and accountability in disclosing the financial status of their organization.

The Act was signed into law by then President George W. Bush on July 30, 2002. The purpose of the Act was to:

- Ensure the transparency and reliability of publicly reported financial information
- Increase confidence in the U.S. capital markets
- Increase duties and penalties for the following:

- Corporate boards
- Executives
- Auditors
- Attorneys
- Security analysts

What is particularly interesting about the Sarbanes–Oxley Act is that it was the government's response to the behavior of one company—Enron. Furthermore, each section of the Act parallels a specific offense at Enron. However, while the legislation was being drafted, WorldCom collapsed into bankruptcy. Consequently, the law was quickly passed and signed by the president five days after the WorldCom incident. Unfortunately, due to the swift reaction of Congress, the law did not address any of the fraudulent events that took place at WorldCom or any other company thereafter.

In short, SOX has four objectives to achieve:

- *Hold management accountable.* Management may be required to return bonuses or be subjected to fines and/or jail time.
- *Increase public disclosure.* Reporting requirements have increased significantly, with a focus on transparency. Examples of where reporting is now required include:
 - Identifying successive resignations of key management personnel
 - More-detailed reporting to uncover inaccurate and unreliable financial information
 - Identifying CEO stock sales during a blackout period
 - Identifying CEO stock sales in general
 - Identifying off-balance sheet transactions that are designed to hide losses
 - Premature or unauthorized document destruction
 - Prevention of rigging the company's financial ratings
- *Increase regular reviews by the Securities and Exchange Commission (SEC).* The SEC is to review companies with greater frequency and depth. Consequently, for many companies this will translate into an increase in internal controls.
- *Hold accountants accountable.* Emphasis was placed on eliminating conflicts of interest; these individuals may be subjected to fines and/or jail time.

Where Lean Six Sigma Meets Sarbanes-Oxley

A key requirement of Sarbanes–Oxley is that of internal control over financial reporting, which is covered under Section 404 of the Act. Further, the SEC has established rules that such internal controls satisfy the following three functions:

- *Recordkeeping.* This function addresses the processes associated with transactions and dispositions of a company's assets. It requires that a sufficient (that is, reasonable) level of detail be kept that is accurate and fair.
- *Compliance.* This function addresses the processes that ensure transactions are actually recorded and receipts and expenditures are accurate and made only when authorized by management. Also, financial statements must be prepared according to the generally accepted accounting principles (GAAP).
- *Prevention and detection.* This function addresses the processes that ensure the prevention and/or detection of unauthorized use or disposition of company assets.

Notice that in each of the three preceding bullets, processes are involved. In fact, it is fair to say that Sarbanes–Oxley is all about processes, not just financial processes, but those processes that provide input to financial processes as well.

From the above discussion, it should be clear that Lean Six Sigma and Sarbanes-Oxley relate to each other through two central ideas:

- *Processes.* For Sarbanes–Oxley to be successful, processes need to be inventoried, mapped, and improved. Simplified processes facilitate the compliance function.
- *Control.* Sustaining the gains is what improvement is all about. Consequently, tools like mistake-proofing and control plans are beneficial and support prevention and detection functions.

Both "processes" and "control" will support the recordkeeping function.

A few remaining points regarding processes should be made. First, Welytok (2008) notes that documenting (that is, process mapping) the company's processes that directly impact the financial statements, controls, and risks is one of the top six most common costs related to Section 404. Note that this doesn't include the cost of processes that provide input to these financial processes.

Second, a Lean Six Sigma professional's involvement with the processes in a Sarbanes–Oxley implementation creates visibility of potential projects that might not otherwise have surfaced. Such projects might eventually be selected to enter the project pipeline.

Part II

Cross-Functional Competencies

| Chapter 10 | Data Gathering |
|------------|-------------------------------------|
| Chapter 11 | Internal Organizational Challenges |
| Chapter 12 | Executive and Team Leadership Roles |

Chapter 10 Data Gathering

Assess the appropriate collection of Voice of the Customer and Voice of the Process data, both internal and external, and develop a customer-focused strategy for capturing and assessing customer feedback on a regular basis. (Evaluate)

Body of Knowledge II.A

Although *The Certified Six Sigma Black Belt Handbook* focused on many of the component elements of voice of the process and voice of the customer, it did not address how these voices should be harmonized to ensure a successful customer-focused strategy.

This chapter will discuss the development of a customer-focused strategy based on knowledge of the voice of the process and voice of the customer. The discussion will include: defining what it means to be customer-focused, designing a customer data collection system, analyzing customer data, and creating an integrated data collection and action process.

VOICE OF THE PROCESS

Typically, control charts are the mechanism by which we discover the voice of the process. In-control and stable processes provide us with estimates of natural process limits.

Natural process limits are known by several names in the literature: *natural process variation, normal process variation,* and *natural tolerance.* In all cases, these terms refer to the $\pm 3\sigma$ limits (that is, 6σ spread) around a process average. Such limits include 99.73% of the process variation and are said to be the "voice of the process" (VOP). Walter Shewhart originally proposed the $\pm 3\sigma$ limits as an economic trade-off between looking for special causes for points outside the control limits when no special causes existed and not looking for special causes when they did exist.

VOICE OF THE CUSTOMER

In contrast to natural process limits, specification limits are customer determined or derived from customer requirements and are used to define acceptable levels of process performance. Said to be the *"voice of the customer"* (VOC), specification limits may be one-sided (that is, upper or lower) or two-sided. The difference between the upper and lower specification limits is known as the *tolerance*.

In some cases, customers provide specifications for products or services explicitly. This is often the situation when the customer is the Department of Defense or any of the military branches. In others, customers express requirements in value terms, the components that influence the buy decision, such as: price, product quality, innovation, service quality, company image, and reputation. In still other cases, customers may spotlight only their needs or wants, thus leaving it up to the organization to translate them into internal specifications. Tools such as quality function deployment, critical-to-quality analysis (also known as customer requirements tree analysis), and so on, often help with the last two situations.

Defining Customer-Focused Organizations

Organizations that recognize the importance of customers and focus on their wants and needs are commonly referred to as being customer-focused or customer-driven. In business literature, the terms *customer-driven* and *market-driven* are often used interchangeably. However, a customer-driven organization focuses more on the care and retention of existing customers, whereas the market-driven organization is more attuned to attracting new customers and serving their needs.

Designing a Data Collection System

Many organizations collect customer data but lack a systematic approach for doing so. Important data are often collected independently via many different departments, but never shared, collated, reconciled, or analyzed to draw meaningful conclusions that drive action.

When designing a customer data collection system, the following key considerations should be at the forefront:

- *Collect data as objectively and consistently as possible.* Essentially, minimize measurement error. This concept should permeate all data collection mechanisms such as surveys, data collection check sheets, and so on. Consider using scripts for interviews to ensure that the same questions are asked the same way each time.
- *Collect data at the right level of granularity.* Consider what type of analysis will be done, what questions you are trying to answer, or the type of requirements you are trying to discover. If data are collected at too high a level, information is lost. Data collected at too low a level is meaningless. Fortunately, it often can be summarized to an appropriate higher level for analysis. However, this creates additional work and consumes time.

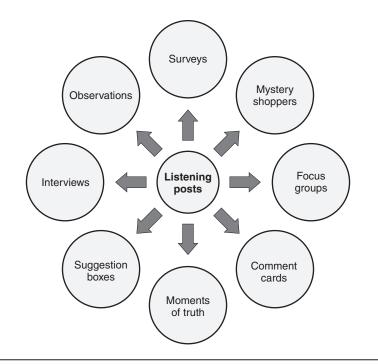


Figure 10.1 Examples of common listening posts.

- *Consider independent sources of data collection.* Multiple data collection mechanisms or listening posts should be used, as shown in Figure 10.1. This permits data regarding each customer category or segment to be validated. If the data can not be validated, this is generally a signal that improvement in the mechanisms (for example, revised questions) is required.
- *Use multiple media for collecting data.* Different customer categories or segments may favor different media for providing feedback, as shown in Figure 10.2. The key is to determine how to best align listening posts with the most appropriate media.
- *Make it easy for the customer.* Customers must have easy access to the organization to provide feedback. Similarly to using multiple media above, this bullet speaks to the ease with which the customer has access to the organization. For example, suppose an organization uses an automated telephone system for collecting customer feedback. Customers may become frustrated with attempting to determine the correct routing path to take to provide the feedback. Consequently, they decide that their input and time is not worth the effort. As a result, the organization loses a source of valuable information regarding the performance of its products and services. Similarly, surveys that are too long or inappropriately designed may also cause customers to abandon participation.

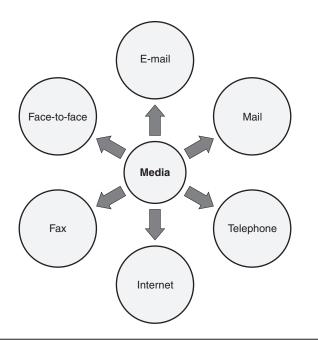


Figure 10.2 Examples of media used for listening posts.

| Internal | External |
|------------------------------|--------------------------------------------------------------------------|
| Complaints | Research (that is, magazines, newspapers, |
| Claim resolutions | trade journals) |
| Warranty and guarantee usage | Public information (that is, customers' and competitors' annual reports) |
| Service records | Advertising media (that is, television, radio, |
| Customer-contact employees | websites) |
| Listening post data | Industry market research |
| Market research | Customers of competitors |
| Transaction data | Industry conferences and forums |
| Defection data | |

Table 10.1Sources of customer data.

The above points focus mainly on collecting customer data from the organization's own internal sources of information. Such data should include both internal and external customers. Table 10.1 identifies these multiples sources of customer data. Notice there are many internal sources of customer data. Actually, many organizations are data rich with regard to such information but often fail to tap into it or even recognize it as an important source.

Analyzing Customer Data

As with any improvement project, one of the first steps is to seek out the voice of the customer, including internal and external customers. These customers are all those who are impacted by the project. In Chapter 15 of *The Certified Six Sigma Black Belt Handbook*, Kubiak and Benbow (2009) provide significant discussion on the following voice of the customer (VOC) topics:

- Methods and techniques for collecting customer data and feedback
- Customer categories and segmentation, including internal customers
- Determining customer requirements using critical-to-quality flow-down, quality function deployment, and the Kano model

As such, the details of each of these tools and techniques will not be addressed here.

Unfortunately, the linear, project-by-project approach limits an organization's ability to fully understand and exploit customer data. Consequently, all customer data must be viewed in its entirety in order for meaningful conclusions to surface. Results from each data source should be compared to determine patterns of reinforcement or contradiction for each customer group or segment for each requirement, need, or want. Only now can an organization begin to translate requirements, needs, or wants into specifications. An added value of this analysis is the insight gained into the effectiveness of the listening posts. Listening posts that are performing poorly can be improved, modified, or even eliminated.

The amount of feedback can be enormous and overwhelming. Westcott (2006) suggests one means of prioritizing customer feedback:

The volume and/or urgency of customer concerns will help determine the requirements that customers deem critical. Some type of criticality rating system should be applied to the feedback data (for example, a phone call from a customer is rated higher than a response to a questionnaire). Suppose, for example, that analysis of customer data identifies six areas where product quality is compromised. With the help of the criticality rating scale, perhaps two of the six would be deemed important enough to motivate immediate improvement projects.

One method to transform this data into information is to document and assess these answers and the impact on the supplier's organization using the QFD-type matrix. A simpler, but less comprehensive analysis may be done using a kind of "priority matrix" for each unique product and service offered, for each type of customer, for example:

- Title a spreadsheet with the name of the product or service and the type of customer
- Set up seven columns, reading left to right:
 - 1. Product/service factor/feature
 - 2. Potential to satisfy customers (brief commentary)

- 3. Estimated rating of customers' perception of value (scale: "0" = none, "1" = low to "5" = high)
- 4. Potential to dissatisfy customers (brief commentary)
- 5. Estimated rating of customers' negative perception (same scale, only with minus prefix for any 1's through 5's)
- 6. Estimated overall rating of impact on customers (arithmetic sum of two ratings from 3 and 5 above)
- 7. Comments about increasing, sustaining, decreasing, or eliminating the factor/feature from the suppliers' perspective
- List every factor/feature of the suppliers' products and services, including those already perceived as negative.
- For each line, rate the positive value perceived, and the negative value perceived, with comments about each.
- Sum the two ratings and make comments about the impact on the supplier's organization if the factor/feature was increased, sustained, decreased, or eliminated, based on the perceptions of the customers.

A scan of the completed table provides an estimate of how the supplier organization anticipates the customers' present expectations, needs, and value of a given product or service. Creating the same type of table for a new product or service, and using data from focus groups or other sources can help head off a potentially disastrous product/service launch. Likewise, using this approach with competitor data may yield valuable insights as to what the supplier must do to meet or exceed the competition.

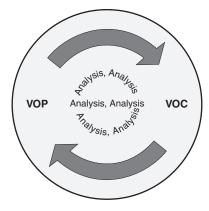
Westcott's approach is one of many. Tools, techniques, and methodologies for analyzing customer data are not new to the area of quality and Lean Six Sigma. Our literature abounds with them. Interested readers may want to pursue resources specifically dedicated to these topics.

Although some organizations consider responding to customer data and feedback a significant step forward, responses may not always be driven inward to affect the processes that produce the products and services.

Creating an Integrated Data Collection and Action Process

So far, the voice of the process and the voice of the customer have been addressed separately. The fact is that many organizations address each separately, and never shall the two meet. This approach is unfortunate in that it limits an organization's ability to meet customer expectations and improve processes.

Kubiak and Benbow (2009) discuss the concept of process capability in Chapter 23 of *The Certified Six Sigma Black Belt Handbook*. In simplest terms, process capability integrates the VOC and VOP through process capability indices. Essentially,





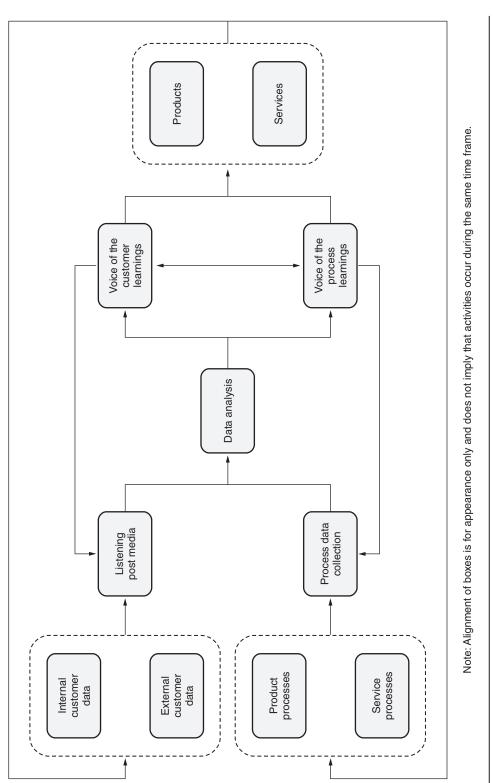
these indices tell us whether our processes are capable of meeting customer specifications. Those that fail to meet specifications must be improved.

Consider Figure 10.3. It depicts a simple, iterative relationship that exists between the VOC and VOP. Briefly, this figure illustrates that:

- *VOC drives the VOP.* When the voice of the customer is fully understood and connected to products and services, the products and services can, in turn, be connected to the processes that produce them. Thus, a line of sight from VOC to VOP is created. Unfortunately, many organizations collect the VOC and can not relate it back to the underlying processes. This constitutes a major disconnect and presents a serious situation for the organization to overcome. Briefly recall Figure 2.2. Notice that customer requirements are integral to the development of the strategic plan, improvement takes place at the individual process level, and we must address the basic question as to whether or not these processes are capable of meeting customer and business requirements.
- *VOP drives the VOC.* Internal processes that produce products or services meeting customer expectations via specifications tend to be reflected positively in the VOC. Similarly, the reverse is true. One way of determining whether internal processes are meeting customers' expectations is through the application of a *critical-to-quality* (CTQ) analysis. This analysis provides insight into the alignment between process outputs and customer needs or wants. Tague (2005) provides an excellent discussion on this topic.

It should be clear from the above that the VOC and VOP are in a symbiotic relationship. Independently, the improvement potential is limited. Working together, and connected through analysis, they can turn mediocre organizations into highperforming ones.

However, most organizations do not integrate the VOC and the VOP, typically because of their organizational affiliations. The VOP is often aligned with



manufacturing, production, or the assembly function while the VOC generally falls under the responsibility of sales, marketing, or similar functional department.

Organizations that have been successful capturing, assessing, and acting on customer feedback on a regular basis have done so through the deliberate introduction of a systematic process. An example of one such process is depicted in Figure 10.4. This process illustrates all of the components we have discussed so far. Notice that the data collection processes are refined based on VOP and VOC learnings, and these learnings are driven into the product and service development and manufacturing processes. Additional elements and considerations of such a process might include:

- Establishing ownership of the process in a new organization to minimize bias and shortsightedness.
- Recognizing the process as a key business process.
- Channeling all VOC input into the new organization.
- Channeling all VOP input into the new organization.
- Ensuring that the organization is staffed with the appropriate talent who can properly analyze the data and draw meaningful conclusions.
- Integrating the output and conclusions into the organizational process for producing current products and services as well as for new product and service development.
- Establishing that process triggers exist—these triggers can be either time or event-based—and are used to generate action. For example: review listening posts for effectiveness, update analysis, and so on. Triggers might include new product introductions by competitors, updated governmental regulations, product recalls, and so on.
- Establishing regular senior management reviews to ensure visibility, responsibility, and accountability of results.

Voice of the process and voice of the customer data are essential to the long-term success of an organization. Although collecting such data is critical, it is not sufficient. It must be analyzed, disseminated, and acted on in a systematic manner. This will only occur when processes have been enacted to ensure that it will happen in a consistent and regular manner.

Chapter 11

Internal Organizational Challenges

This chapter will deal with many of the soft skills and related topics that Master Black Belts must understand and master to be able to work effectively in their roles. Such topics include: organizational dynamics, conflict management, negotiation skills, and various intervention, communication, influence, and leadership styles.

ORGANIZATIONAL DYNAMICS

Use knowledge of human and organizational dynamics to enhance project success and align cultural objectives with organizational objectives. (Apply)

Body of Knowledge II.B.1

Defining Organizational Dynamics

Organizational dynamics deals with the behavioral nature of organizations, specifically with regard to the following elements:

- Organizational culture
- Organizational structure and alignment
- Theories of motivation
- Leadership styles
- Group or team dynamics
- Conflict management and resolution
- Change management

More importantly, organizational dynamics deals with the interaction of the above elements when they are all brought together.

The remainder of this subsection will specifically address organizational culture, organizational structure and alignment, theories of motivation, and leadership and management styles. Any of the above bullets not addressed here will be covered elsewhere in this book or have been covered in *The Certified Six Sigma Black Belt Handbook*.

Organizational Culture

Schein (1996) provides a formal definition of *culture* as "A pattern of shared basic assumptions that the group learned as it solved its problems of external adaptation and internal integration, that has worked well enough to be considered valid and, therefore, to be taught to new members as the correct way you perceive, think, and feel in relation to those problems."

Westcott (2006) adds that the organization's *culture* "refers to the collective beliefs, values, attitudes, manners, customs, behaviors, and artifacts unique to an organization."

Similarly to Schein, Westcott sees culture as "a function of the values, norms, and assumptions shared by members of the organization." Communicating what standards of behavior are expected, and ensuring that policies, procedures, promotions, and day-to-day decisions are appropriately aligned can therefore shape it. Culture is manifested in ways such as:

- How power is used or shared
- The organization's orientation toward risk or safety
- How outsiders are perceived and treated
- Artifacts, layout, and amenities (for example, furnishings, artwork, open versus closed work spaces, signage, and so on)

New members of an organization are soon acclimated to the culture, sometimes through training but usually through formal reinforcement and peer influence.

Organizational Structure and Alignment

The organizational structure can have a significant impact on culture. Examples include:

- Hierarchical structures restrict cross-functional communications. Consequently, close relationships between interrelated functions become more difficult.
- Complex organizational structures with multiple lines of reporting make it difficult for employees to determine how to navigate or communicate to the organization, or even identify responsibilities and accountabilities.
- Placing key organizational units too low in the structure to be effective. This is often the case with Lean Six Sigma organizations. They are touted by senior leadership as a strategic initiative or approach, but frequently buried in the organization. This sends

conflicting messages to organizational members about its level of importance and longevity.

The Impact of Culture on Quality. Culture can have a significant impact on quality, and is shaped by the words and actions of leadership, how work systems are designed, and what is rewarded. If the culture is not proactive, is not focused on customers, and does not use data to guide decision making, the organization is not likely to be highly successful in the continual improvement of quality.

Gryna et al. (2007) list five steps for changing to a quality culture:

- 1. Create and maintain an awareness of quality
- 2. Provide evidence of management leadership on quality
- 3. Provide for self-development and empowerment
- 4. Provide participation as a means of inspiring action
- 5. Provide recognition and rewards

Changing an organization's culture is difficult and requires time. Fear of change must be removed, poor labor–management relations must be resolved, and the company's focus must change from the status quo. Employees should be convinced of the benefits that a quality management approach will provide and should "buy in" to the changes. However, this often means that employees at all levels will need to change behaviors or perform tasks in a different manner. If strong leadership, motivation, and enthusiasm are lacking, frustration and stress will result.

Changing an Organization's Culture. Changing an organizational culture is no simple task. It requires committed, engaged, involved, and visible senior leadership serving as role models. Though Master Black Belts can serve as liaisons to senior leadership and execute activities in support of change, they are neither responsible nor accountable for changing an organization's culture. Change does not happen from the middle.

Gryna et al. (2007) present an organizational change curve that demonstrates the life cycle of a change initiative. This curve is depicted in Figure 11.1. Notice that change is rooted in emotion. As such, it can only be dealt with effectively at the top levels of an organization.

Aligning Cultural Objectives with Organizational Objectives. Why do many organizations hesitate to change their culture? Difficulty explains part of the reason. The organizational change curve in Figure 11.1 provides additional explanation.

However, could it be that to change an organization from its present state to a desired future state, an organization must be able to characterize its present state? Not only that, but the view of the present state of the culture of an organization depends on who is asked. For example, an executive might characterize the organization as collaborative, sharing, friendly, open, innovative, and so on. Ask an individual who works their day in a cubicle and they might characterize it as selfish, hostile, closed-door, stifling, and so on. This positional view of the organization's culture is particularly interesting since most organizations consider "cultural fit" as a key criterion for hiring. Paradoxically, one of the top reasons given for firing is "cultural fit." Adams and Schuker (2011) note the recent

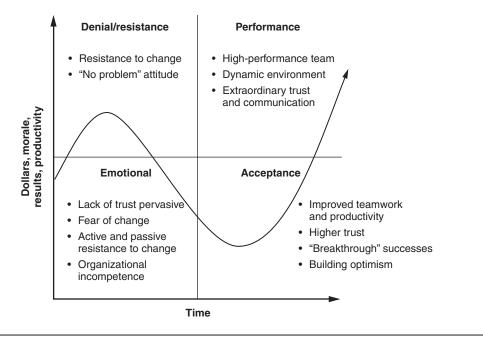


Figure 11.1 Organizational change curve. *Source:* Adapted from Gryna et al. (2007).

firing of Jack Griffin, former CEO of Time Warner. Mr. Griffin was fired after less than six months on the job because his leadership style "did not mesh with Time Inc. and Time Warner." Complaints against Mr. Griffin ran the entire spectrum from bucking a long-standing tradition of decision making traditionally left to the individual editors to calling 7:30 a.m. meetings. An individual close to Mr. Griffin defended his approach with, "Jack's exit had nothing to do with management style and everything to do with the question of whether Time is manageable so long as entrenched interests fiercely resist the change necessary to position the organization for the future." Additional people said, "Mr. Griffin appeared to be on a mission to reinvent the business and the culture, when people in the senior ranks at Time Warner weren't looking for dramatic changes." What makes this statement so interesting is that Mr. Griffin won praise for essentially reinventing his former company.

Clearly, before any alignment between cultural objectives and organizational objectives can occur, the present state of the culture must be assessed and properly characterized. Recall that the concepts of cultural and organizational assessments were addressed in Chapter 3. When such assessments are completed and the results understood, only then can alignment between cultural objectives and organizational objectives be addressed. Of course, this is all predicated on the assumption that an organization is sufficiently mature and willing to establish cultural objectives in the first place.

Establishing an Organization's Culture. Before any organization can effectively develop its culture, it must establish its founding principles. Covey (1989) defines

principles as "guidelines for human conduct that are proven to have enduring, permanent value. They're fundamental. They're essentially unarguable because they are self-evident. One way to quickly grasp the self-evident nature of principles is to simply consider the absurdity of attempting to live an effective life based on their opposites. I doubt that anyone would seriously consider unfairness, deceit, baseness, uselessness, mediocrity, or degeneration to be a solid foundation for lasting happiness and success."

An example of a principle proffered by Deming was his point 5: "Improve constantly and forever the system of production and service, to improve quality and productivity, and thus constantly decrease costs." Such a principle would support a cultural objective of being data-driven. This, in turn, would support organizational practices of surfacing, not hiding, problems. Consequently, alignment between principles, culture, and practices facilitates the introduction of tools and methodologies that are consistent and supportive of these principles, culture, and practices. For example, Six Sigma and Lean would be able to thrive in such an organization.

If such alignment did not exist, it is highly likely that Six Sigma and Lean would be doomed to failure. For example, suppose the culture was one of blame, not data-driven, and one of the organization's practices was to chastise employees for surfacing problems. (How many times have you heard, "Bring me solutions, not problems!") Do you think Six Sigma or Lean would survive and flourish even if senior leadership provided support? Ironically, senior leadership often introduces Six Sigma and Lean into their organization specifically as a culture-changing initiative, only to find substantial forces opposing it. Many of these forces are defined in Figure 11.2.

This flow-down of principles through tools is illustrated in Figure 11.3. Notice that the principles, culture, and practices define the organization's identity. Also, it should be obvious from the figure that one can not simply align the cultural objectives to the organizational objectives without considering the relationship that exists between the founding principles and the organization's practices.

Theories of Motivation

Motivation is derived from within a person. However, a person needs to feel motivated. Therefore, motivating a person can only be done by creating an environment in which the person feels motivated. When a manager says he or she is going to motivate a subordinate, he or she means (or should mean) doing something that will cause the subordinate to become motivated.

Two types of motivation have been identified:

- *Extrinsic motivation*. The satisfaction of either material or psychological needs that is applied by others or the organization through pre-action (incentive) or post-action (reward).
- *Intrinsic motivation*. The qualities of work itself or of relationships, events, or situations that satisfy basic psychological needs (for example, achievement, power, affiliation, autonomy, responsibility, creativity, and self-actualization) in a self-rewarding process.

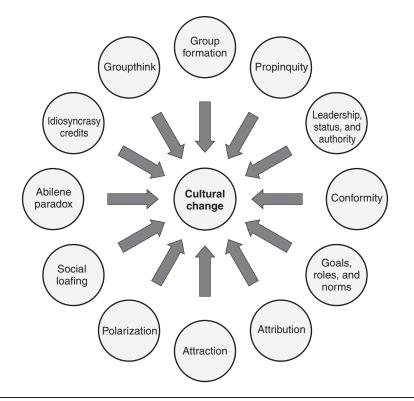


Figure 11.2 Forces opposing cultural change.

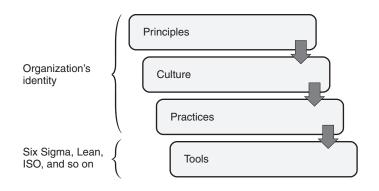


Figure 11.3 Fitting tools into the organization's identity.

Maslow's Hierarchy of Needs. Abraham Maslow developed a model demonstrating a hierarchy of needs through which he believed people progressed. This model is illustrated in Figure 11.4. It assumes that once humans satisfy the basic, physiologically driven needs, they will then be motivated by higher-level needs. This process continues until achieving self-actualization.

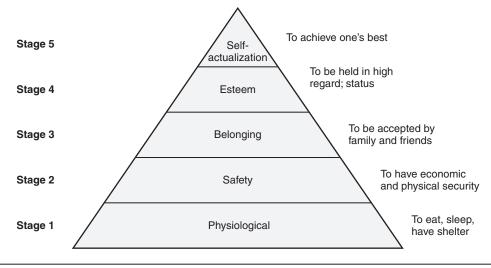


Figure 11.4 Maslow's hierarchy of needs. Source: Adapted from Westcott (2006).

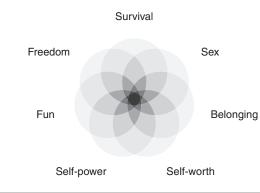


Figure 11.5 The seven inborn human needs. *Source:* Adapted from Lareau (2011).

Lareau (2011) offers a modification of control theory that is similar to Maslow's hierarchy of needs. This is depicted in Figure 11.5. However, unlike Maslow's hierarchy, Lareau believes that all seven inborn human needs must be pursued simultaneously.

Herzberg's Motivational Factors. Frederick Herzberg identified two categories into which work motivation factors could be classified—satisfiers and dissatisfiers. *Dissatisfiers* included things such as work conditions, salary, company policies, and relationship with one's supervisor. He also called them *hygiene factors*, since although they create dissatisfaction if not adequately addressed, correcting the deficiencies would not create satisfaction. *Satisfiers* included items such as responsibility, achievement, advancement, and recognition. These satisfiers and dissatisfiers are depicted in Table 11.1.

| Satisfiers (Motivational factors) | Dissatisfiers (Hygiene needs) |
|-----------------------------------|-----------------------------------|
| Achievement | Company policy and administration |
| Recognition | Supervision |
| Work itself | Relationship with supervisor |
| Responsibility | Work conditions |
| Advancement | Salary |
| Personal growth | Relationship with peers |
| | Personal life |
| | Relationship with subordinates |
| | Status |
| | Security |

Table 11.1 Hertzberg's satisfiers and dissatisfiers in the workplace.

Motivational Theories of Rewards. There are also three motivational theories specifically related to rewards:

- *Equity theory* (John Stacey Adams). Job motivation depends on how equitable the person believes the rewards (or punishment) to be.
- *Expectancy theory* (Victor Vroom). What people do is based on what they expect to gain from the activity.
- *Reinforcement theory* (B. F. Skinner). What people do depends on what triggers a behavior initially (the antecedent) and the consequences that have in the past resulted from such behavior, or the consequences the performer believes will happen as a result of a behavior.

McClelland—One Size Doesn't Fit All! What motivates one person may not motivate another. David McClelland and others have posited that:

- An individual who enjoys working closely with other people is motivated by affiliation.
- Someone who works in order to accomplish personal goals is motivated by achievement.
- A person who works in order to contribute to the well-being of others is motivated by altruism.
- Someone who wants to have control over their work is motivated by power.

These motivations are not mutually exclusive, nor is any one person driven only by a single factor. In fact, people are often involved in several different activities at work, in their local community, and in other organizations in order to satisfy their many different needs.

EXAMPLE 11.1

Work is backing up and the manager tells her work unit personnel that the unit will have to work overtime each day for the next ten days. Ray, a young man who is saving money to buy a dirt bike, is delighted about the prospect of extra compensation. Rose, a single mother whose baby is cared for during the day by a babysitter, is distraught because she has to be home by 6 p.m. to relieve the babysitter and realizes that any other option would cost more than she will earn on overtime. Ray is motivated, Rose is not. Six months later the same need for overtime occurs. Ray resents the mandate since he has purchased his dirt bike and wants to ride it in the early evenings before the sun sets. Rose's aunt has moved in with her and now cares for the baby. Rose needs the additional compensation to help with her expanded family. Rose is motivated, Ray is not.

Some people are driven more by their own internal needs or desires while others are motivated primarily by external factors. This is partially a function of whether the person has an internal or external locus of control (that is, whether they believe their future is impacted more by their own actions or by actions and decisions of others).

One should never assume that what stimulates motivation for an individual in one situation, or time frame, will continue to motivate the next time. Also, that which provides motivation for one person may not do so for another.

This subsection has presented multiple theories of motivation. Fundamentally, many of these theories are similar and are based on a hierarchal structure. As Master Black Belts, you may want to consider reflecting on these theories from time to time as you work through your Lean Six Sigma deployment and engage all levels of the organization in doing so. When you run into a motivation issue, ask yourself, are the individual's needs being met, where are they in the hierarchy, and how from a leadership perspective can you provide an environment where motivation can flourish?

Leadership (Management) Styles and Models

Numerous leadership styles and models have been proffered over the years. This subsection will address some of the more common models and define the associated terminology.

The Managerial Grid. Blake and Mouton (1972) defined five leadership styles according to their managerial grid. See Figure 11.6. The grid pits the concern for people against the concern for production. In the general sense, "production" may be interpreted to mean "tasks" or "results." These five leadership styles reflected on the grid are:

• *Impoverished*. The impoverished leader has both a low concern for people and production. Such a leader uses this style to avoid getting into trouble and being held responsible for any mistakes.

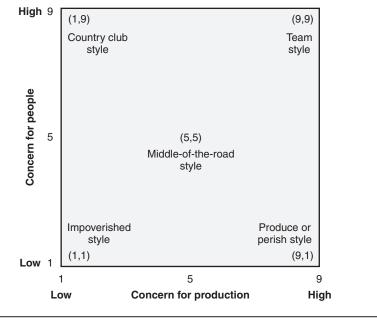


Figure 11.6 The managerial grid. *Source:* Adapted from Blake and Mouton (1972).

- *Country club.* The country club leader has a high concern for people, but a low concern for production. Leaders using this style pay attention to the comfort and security of their employees with the objective of increasing performance. This creates an atmosphere that is friendly, but not always productive.
- *Middle-of-the-road.* The middle-of-the-road leader seeks to balance concern for people and production with the hope of achieving acceptable performance.
- *Team.* The team leader has both a high concern for people and production. Such a leader uses this style to encourage both commitment and teamwork among employees. This style emphasizes making employees feel like contributors to the organization.
- *Produce or perish.* The produce or perish leader has a low concern for people and a high concern for production. Such a style is akin to a dictatorship. Leaders using this style expect performance for pay. They frequently pressure their employees through adherence to rules and application of negative reinforcement to accomplish organizational goals. This style readily surfaces during times of crisis.

Kahler's Interaction Styles. Pauley and Pauley (2009) describe Kahler's set of four leadership styles, which is based on six types of individuals with whom the leader is dealing. This set of interaction styles is depicted in Table 11.2. Each of these leadership styles will now be described:

| Leadership style | Type of individual |
|------------------|--------------------|
| Benevolent | Reactor |
| Democratic | Workaholic |
| Democratic | Persister |
| Autocratic | Dreamer |
| Laissez faire | Rebel |
| Autocratic | Promoter |

| Table 11.2 | Kahler's interaction styles. | |
|------------|------------------------------|--|
| | Runner 3 mileraction styles. | |

Source: Adapted from Pauley and Pauley (2009).

- *Benevolent.* This leadership style assumes that when people feel good, their performance improves. Leaders who use this style try to create a sense of belonging in others by interacting in a nurturing way. This style works well with individuals who require unconditional acceptance.
- *Democratic.* This style is predicated on the foundation of group participation and decision making. Leaders who use this style encourage interaction between employees, solicit feedback, and foster independent thinking. The advantage of this style is that it promotes group cohesion and morale.
- *Autocratic.* This leadership style is directive and commanding and encourages people to respond directly to the leader. Such a leadership style is task-oriented and is useful with individuals who require direction, structure, definition, or training. This approach is often visible in organizations facing crises, and has been prevalent since the industrial revolution.
- *Laissez faire.* This style invites others to assume as much responsibility as they can handle. It works well with individuals who resent authority, rules, and regulations. Such a style is often found in high-tech start-up organizations.

Each of the six types of individuals will now be described:

- *Reactor*. The reactor is the type of individual who feels first. They are typically sensitive and warm, and have strong interpersonal skills. Their goal is to help everyone feel good, and they prefer to work in groups with people with whom they are comfortable.
- *Workaholic.* Such individuals are goal-oriented self-starters who thrive on data. They are considered responsible, logical, and organized, and expect others to be goal-oriented and to think clearly as well.
- *Persister.* These are individuals who have a strong sense of right and wrong and judge individuals according to their value system. They set

high standards and values and expect the same of others. Persisters are often described as conscientious, dedicated, observant, and goal-oriented. Such individuals quickly rise to leadership positions in organizations.

- *Dreamer.* This type of individual is often seen as reflective, imaginative, and calm, and requires time to ponder tasks before taking action or contributing to a discussion. The dreamer is insightful and generally thinks outside the box.
- *Rebel.* These individuals are the most creative and can have wide mood swings. They are also considered spontaneous and playful, and thrive in an environment that provides great freedom and autonomy.
- *Promoter.* This type of individual is the natural entrepreneur who is action-oriented and thrives on challenges. They are described as being persuasive, adaptable, and charming, and they love to sell. Promoters are risk-takers and respond to short-term challenges and quick rewards. They are natural leaders and often become leaders of start-up organizations.

No one leadership style works with all types of individuals. Leaders must be able to quickly assess an individual's type and adapt their interaction style as they go. Based on the above descriptions and recognizing that most organizational units are heterogeneous compilations of the above individual types, it would appear this model is best suited for one-to-one interactions.

U.S. Army Model. Lewin et al. (1939) identified what they believed to be three types of leadership styles. Briefly, these styles are:

- *Authoritarian (Autocratic).* This leadership style is characterized by telling employees what to do and how to do it. It is effective when time is short, the leader has all the information necessary to address the situation at hand, and the employees are motivated. This style is often confused with threats, abuse of power, and demeaning language, which the authoritarian style, however, is not.
- *Participative (Democratic).* In contrast to the authoritarian style, the participative style involves the employee in the decision-making process with regard to what must be done and how it must be done. It is used when the leader has only partial information on which to proceed and knowledgeable and skilled employees are available. This style provides specific emphasis on respect for the employee.
- *Delegative (Free reign).* This style, sometimes known as *laissez faire*, shifts decision making from the leader to the employees. Employees determine what must be done and how it must be done. However, the leader retains responsibility for the results.

Interestingly, the U.S. Army adopted these leadership styles and incorporated them into the U.S. Army Handbook (1973).

Situational Leadership Model. Blanchard et al. (1999) developed what is described as a *situational leadership* model. This model is illustrated in Figure 11.7. In this model, the leadership style is categorized by quadrant according to the leader's degree of supportive and directive behavior. *Supportive behavior* involves two-way communication and focuses on emotional and social support while *directive behavior* is to be done, how it is to be done, and who is going to do it.

Each leadership style now will be addressed:

- *Directing.* This style is characterized by high directive and low supportive behavior. It is suited for individuals who possess low competence and high commitment. Such individuals are often described as the *enthusiastic beginner*. They are new to a task and motivated about it.
- *Coaching.* This style is characterized by high directive and high supportive behavior. It is suited for individuals who possess moderate competence and low commitment. Such individuals are often described as the *disillusioned learner*. They are new to a task, but not motivated about it.
- *Supporting.* This style is characterized by low directive and high supportive behavior. It is suited for individuals who possess mid competence and moderate commitment. Such individuals are often described as the *capable, but cautious, performer.* They know their task, but are not motivated about it.

| High | | |
|---------------------|------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------|
| mgn | | |
| | Supporting | Coaching |
| Supportive behavior | For individuals who have:Mid competenceModerate commitment | For individuals who have:Moderate competenceLow commitment |
| Supportive | Delegating (Empowering) | Directing |
| | For individuals who have: | For individuals who have: |
| | High competence | Low competence |
| | High commitment | High commitment |
| Low | | |
| | Low Directive | behavior High |

Figure 11.7 Situational leadership model. *Source:* Adapted from Blanchard et al. (1999).

• *Delegating (Empowering).* This style is characterized by low directive and low supportive behavior. It is suited for individuals who possess high competence and high commitment. Such individuals are often described as the *self-reliant achiever*. They are proficient in their task and are highly motivated about it.

It is important to note that leadership styles are highly dynamic and depend on the competency or commitment level of an individual at any given time. For example, an individual who is competent at a specific task may vary in their commitment toward that task over time depending on their motivation. Similarly, an individual who is committed to their task will likely demonstrate an increase in competency as they gain proficiency over time. Consequently, leaders should recognize these dynamics and adjust their styles accordingly.

Goleman's Six Leadership Styles Based on Emotional Intelligence. Goleman et al. (2004) introduced an innovative leadership model that integrated the concept of emotional intelligence. Goleman's six leadership styles are depicted in Table 11.3. Notice that of the six styles, four have a positive impact while two have a negative impact. In addition to the six styles, Goleman identified six elements of the climate of an organization that he believes are impacted by these styles:

- *Flexibility*. This represents how free the people of the organization are to innovate. An associated style might be *visionary*.
- *Responsibility*. This represents the sense of responsibility the people feel to the organization. Associated styles might include *democratic* and *pace-setting*.
- *Standards.* This represents the standards the people set. Associated styles might include *pace-setting* or *commanding*.
- *Rewards.* This represents the accuracy of the performance feedback. An associated style might be *coaching.*
- *Clarity.* This represents how clear the people are regarding the organization's mission and values. An associated style might be *visionary.*
- *Commitment*. This represents how committed the people are to a common purpose. Associated styles might include *affiliative* and *democratic*.

Transactional Leadership. A transactional style is one in which the leader views the relationship as one of getting the work done through clear definition of tasks and responsibilities, and providing whatever resources are needed. This view might be likened to a contractual relationship, with rewards (positive or negative) being associated with achieving the desired goal.

Furthermore, transactional leadership recognizes the need for pay for performance, job descriptions and grading, performance appraisals, management by objectives, and the delegation of responsibility.

Transformational Leadership. Transformational leadership is a style whereby a leader articulates the vision and values that are necessary in order for the

| | | | Leadership style | uip style | | |
|-------------------------------------------|-------------------------------------------------------------------|-----------------------------------------------------------------------------------------------|-----------------------------------------------------------------|---------------------------------------------------------------------------------|--------------------------------------------------------------------------------------|-----------------------------------------------------------------------------|
| | Visionary | | | | : | Commanding |
| | (Authoritative) | Coaching | Attiliative | Democratic | Pace-setting | (Coercive) |
| Nickname | The Visionary | The Nurturer | The People Person | The Listener | The Superman/ Superwoman | The Dictator |
| Impact on organization | Extremely positive | Very positive | Positive | Positive | Often very negative | Negative when not in crisis |
| Phrase used | Come with me | Try this | People come first | What do you think | Do as I do | Do what I tell you |
| Characteristics | Inspirational, empathetic, open, sharing | Listens, counsels, encourages | Collaborative, creates harmony | Collaborative, influencer, team player | Hands-on, impatient, data-driven | Threatening, demanding, controlling |
| Emotional intelligence competencies | Self-confidence, empathy, change catalyst | Developing others, empathy, self-awareness | Empathy, building relationships, communication | Collaboration, team leadership, communication | Conscientiousness, drive to achieve, initiative | Drive to achieve, initiative, self-control |
| Appropriate to use when | New direction is required or goals require clarification | Improving people's strengths and building and developing skills in future leaders | Creating teams and healing dysfunctional relationships | Seeking to involve a group of people in the decision-making process | Raising the standard when a competent and motivated team is working well | Emergencies occur, time is short, severe situations have set in |

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Source: Adapted from Goleman et al. (2004).

organization to succeed. It is sometimes equated to charismatic leadership, but is aimed more at elevating the goals of subordinates and enhancing their selfconfidence to achieve those goals.

Central to the transformational leadership approach is the concern for people. The transformational leader is change-oriented, future-focused, and spends time with employees trying to learn about their goals and problems. Not only does the transformational leader bring a vision of the future to the table, but he or she also brings along a plan for achieving the vision and accompanying future organizational designs, and plans for products and processes.

Management by Fact. A key contribution of quality management to business philosophy is that decisions should be based on data. This is reinforced by the Baldrige Award's category of "Measurement, Analysis, and Knowledge Management" and the ISO/TS16949 requirements of "Analysis of data." Quality management focuses on fact-based decision making and the use of quality tools for performance measures and process improvement.

Leaders who manage by fact lead organizations into change by establishing expected behaviors and by setting examples. Leaders who ask for data are telling employees that "guessing" and "feeling" are no longer acceptable ways of managing an organization. Setting expectations that facts and data are required and holding to these expectations will eventually cause them to take root in the organization's culture.

Coaching. As competing organizations attain the same levels of technology, a key differentiation in organizational performance is their ability to develop and effectively empower employees. Providing guidance and constructive feedback to employees to help them better apply their natural talents and to move beyond previous limitations has become a style of management. In this role, leaders ask questions that will help others reach a conclusion rather than the leader or manager making the decision for them.

Theories X, Y, and Z. Douglas McGregor formulated the *theory X versus theory Y model. Theory X* is a negative view of human nature, assuming that most employees do not want to work, need to be told what to do, and will require continual prodding. *Theory Y* is a positive view, believing that employees want to work, are self-motivated, and will seek responsibilities that can offer solutions to organizational problems and their own personal growth.

William Ouchi's theory Z is a hybrid approach that combines Japanese management philosophies with a U.S. culture. *Theory* Z is characterized by long-term employment, slow evaluations and promotions, consensus decision making, individual responsibility, informal control with formalized measures, moderately specialized career paths, and a holistic concern for the employee that extends beyond the workplace. Theory Z organizations have cultures characterized by a consistent set of values, beliefs, and objectives. Such consistency is expected to promote greater organizational commitment by employees.

Which Model Is Best? Numerous models and theories have been presented, all of which are valid to some degree. Hopefully, it is clear there is no one right way to lead or manage. Furthermore, models serve only to provide guidance and direction. Leaders must have a solid understanding of organizational theory, moti-

vational theory, and learning theory if they are to effectively utilize the human resources that make up their organizations. Matching an individual to the job, matching training design to the learners, and matching one's leadership or management style to a specific individual and/or situation is fundamental to organizational leadership.

INTERVENTION STYLES

Use appropriate intervention, communications, and influence styles, and adapt those styles to specific situations (i.e., situational leadership). (Apply)

Body of Knowledge II.B.2

Heron's Intervention Approach

Heron (1999) introduced two types and six categories of intervention, the purpose of which is to increase the effectiveness of communication by a leader or mentor. These are summarized in Table 11.4. Let's briefly discuss the details associated with this table:

- *Authoritative intervention*. A leader using this type of intervention approach is providing information, challenging an individual or group, or suggesting what should be done.
 - Prescriptive. The leader or mentor using this style will advise, provide opinions, or otherwise attempt to directly influence an individual or group. Sometimes, the leader will provide explicit advice or direction. This may take the form of, "You need to"
 - *Informative.* The leader or mentor using this style is providing information based on knowledge and experience, generally to help

| Туре | Category | Also described as | |
|---------------|--------------|-------------------|--|
| Authoritative | Prescriptive | Planning | |
| | Informative | Meaning | |
| | Confronting | Confronting | |
| Facilitative | Cathartic | Feeling | |
| | Catalytic | Structuring | |
| | Supportive | Valuing | |

Table 11.4 Heron's types and categories of intervention.

an individual or group gain a better understanding. This may take the form of, "That book will be very useful to you."

- *Confronting.* The leader or mentor using this style is attempting to provide direct feedback in order to raise awareness of an attitude or behavior of which the individual is unaware. For example: "I noticed you were nervous during the meeting with the vice president."
- *Facilitative intervention*. A leader using this type of intervention approach is attempting to draw out ideas, solutions, and self-confidence from an individual or group. The emphasis is on reaching one's own solutions and decisions.
 - *Cathartic.* The leader or mentor using this style wants to help an individual express their emotions, usually through empathy. This may take the form of, "What do you feel about this situation?"
 - *Catalytic.* The leader or mentor using this style seeks to help an individual solve problems and learn through self-discovery. This may take the form of, "How did you deal with that similar problem previously?"
 - Supporting. The leader or mentor using this style is attempting to boost an individual's sense of self-worth through approval, confirmation, and validation. This may take the form of, "It sounds like you handled that problem very well."

Notice the comprehensiveness of Heron's model. In particular, inclusion of the facilitative type should work well for those leaders who must function without designated authority.

Communication Styles

Just as numerous leadership styles and models have been developed over the years, likewise so have communication styles.

One of the more commonly recognized classifications of communication styles is shown in Table 11.5. Each of these styles will now be addressed briefly:

- *Passive*. The individual who adopts this style of communication avoids confrontation, focuses on compliance, does little, and asks the minimum number of questions possible. These individuals try not to rock the boat and hope not to be noticed. Would this style be suitable at any time for a Master Black Belt?
- Assertive. This style of communicator avoids games, manipulation, and hidden agendas. Communication is clear, open, and honest, with the communicator seeking a win/win scenario. Although considered the healthiest form of communication, this style is used the least in most organizations. Combined with techniques of negotiation, this style can be very powerful in moving organizations toward their stated objectives.

| | Passive | Assertive | Aggressive |
|---------------------------|--------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------|----------------------------------------------------------------------|
| Definition | You put the rights of others before your own, thus minimizing your own self-worth | You stand up for your rights while maintaining respect for the rights of others | You stand up for your rights, but violate the rights of others |
| Behavior | • Sighs a lot | Action-oriented | • Bossy |
| | • Rides the fence | • Fair, just | Never wrong |
| | Asks permission | Consistent | Puts others down |
| Implications to others | My feelings are not important | • We are both important | Your feelings are not important |
| | • I don't matter | • We both matter | • You don't matter |
| | • I think I'm inferior | • I think we are equal | • I think I'm superior |
| Potential | Lowered self-esteem | • Higher self-esteem | Lowered self-esteem |
| consequences | • Anger at self | • Self-respect | Anger from others |
| | False feelings of inferiority | | • Guilt |
| | Disrespect from others | Respect of others | • Disrespect from others |
| | Pitied by others | Respect from others | • Feared by others |
| Verbal styles | Apologetic | • "I" statements | • "You" statements |
| | Overly soft or tentative voice | Firm voice | Loud voice |
| Nonverbal styles | Looking down or away | Looking direct | • Staring, narrow eyes |
| | • Stooped posture, excessive head nodding | • Relaxed posture, smooth and relaxed movements | • Tense, clenched fists rigid posture, pointing fingers |

| Table 11.5 | Heffner's | communication | styles. |
|------------|-----------|---------------|---------|
| Tubic 11.5 | Fiermer 5 | communication | Styres. |

Source: Adapted from a table by Christopher L. Heffner on http://www.cedanet.com/meta/ communcation_styles.htm.

• *Aggressive*. Individuals practicing this type of communication are generally organizational bullies. They work through manipulation, guilt, anger, and intimidation. This style is not suitable for building relationships. How far would a Lean Six Sigma implementation progress with a Master Black Belt using this style of communication?

In addition to the above three styles, a fourth style will occasionally surface. That is the style of "passive-aggressive." This style avoids direct confrontation, but relies on manipulation that often leads to office politics and rumors. The individual using this style is particularly difficult to deal with since they may be working under the radar.

Influence Styles and Strategies

Influencing individuals and leaders within an organization is difficult, particularly when that influence must occur without accompanying authority. Useful approaches others have found successful include:

- Go out of your way to assist others. Volunteering and helping others will motivate them to work with you. Reciprocity is generally a universal norm.
- *Share credit with others.* Sharing credit with others will help motivate them to work with you.
- *Don't pull rank.* Doing so will create resentment and undermine your efforts.
- *Develop networks*. Build a power base of relationships both horizontally and vertically.
- *Seek a win/win outcome.* Ensure you address the needs of others. It's not just all about you.

In addition to the above approaches, McClaran et al. (2009) provide the following recommendations for influencing without authority:

- *Be knowledgeable.* This includes being knowledgeable about the organization and about your technical field of study.
- *Build credibility.* This can be achieved in many different ways, including delivering on promises, technical knowledge, working toward the greater goal, and so on.
- *Exhibiting strong personal characteristics.* These would include not only a sense of integrity, selflessness, generosity, and willingness to share resources, but also the ability to say "no."
- *Be a team player.* Doing otherwise undermines one's efforts to fit into most organizations.
- *Lead by example.* Serve as a role model.
- Understand those whom you want to influence. Recall the stakeholder analysis discussed in Chapter 5. This is a useful tool for understanding stakeholder concerns and positions toward an initiative or a specific project. Once understood, a plan can be developed for overcoming them. Also, this is where the force field analysis tool becomes useful.
- *Demonstrate the benefits.* This is a key point in any discussion where influencing without authority must take place. Benefits must be communicated clearly. Don't assume they are obvious.

Influencing without authority can be difficult. However, most organizations operate this way today. Thus, it is critical that every Master Black Belt be comfortable working without the proverbial net of authority. This is particularly important since many organizations expect their Master Black Belts to work horizontally and vertically throughout the entire organization.

Bringing It All Together

So far in this chapter, we have addressed various styles of intervention, communication, influencing without authority, and, of course, situational leadership. Recall that:

- *Intervention styles* or approaches give us the necessary ability to deal with individuals or groups who require a stronger or gentler guidance.
- *Communication styles* permit us to deal with a wide variety of audiences, but nonetheless can be quite destructive if not used properly.
- *Influencing without authority* concepts give us guidance when the opportunity to work with individuals or groups, both horizontally and vertically throughout the organization, presents itself.

These styles, approaches, or whatever you wish to call them, provide the Master Black Belt the proper skills to operate in a complex environment without organizational authority. However, skills must be continually practiced and refined in order to move your organization toward higher levels of success.

INTERDEPARTMENTAL CONFLICTS

Address and resolve potential situations that could cause the program or a project to under-perform. (Apply)

Body of Knowledge II.B.3

Understanding Conflict

Sources of Conflict. Conflict exists because people are involved. Consequently, every Lean Six Sigma initiative should expect interdepartmental conflicts. Inevitably, such conflicts will become evident at the project level. If your implementation is moving along and conflicts are not apparent, beware!

Conflict can occur for many different reasons. It can occur in terms of disagreements, misunderstandings of underlying assumptions, or due to differing needs, styles, perceptions, goals, pressures, roles, and values. Also, highly dynamic or unpredictable environments can create conflicts. This is particularly true when policies are absent, constantly changing, or inconsistently applied.

Conflict occurs when two or more options appear to be mutually exclusive and a viable alternative is felt to be absent. Conflict can not be resolved when the parties involved firmly believe that what each wants is incompatible with what the other wants.

Conflict should never be ignored and left to fester. Conflict that is ignored can create irreparable damage to individuals, departments, or to the organization as a whole. It can jeopardize the image of the organization with the customer, or its ability to deal with suppliers. It can undermine the ability of the organization to achieve its objectives. It is always in the best interests of the organization to identify and address conflict at the earliest possible opportunity.

Resolving Conflict. No single best approach exists for resolving conflict. Instead, the approach to be used depends on the particular type of conflict in a given situation. If an agreeable solution can simply be identified, and the parties involved are not adamantly opposed to each other, then simple discussions utilizing brainstorming, multi-voting, and consensus may be sufficient. The brainstorming process can be used to identify several possible solutions, followed by a multi-voting process that narrows the list to a smaller, more viable list. Looking at the positive and negative attributes of each option may then result in agreement of all parties on what actions will be taken. This discussion can be aided by looking at each of the activities that would be required of each party in the conflict, the effort it would require, and the impact it would have, both on resolving the issues as well as on the relationship between the parties.

A more difficult situation arises when there may be several viable solutions but the parties involved do not appear to be prepared to work cooperatively in order to reach a joint decision. A process such as *interest-based bargaining* (also called *principled negotiation*) might then be used to attempt to satisfy as many interests as possible using the following steps:

- 1. Define the problem in a way that distinguishes it from the people involved.
- 2. Clarify the interests of the parties (as opposed to their positions on the issue).
- 3. Identify new and creative options beneficial to all parties.
- Determine objective criteria to be used to evaluate fairness of the outcomes for all parties.

Conflict resolution may also involve intervention by a third party. For example, a person trained in human process interventions might help parties involved in a conflict by engaging them in dialogue. Possible strategies that might be used include:

- Helping the parties avoid the factors that trigger conflict
- Setting guidelines for interaction of the parties
- Helping the parties find ways to cope with the conflict
- Identifying and eliminating the underlying issues
- Understanding each party's values
- Understanding and surfacing underlying assumptions

The last bullet above is frequently a source of conflict that can be easily overlooked. The parties involved in the conflict make assumptions that are typically based on perceptions, poor communications, rumors, and so on. Once these questions are surfaced, usually through continual probing by a facilitator, the assumptions can be acted on. They can be clarified or corrected to help reduce or eliminate the source of conflict.

Important Considerations When Dealing with Conflict. Conflict management is difficult when people take a "What's in it for me?" attitude. By contrast, a win/win approach offers the following benefits:

- Unified direction and platform for achieving the organization's goals and objectives
- Higher employee satisfaction, especially when active listening is used and the search for alternatives is expanded
- Improved health and safety of employees due to encountering less stress in their lives

When attempting to resolve conflict, the following important considerations should be addressed for the mutual benefit of all parties involved:

- Define the conflict as a mutual problem.
- Identify goals common to all parties.
- Find creative alternatives that satisfy all parties.
- Ensure that all parties understand their own needs and communicate them clearly.
- Emphasize mutual interdependence.
- Be certain that contacts are on the basis of equal power.
- Communicate needs, goals, positions, and proposals openly, honestly, and accurately.
- State needs, goals, and positions in the opening offer.
- Empathize with and understand others' positions, feelings, and frames of reference.
- Reduce defensiveness by avoiding threats, harassment, or inconveniencing other parties.

A conflict is not likely to be resolved successfully when either party does any of the following:

- Defines the conflict as a win/lose strategy
- Pursues his or her own goals or hidden agendas
- Forces the other party into submission
- Increases power by emphasizing independence from the other party and the other party's dependence on them

- Tries to arrange contacts based on power relationships
- Uses inaccurate or misleading communications
- Overemphasizes needs, goals, and position in the opening offer
- Avoids empathy and understanding of others' positions, feelings, and frames of reference
- Would rather both parties lose (lose/lose) than have the other party get his or her way

Negotiation

In order for multiple parties, whether individuals or groups, to work together to achieve common goals, there must be agreement on the goals, methods for achieving them, and what will occur when difficulties arise. Each party will typically have different values and priorities that need to be addressed, and that calls for negotiations resulting in an agreement acceptable to all.

Management–union negotiations are perhaps the most widely cited example, but negotiations occur as part of many other normal business processes such as:

- When establishing specifications for purchased components for a new product
- Setting performance measures for a process or department
- · Defining the desired outcomes for an improvement project
- Identifying personal development goals.

Examples of parties involved in negotiations include customers and suppliers, senior and middle management, project sponsor and team leader, Master Black Belt and Black Belt, and Black Belt and the project team members.

A key difficulty in the negotiation process occurs when the two parties approach the task as though there are only two sides to the situation, when in fact there are multiple views that could be taken. This is exacerbated when one or both parties take a win/lose attitude. When this occurs, the process becomes divisive and usually results in outcomes that are not seen as beneficial by either party.

There are four possible outcomes to a negotiation:

- Win/win
- Win/lose
- Lose/win
- Lose/lose

When both parties approach negotiation with an anticipation of achieving mutual benefit, the outcome is usually win/win. The other three orientations leave the parties striving not to lose, at any cost.

The concept of a principled negotiation that is based on a win/win orientation includes:

- Separating the people from the problem
- Focusing on interests, not position
- Understanding what both sides want to achieve
- Inventing options for mutual gain
- Insisting on objective criteria

Timing also affects negotiations, both long-term and short-term. For negotiations that recur on a long-term periodic basis (for example, labor agreements), the years between sessions allow the parties to "store up" their frustrations, and release them just as they come together to forge a new contract. At the same time, allowing a break between negotiation sessions allows the parties to back off from what may be an emotional discussion and reflect on their primary purposes.

Negotiations should take place in an environment that is conducive to open discussion and allows all involved to see each other face-to-face. Rather than presentations being made to each other, both parties should focus on conversations with each other. This means listening is critical in order to understand the other person's viewpoint. A set of ground rules or a third-party mediator may be useful if the negotiations involve a highly controversial or emotional subject.

Some techniques that have been found to improve the negotiation process include:

- Focusing on common objectives before discussing areas of differences
- Avoiding power strategies such as lying about one's priorities in order to get the other party to submit to lowered expectations
- Doing something for the other party, even if symbolic, to create positive energy
- Separating out discussion issues that are not interconnected so that they can be discussed based on their own merit
- Bringing in other parties that may have additional or different information about the situation

Additional techniques include:

- Identifying, up front, a range of acceptable outcomes—and why you want what you want.
- Determining the real intentions of the other party: the goals, objectives, and priorities.
- Being prepared with supporting information.
- Not rushing the process.
- Keeping the hardest-to-resolve issues for last.
- Being sensitive to face-saving needs of the other party.
- Being known as firm, fair, and factual.

- Always controlling your emotions.
- Evaluating each move against your objectives and assessing how it relates to all other moves.
- Being adept at formulating a win/win compromise.
- Being aware of the outcomes of the present negotiation on future negotiations (for example, periodic labor agreements).
- Actively listening and seeking clarity of expectations.
- Being flexible with your position and being able to step back and look at your position. Recognize that there may be multiple ways the same overall objective can be achieved.

Negotiating is a skill very much needed in the project management arena. Often, the project manager or leader has less authority than ongoing operations managers, and that necessitates negotiating for needed resources. This is particularly problematic when the project team is not solely dedicated to the project. Then one of the largest problems is ensuring that the project team members get sufficient time to work on the project.

Having effectively completed negotiations usually means that the relationship between parties will continue, and that the agreement should be documented. The agreement should include the standards against which compliance for each party will be compared, a frequency or process for reviewing performance, and what is to take place if a violation should occur.

Getting Back on Track

Even with the most diligent of project leaders, projects go awry. Many times, this derailment is due to conflict at the project, department, or even a higher level. This subsection has discussed the sources of conflict, conflict resolution, and additional important considerations associated with conflict.

However, dealing with conflict is not sufficient. To manage conflict successfully, it must be coupled with the understanding and application of the techniques of negotiation. The Master Black Belt must be skilled and adept at both of these techniques and must be able to apply them with ease at all levels of the organization.

Chapter 12

Executive and Team Leadership Roles

EXECUTIVE LEADERSHIP ROLES

Describe the roles and responsibilities of executive leaders in the deployment of six sigma in terms of providing resources, managing change, communicating ideas, etc. (Analyze)

Body of Knowledge II.C.1

It is critical during any strategic initiative implementation that the roles and responsibilities of executive leadership be explicitly identified and communicated to the leaders. Leaders must have full understanding and buy-in; otherwise, failure is assured. This section will detail the roles and responsibilities of executive leadership for a Lean Six Sigma implementation.

Executive Management

The definition and role of leadership have undergone major shifts in recent years. The leadership model that is most effective in the deployment of Lean Six Sigma envisions the leader as a problem solver. The leader's job is to implement systems that identify and solve problems that impede the effectiveness of processes.

This concept of leadership implies an understanding of team dynamics and Lean Six Sigma problem-solving techniques. Deployment of any culture-changing initiative such as the adoption of Lean Six Sigma rarely succeeds without management support. Initiatives starting in the rank and file seldom gain the critical mass necessary to sustain and fuel their own existence.

Successful implementations of Lean Six Sigma have unwavering support from the organization-level executives (also known as *top, senior,* or *executive management*). Therefore, it is imperative that executive management be involved in at least the following activities:

- Demonstrating support for the initiative by serving as a role model and exhibiting positive behaviors
- Communicating the importance of the initiatives
- Setting the vision and overall objectives for the initiative
- Setting the direction and priorities for the Lean Six Sigma organization
- Approving the project selection methodology
- Selecting/suggesting projects
- Ensuring projects are aligned to organizational strategy
- · Selecting project champions
- Allocating resources for projects
- Monitoring the progress of the initiative
- Modifying compensation systems/(may reward successful projects)
- Establishing new policies related to or brought about by Lean Six Sigma

Occasionally, it may be necessary to assess the level of support received from executive management. Figure 12.1 provides an example of a radar chart that identifies several dimensions of management support. Criteria for scoring each dimension can be developed easily to accommodate most organization's needs. Where necessary, plans may be put in place to fortify those dimensions that require strengthening.

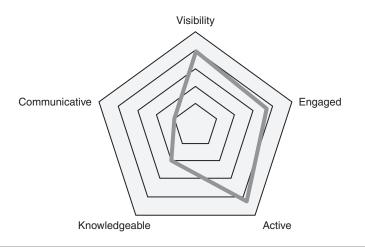


Figure 12.1 Dimensions of management support.

Champions

A *champion* is a Lean Six Sigma role associated with a senior manager (usually an executive) who:

- · Assumes ultimate responsibility for the success of the project
- · Serves as the liaison with executive management
- Serves as the team's backer
- Ensures that his or her projects are aligned with the organization's strategic goals and objectives
- Terminates projects that no longer align with strategic goals and objectives
- Provides the Lean Six Sigma project team with resources
- Removes organizational barriers for the project team
- · Participates in project tollgate reviews
- Asks appropriate questions of the project team
- Approves completed projects
- · Provides reward and recognition to project team members
- · Leverages (replicates) project results

A champion is also known as a *sponsor*. [Special note: Some authors make a distinction between a champion and a sponsor. They consider the champion as the role defined here. However, they consider a sponsor to be an executive who assumes the roles and responsibilities identified with executive management above.]

Process Owners

A *process owner* is a Lean Six Sigma role associated with an individual who coordinates the various functions and work activities at all levels of a process, has the authority or ability to make changes in the process as required, and manages the entire process cycle so as to ensure performance effectiveness. Process owners should be sufficiently high in the organization (ideally an executive) to make decisions regarding process changes. In many cases, the process owner is the same as the champion. Note that issues implementing improvements quickly surface when "process owners" are too low in the organization such that they must seek permission to make changes. When this occurs, projects can stall easily and gains may never be achieved. This is not to say that process owners should implement changes in a unilateral way or dictatorial manner. As always, careful coordination and communication are required.

It is only natural that managers responsible for a particular process frequently have a vested interest in keeping things as they are. They should be involved with any discussion of change. In most cases, they are willing to support changes but need to see evidence that recommended improvements are for the long-term good of the enterprise. A team member with a "show me" attitude can make a very positive contribution to the team. Process owners should be provided with opportunities for training at least to the Green Belt level.

Specific roles and responsibilities of the process owner include:

- Providing process knowledge
- Reviewing process changes
- Implementing process changes
- Assuming responsibility for the execution and maintenance of the control plan
- Ensuring improvements are sustained

In some cases, when the process is a large, cross-functional business process, the process owner may have lower-level process owners within each functional organization reporting indirectly to him or her. Consider the business processes shown in Figure 4.6. In these instances, the process owner will have the additional responsibility of providing coordination between the lower-level process owners.

LEADERSHIP FOR DEPLOYMENT

Create action plans to support optimal functioning of master black belts, black belts, green belts, champions, and other participants in the deployment effort. Design, coordinate, and participate in deployment activities, and ensure that project leaders and teams have the required knowledge, skills, abilities, and attitudes to support the organization's six sigma program. (Create)

Body of Knowledge II.C.2

This section will discuss the purpose of action plans as they relate to strategic plan deployment, provide an example of an action plan, address how action plans can be used to support resource allocations, and how they can be beneficial to the deployment of Lean Six Sigma.

Action Plans

Action plans, sometimes referred to as operational plans, are short-term (that is, accomplishable within a year). Projects too large for action plans because of scope and duration may require the establishment of a project office and a full-scale project plan. Of course, many components of the project plan can be sized appropriately to be executed using actions plans.

Westcott (2006) defines an *action plan* as "a detail[ed] plan to implement the actions needed to achieve strategic goals and objectives." He goes on to say that it is "similar to, but not as comprehensive as a project plan."

To transform the organization's strategic objectives into action, an action plan is needed. The level of planning detail that comprises an action plan is equivalent to a mini-project plan. Several action plans may be generated from one strategic objective. It is the systematic approach to initiating actions, and the follow-up of those actions, that closes the loop toward achieving the organization's strategy. An action plan may include:

- *Project identification and description.* The project description should be concise but sufficiently clear, as it is likely to be compared with other action plan documents. Both concise and accurate descriptions provide visibility into redundant projects and/or gaps in the deployment of strategies.
- *Project completion date.* This date should reflect all of the information about the project, including the scope, assumptions, availability of resources, start date, and so on. An accurate completion date allows for the reallocation of resources in a timely manner.
- *Linkage to the strategic objectives (and any pertinent mandates).* This linkage should be directly apparent and not implied.
- *Project objective.* The project objective, like the project description, should be concise and accurate. Although any given action plan may have multiple objectives, plans with multiple objectives should be reviewed carefully to determine whether they should be split into multiple action plans.
- *Project scope*. Include any boundaries on authority. Specify where and for whom the solution/implementation will be applied. Identify any expected limitations or constraints.
- *Deliverables.* Include both outputs pertaining to the project objective and outputs for managing the project and ultimate outcomes.
- *Criteria and measures by which progress will be assessed.* This might be in the form of milestones, events, activities, or any reasonable form, including metrics. If the criteria reflect milestones, events, or activities, these should be visibile on the project plan discussed below.
- Assumptions affecting the project. Be specific and as inclusive as possible. For example, the assumption regarding the availability of certain personnel with specific *knowledge*, *skills*, *and abilities* (KSAs) may be required to establish the project start date.
- *Description of the overall approach.* This need not be overly elaborate, but sufficient detail should be provided to assure the reviewers that the general direction of the project is sound.

- *Project start date.* Provide the best estimate of the project start date possible. This start date may be linked to some of the project assumptions discussed previously.
- *Estimated resources required.* Include time, personnel, facilities, equipment, tools, materials, and money. Specify KSAs requirements for personnel.
- Assessment of capacity and resources. This is an important issue but may not be known at the time the action plan is written. If known, be sure to address whether there are sufficient organizational capacity and resources to acomplish the objectives as this may affect the order in which the action plans are carried out.
- *Estimated benefits versus costs.* Attach a cost–benefit analysis as a supplementary document, if one has been completed. This should be generated with cooperation from the organization's financial department.
- *Project plan.* This includes a timeline and identification of each person responsible for each step in the project. The project plan might take the form of mini Gantt chart.

Figure 12.2 shows an example of an action plan for one of several projects linked to a strategic "cycle time reduction" objective. Note that the Action Plan form can also be used for other small projects such as corrective or preventive action, process changes resulting from regulatory changes, a kaizen blitz, an internal audit, and so on. This is because the Action Plan form is simply a means of generating action and making assignments.

Resource Availability

One of the principal concerns is how to superimpose and/or integrate action plans resulting from the strategic planning process with everyday business activities. For this to take place effectively, there must be a high level of management commitment and involvement. There also needs to be an organizational culture that permits lower-level individuals to challenge the actions they are being asked to accept. Such individuals must be heard, and receive respectful answers to their challenges without fear of retribution.

One of the several advantages of hoshin kanri planning, discussed in Chapter 1, is that care is taken to ensure that the plans are acceptable and doable by all involved. This includes a consensus on resources required.

The question is: Can time be made in the day-to-day activity schedule to integrate the work involved in carrying out the action plans? Or, is it just assumed that the time will somehow be found for these new actions, and nothing will suffer? In today's world, few organizations have the time to assimilate many new activities without sacrificing something. The strategic planning team needs to understand how far the organization can be stretched before something breaks.

Action Plan

| Project Title: Replace existing laser cutting machine | Plan No: Q-5 |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------|
| Troject Thie. Replace existing laser euting machine | Date Initiated: 032910 |
| Description: Determine, select, purchase, and install an appropriate laser cutting | Date Initiated: 032910 Date Needed: 091010 |
| machine that will meet our needs through 2015 | |
| machine that will meet our needs through 2015 | |
| | Team Leader: TMK |
| Linked to what (strategic objective, contract, policy, procedure, process, | Team Member: FAS |
| corrective or preventive action, customer mandate, or regulatory | Team Member: WPO |
| requirement)?: | Team Member: |
| Strategic objective: Cycle Time Reduction Project Objectives: To replace existing cutting machine with the most cost-effective | us lagon autting machine that maste |
| engineering specifications, capital equipment purchase policy, and production criter | |
| SMED-type setup requirements and accommodate manufacturing cell configuration | |
| thinking is applied in the plant. | r changes as needed when reall |
| Scope (Where & for whom will the solution/implementation be applied? What | limitations & constraints?); |
| Up to ten equipment vendors will be contacted with our Request for Proposal. Up to | |
| evaluated in detail. Up to three vendors' machines will be subjected to a live demor | |
| specifications. Preference will be given to U.S. vendors, if all other factors are equa | |
| Deliverables (include Outputs re Content, Outputs re Project Management, an | |
| Evaluation criteria | iu Outcomes). |
| | |
| Engineering specifications | |
| Request for Proposal | |
| Review of project status at weekly management meeting (time and costs exp | |
| By what criteria/measures will completion and success of project be measured | ?: |
| Deadline met | |
| Equipment meeting all specifications and production requirements selected, | , purchased, installed, tested and |
| turned over to production | |
| Purchase within capital project policy parameters | |
| Assumptions made that might affect project (resources, circumstances outside | this project): |
| Machine selected will not require pre-delivery design modifications. | |
| Describe the overall approach to be taken, data needed, processes to apply: | |
| A three-person team will be formed (leader from quality, members from engineerin | g and production). Within an |
| extended work-week (48 hours), the team will: | |
| Research appropriate vendors and their equipment | |
| Prepare requests for proposal from selected vendors | |
| Evaluate proposals and select machines to subject to demonstration testing | |
| Select and place purchase order for equipment | |
| Arrange for appropriate operator and supervisor training | |
| Provide necessary changes to procedures and work instructions Install, test, accept, and turn over machine to production | |
| instant, test, decept, and tain over indefinite to production | 0. 2011 |
| When should the project be started to meet the date needed/wanted?: March 29 | / |
| Estimate the resources required (time, KSAs for personnel, facilities, equipment | nt, tools, materials, money): |
| Quality Engineer for 16 hours per week for 22 weeks | |
| Production Engineer for 16 hours per week for 10 weeks | |
| Production Planner for 16 hours per week for 18 weeks T & L approace for trian to machine demonstrations \$5,000 | |
| T & L expenses for trips to machine demonstrations \$5,000 T & L expenses for triping for the provided and the provi | |
| T & L expenses for training first operator and supervisor \$3,000 Machine replacement (transportation, rigging, etc.) \$4,000 | |
| | |
| Machine cost \$850,000 Other expenses \$2,000 | |
| | |
| | |
| Is there sufficient organizational capacity and are resources sufficient to meet | |
| Is there sufficient organizational capacity and are resources sufficient to meet Estimate the benefits vs. costs value: Annualized decrease in setup costs = \$75,00 | 00. Annualized decrease in cycle tim |
| Is there sufficient organizational capacity and are resources sufficient to meet | 00. Annualized decrease in cycle tim Annual amortized machine cost = |

Outline the major steps and dates on page 2.

Figure 12.2 Example of an action plan document. *Source:* Adapted from Westcott (2006).

| | major steps to be taken, a projected start and responsible for each step. Attach any backup | data. | eacn step | and the | Plan No: Q-: |
|----------|------------------------------------------------------------------------------------------------|--------------------|---------------|----------------|-----------------------|
| Step No. | Activity/Event Description | Depends on Step | Start Date | Finish Date | Person Responsible |
| 1 | Form project team and launch project | | 032911 | 040111 | TMK |
| 2 | Prepare machine specifications and request for proposal | 1 | 040111 | 041211 | FAS |
| 3 | Research vendors | 1 | 040111 | 050311 | TMK |
| 4 | Select vendor and order machine | 2,3 | 050311 | 051011 | FAS |
| 5 | Train initial operator | 4 | 052411 | 052811 | WPO |
| 6 | Install machine | 4 | 062111 | 062511 | WPO |
| 7 | Conduct trial runs | 5,6 | 062811 | 071211 | WPO |
| 8 | Revise documented procedures and operator training materials | 7 | 072611 | 080211 | TMK |
| 9 | Train supervisor | 8 | 080211 | 083111 | WPO |
| 10 | Place machine in production schedule | 9 | 082311 | 083111 | WPO |
| 11 | Document project and disband team | 10 | 083111 | 091011 | ТМК |
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Action Plan — Steps

Figure 12.2 Continued.

Aside from daily activities that must take place to maintain the business functions, there are other potentially conflicting activities and programs. Consider the following examples:

- Several key business processes are being reengineered.
- A new ISO 2001 quality management system is being implemented.
- A new company-wide information processing system is being implemented.
- An outsourcing initiative that will result in downsizing or the elimination of existing functions is under way.
- A merger, acquisition, or divestiture of part of the business is scheduled for the second and third quarters.
- The introduction of a new product/service line of business is expected in the second half of the year.
- Upgraded machines and processes will be introduced.
- Heavy turnover of personnel due to new, large employers entering the same labor pool area will be taking place during the first half of the year.
- Introduction of Six Sigma and/or lean management principles and technology is under way.
- A major shift in marketplace (for example, new products/services threatening to obsolete present products/services) is under way and not expected to stabilize for at least two years.
- New restrictive regulations requiring mass retraining, new procedures, and controls will be released during the first quarter.

Obviously, if any of these or similar conflicts are under way, or soon will be, this must be taken into consideration in the deployment process. For example, assume one of last year's multi-year objectives (in response to an objective to "improve the effectiveness of organization business decisions") was to replace the present information system with a system fully integrated with all business functions, within the next three years. That means the system design, programming, testing, and implementation is still going on. And, like many of these total enterprise systems, it is placing a severe strain on existing resources. The strategic planning team must be able to make a clear assessment of the impact the information system that will conflict. And, if the persons responsible for the means do not challenge any conflicts, then it is those at the action level who will most feel the conflict in priorities and shortage of resources. Hence, they must be able to challenge the feasibility of completing the action.

Allocation of Resources

So far, we have seen that resources need to be allocated to the following three categories of work:

- Strategic objectives via the action plans
- Daily activities
- Specially planned or ongoing activities (see list above for examples)

Before action plans are actually launched, it may be beneficial to prepare a spreadsheet showing the various types of resources required against each of the three categories above, along with when and for how long a resource will be required. (Note: Many project planning software programs have the capability to provide such a document, as well as the ability to play out "what if" scenarios to determine the best mix with the least conflict.) Such a spreadsheet will provide insight into time gaps, conflicts, excessive manpower demands, and so on. When these issues occur, adjustments can be requested and a revised resource allocation plan prepared. This process can continue until priorities and conflicts are sufficiently resolved.

A key aspect of the above method for managing resource allocations is to recognize that the allocations must be revisited periodically to account for changes in plans. People leave, training replacements takes time, equipment breaks down, funds are jeopardized by external aberrations, and so on. No plan is ever stationary. Consequently, resource allocations must change as well.

Deployment of Action Plans

Action plans also provide the last opportunity to assess both the vertical and horizontal integration and alignment of higher-level plans. This is critical as some of the more traditional strategic planning processes often move directly from the objectives to action plans. This causes great difficulty in conveying a clear understanding of the linkage between objectives and action plans.

In traditional strategic planning, it is implied that action planning is a natural extension of the deployment process. Unfortunately, this is where the traditional approach has sometimes faltered for lack of commitment to support and execute the plans. The hoshin kanri planning process is intended to remedy this potential shortfall. The hoshin kanri approach literally involves everyone in the organization with the action planning that pertains to his or her function. The action plans must be prepared and sent back up the planning hierarchy for acceptance and integration before the implementation of the plan can take place. This catch-ball technique assures not only involvement of those persons responsible for implementing the action plans, but also achieves better buy-in to the plans.

The summation of all the action-planning details (that is, time and resources required, including dollars) is reviewed by successive organizational levels back up to the strategic planning team. At this point, the plans are given final approval or adjusted and sent back down the organization for replanning and return. This iterative process aids in communicating the appropriate elements of the strategic

plan to all employees and allows each employee to clearly understand what their contribution is to the overall plan.

When Action Plans Meet Lean Six Sigma

Generally, implementing any strategic initiative is difficult. Implementing a strategic initiative that inherently brings about change can be challenging and be perceived as threatening. Action plans can be helpful in this regard and, in particular, to Lean Six Sigma.

As we have seen above, action plans engage all levels and are designed to secure buy-in both horizontally and vertically throughout the organization while creating strategic and resource alignment. In addition, they formally transfer responsibility and accountability.

EXAMPLE 12.1

Consider an action plan for driving improvement that is deployed to a champion. For the champion to execute the action plan successfully, he or she may find it necessary to employ the services of the Lean Six Sigma organization to undertake a series of projects to achieve the level of improvement designated by the action plan. Subsequently, these projects will likely involve a Master Black Belt as a coach or mentor, Black Belts to lead teams, Green Belts to participate on teams, and most likely other participants, such as subject matter experts, who may come and go depending on the needs of the team. (Note: Some organizations use a Green Belt to lead project teams within his or her own organization.)

EXAMPLE 12.2

Consider an action plan for developing the internal training material that will be used to train Black Belts. Such a plan might specify a Master Black Belt as the team leader. Additional members might include other Master Black Belts, an outside consultant, or an instructional designer. The overall approach might be to decide on the content, sequence of material, source of material, who will write each module, and so on. This action plan would support the strategic initiative of deploying Lean Six Sigma.

EXAMPLE 12.3

Similarly to Example 12.2, an action plan is launched to acquire training material that will be used to train Black Belts. This action plan might specify a Master Black Belt as the team leader and other Master Black Belts as members. The overall approach taken might be entirely different in that the team might wish to develop a list of potential Lean Six Sigma consultants, narrow the list down to six, send a request for proposal to three and invite them in for on-site presentations, and so on. This action plan would support the strategic initiative of deploying Lean Six Sigma.

Part III Project Management

| Chapter 13 | Project Execution |
|------------|-----------------------------------|
| Chapter 14 | Project Oversight and Management |
| Chapter 15 | Project Management Infrastructure |
| Chapter 16 | Project Financial Tools |
| | |

Chapter 13

Project Execution

CROSS-FUNCTIONAL PROJECT ASSESSMENT

Appraise interrelated projects for scope overlap and refinement and identify opportunities for leveraging concomitant projects. Identify and participate in the implementation of multi-disciplinary redesign and improvement projects. (Analyze)

Body of Knowledge III.A.1

As the project pipeline fills up and the number of active projects grows, it is inevitable that project scopes begin to overlap. This can be minimized to some degree by strict adherence to the project qualification process. Recall that a key step in this process was the presentation of all projects in a common format. This positions all the projects in the same light and enhances the chance of detecting duplicated or overlapping scopes.

For example, some organizations have established project databases that permit keywords to be assigned to each project. This allows projects to be sorted by category, subcategories, and so on. This multiple sort capability provides the further refinement required to zero in on potential concomitant projects. Once discovered, these projects can be evaluated against others being submitted for qualification, in the pipeline, or currently active. If necessary, project scopes can be adjusted to avoid overlap. However, caution is recommended against adjusting the scope of active projects. Doing so may effectively derail the project, eventually causing it to be canceled or postponed.

An added advantage of the keyword approach is that similar projects in different business units, functional groups, sites, departments, and so on, can be identified. This provides the opportunity for different organizations to work together toward a common goal that each has identified independently.

EXAMPLE 13.1

Four functional groups are responsible for eight projects. These projects are aligned with each functional group as shown in Figure 13.1. Without regard to scope overlap or relationships to other projects, the projects were scheduled for execution according to Figure 13.2. Notice that project 4 is not scheduled during the specified time horizon and projects 3 and 5 are scheduled to start simultaneously. This haphazard manner (as we will see shortly) of scheduling projects may be due to several reasons including the existence of communication walls between functional groups, the inability to determine relationships between projects, or simply a lack of proper project oversight.

Now consider how the projects might be scheduled for execution if similar projects can be identified using a method such as the keyword assignment mentioned above. Once this occurs, the projects can be reviewed for scope overlap, and predecessor-successor relationships, as shown in Figure 13.3, can be determined. Scopes can be adjusted or projects divided into multiple projects to keep scopes synchronized. In some cases, projects might even be combined. Simply put, information is now available to make such informed decisions. Figure 13.2 can now be transformed into something similar to Figure 13.4. Figure 13.4 addresses the predecessor-successor relationships depicted in Figure 13.3.

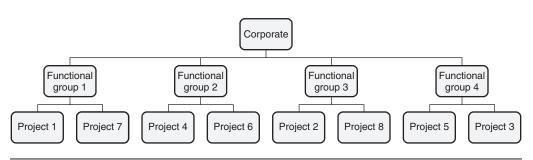


Figure 13.1 Projects aligned to functional groups.

It is important to note that the project databases just described need not be elaborate or costly. Usually, off-the-shelf software such as Microsoft Access or Excel is often sufficient to manage an organization's projects.

However, one drawback of using keywords is that the method relies heavily on a set of standardized keywords. If the time has not been taken to create a standardized set, the likelihood of surfacing potentially concomitant projects is diminished.

Managing the Megaproject

The concept of super- or megaprojects was raised in Chapter 3 in the context of multiphase planning. Recall that *megaprojects* are projects that are so large they can't

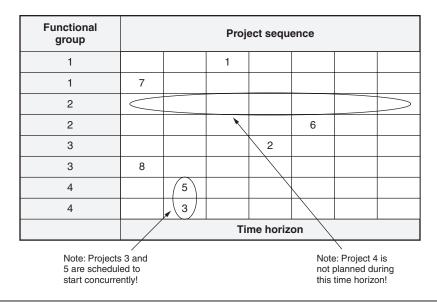


Figure 13.2 Project sequence as determined without regard to relationships between projects.

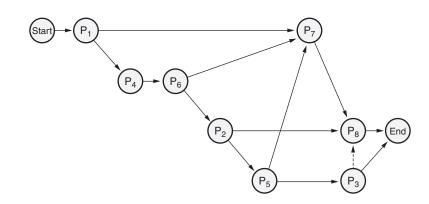


Figure 13.3 Activity network diagram illustrating predecessor–successor relationships.

| Functional group | Project sequence | | | | | | |
|------------------|------------------|---|---|---|---|---|---|
| 1 | 1 | | | | | 7 | |
| 2 | | 4 | 6 | | | | |
| 3 | | | | 2 | | | 8 |
| 4 | | | | | 5 | | 3 |
| | Time horizon | | | | | | |

Figure 13.4 Project sequence as determined with regard to relationships between projects.

be managed effectively. Consequently, they are decomposed into smaller, more manageable projects that can be completed in shorter times with fewer resources. When this occurs, it is common for the smaller projects to develop predecessor–successor relationships to each other.

Megaprojects often require the use of multidisciplinary teams because they frequently involve multiple functional groups, departments, and so on. Such teams may address the need to improve existing processes as well as develop new processes. Because megaprojects involve a number of projects that must be managed closely, they are frequently assigned a program manager. The program manager, in turn, has project managers for each project. This concept is illustrated in Figure 15.2.

EXECUTIVE AND MID-LEVEL MANAGEMENT ENGAGEMENT

Formulate the positioning of multiple projects in terms of providing strategic advice to top management and affected mid-level managers. (Create)

Body of Knowledge III.A.2

Chapter 2 described at great length how projects should be aligned with the organization's strategic plan. These concepts were illustrated in Figures 2.2 and 2.4. Chapter 12 introduced the concept of action plans and created the linkage between strategies and projects.

Consider Table 13.1. The first three columns outline the strategic plan breakdown from strategies to tactical plans to operational or action plans. The exact content of the plans is not necessary at this point, only the structure. Table 13.1 indicates that the first strategy requires two tactical plans and three operational (action) plans spread across four functional group owners (that is, A, B, C, and D).

EXAMPLE 13.2

Senior management sought strategic advice from a consultant regarding the best way to achieve its goals and objectives. After careful review and consideration, the consultant suggested a megaproject be established that focused on achieving each of the strategies. These are depicted in the fourth column in Table 13.1. Notice there is a single point of accountability for each project. (Strategy and tactical plan owners have not been shown for the sake of simplicity. Senior leaders often own the strategies while mid-level management often owns the tactical plans.) However, this does not preclude other functional groups from assuming support roles on projects and, subsequently, strategies.

| Strategy | Tactical plans | Operational (action) plans | Projects | Functional group (owner |
|----------|-------------------|-------------------------------|----------|----------------------------|
| 1 | 1 | 1 | 1 | A |
| | | | 2 | В |
| | | 2 | 3 | C |
| | | | 4 | А |
| | | | 5 | С |
| | 2 | 3 | 6 | С |
| | | | 7 | С |
| | | | 8 | D |
| | | | 9 | А |
| | | | 10 | D |
| 2 | 3 | 4 | 11 | А |
| | | | 12 | В |
| | | 5 | 13 | Е |
| | | | 14 | F |
| | | | 15 | C |
| 3 | 4 | 6 | 16 | F |
| | | | 17 | F |
| | | 7 | 18 | Е |
| | / | 1 | 19 | D |
| | 5 | 8 | 20/ | В |

Table 13.1 A structure for linking multiple projects to strategies for Example 13.2.

Each megaproject comprises a coordinated set of projects that focus on a particular strategy. The first strategy, for example, calls for ten projects. Each project will be assigned a project manager and a program manager who will oversee the megaproject along with the ten project managers. From III.A.1 above, the projects will be properly sequenced to fit within the time frame allotted to accomplishing the strategy. However, it must be recognized that projects outside of the ten comprising the megaproject may be introduced into the project-sequencing task. Consequently, program managers and project managers need to work together to ensure a holistic view is taken and all projects are scheduled properly within that view.

Table 13.1 raises another consideration of which program and project managers should be aware. Though not depicted in the table, it is quite possible that any

given project will be applicable to more than one strategy. For example, project 3 may support the achievement of strategy 2. Thus, it must be considered part of the second megaproject as well. As such, additional coordination is required between the supporting program and project managers for each of those megaprojects.

Megaprojects provide a focused way of advancing an organization's strategies. However, they require additional coordination among projects and functional groups to be successful.

PROJECT PRIORITIZATION

Prioritize projects in terms of their criticality to the organization. (Apply)

Body of Knowledge III.A.3

The prioritization of projects in terms of their criticality to the organization is addressed in Chapters 5, 6, and 16. Consequently, it will not be reiterated here. The reader is referred to those chapters.

Chapter 14

Project Oversight and Management

D very Lean Six Sigma project leader at one time or another will find it necessary to employ the principles of project management. Each phase of the DMAIC methodology will likely see the start and close of many non–Lean Six Sigma mini projects. For this reason alone, it is essential that the project leader understands these basic principles and uses them efficiently and effectively in the execution of the overall Lean Six Sigma project.

This chapter will address those basic principles, not from the Lean Six Sigma point of view, but from the project management point of view. What does that mean exactly? Recall Figure 4.1. This figure provided a simple flowchart for selecting the proper project methodology. Notice that project management is selected when the solution path is known. A known solution path means that each step of the project can be defined and executed, resources can be defined, risks assessed, and a plan established. If this is done well, the desired results will follow.

The mini projects mentioned above should have known solution paths to employ the project management approach. When they do, the tools, techniques, and approaches discussed in this chapter can be readily applied. When they don't, there may be an opportunity to spin off another Lean Six Sigma project.

PROJECT MANAGEMENT PRINCIPLES

Oversee critical projects and evaluate them in terms of their scope, goals, time, cost, quality, human resources requirements, communications needs, and risks. Identify and balance competing project demands with regard to prioritization, project resources, customer requirements, etc. (Evaluate)

Body of Knowledge III.B.1

What Is a Project?

Projects exist for producing results. Portny (2010) states that three components must be present for an activity to be defined as a project. These components are

depicted in Figure 14.1. Resources include both the financial component and the human capital component. These components define boundaries or constraints for the project. Without any one of them, the concept of a project evaporates. For example, consider a project without a scope. What would be accomplished? How about a project without a schedule? When would it be complete? Without any resources, who would do the project and how would it be paid for?

Williams (2008) offers what she calls the *balanced quadrant* concept depicted in Figure 14.2. Each quadrant represents one aspect of a project. Managing a project requires maintaining a balance among the four quadrants. Consider the following examples:

- An increase in scope results in an increase in time and cost
- An increase in quality results in an increase in cost and perhaps time
- A decrease in cost results in a decrease in scope, time, and/or quality

In most cases, the project manager can not make balancing decisions alone. Often, the internal and external stakeholders must be consulted, particularly when cost is a consideration.

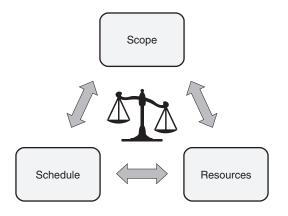


Figure 14.1 The components of any project. *Source:* Adapted from Portny (2010).

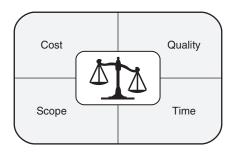


Figure 14.2 The balanced quadrant. *Source:* Adapted from Williams (2008).

Fundamental Principles

In *The Certified Six Sigma Black Belt Handbook,* Kubiak and Benbow (2009) define project management as "the entire process of managing activities and events throughout a project's life cycle." While this definition was suitable for the *Black Belt Handbook,* an expanded definition is needed for the *Master Black Belt Handbook.*

Project management is the discipline of planning, organizing, and managing resources to bring about the successful completion of specific project goals and objectives and is categorized into five component processes (sometimes called life cycle phases): *initiating, planning, executing, controlling,* and *closing*. The Project Management Institute notes that project management is the "application of knowledge, skills, tools, and techniques" to activities to meet the requirements of a specific project.

Occasionally, *project management* is used interchangeably with *program management*. However, program management is actually a higher-level activity often involving the management of multiple projects with a common goal.

In addition to the five life cycle phases, each project progresses through four stages from start to close. The relationship between the stages and life cycle phases is depicted in Table 14.1. Notice the iterative relationship that exits between the second and third stages. The center column in this figure identifies the various outputs from each stage. Many of these outputs make use of specific tools, forms, or templates to help the project manager move efficiently through the project.

The Five Processes

As mentioned above, projects are conducted through the application of a series of five processes. Each of these processes will now be discussed in detail.

Initiating. This is a critical process in any project as it sets up the project for success or failure. This process includes creating the project charter, establishing the business need, clarifying the project goals, defining the scope, estimating resource requirements, and performing a cost–benefit analysis. Let's address each of these in more detail:

- *Creating the project charter.* The charter is a formal document that legitimizes the project, establishes the team as an organizational entity, and provides authority to the team leader to request resources and make assignments. Also, it typically includes most of the elements in the following bullets.
- *Establishing the business need.* This is similar to the problem statement used in the creation of a Lean Six Sigma charter.
- *Clarifying goals.* What is the project trying to accomplish? Goals, in the general context, will not necessarily be stated in terms of reducing cycle time or defect levels. However, they must be very specific and measureable. Ideally, they will be SMART!

| Stage | Outputs | Life cycle phases | | |
|-------------------------|-------------------------------------------------------|--------------------|--|--|
| Starting the project | • Charter | Initiating process | | |
| | Documentation of need | | | |
| | • Clear goals | | | |
| | Quality requirements | | | |
| | • Scope | | | |
| | Initial resource requirements | | | |
| | Cost–benefit analysis | | | |
| | • Approval to proceed to next stage | | | |
| Organizing | Communications plan | Planning process | | |
| and preparing | • Quality plan | | | |
| | • WBS | | | |
| | • Resource plan | | | |
| | • Project plan | | | |
| | Risk management plan | | | |
| | Approval to proceed to next stage | | | |
| Carrying out | Project results | Executing process | | |
| the work | Progress reports | Controlling and | | |
| | Communications | monitoring process | | |
| | Develop tracking systems | | | |
| | Team management/development | | | |
| | Approval to proceed to next stage | | | |
| Closing out | • Demonstrating results | Closing process | | |
| the work | • External and internal approvals for project results | | | |
| | • Lessons learned | | | |
| | • Transitioning team members to new assignments | | | |
| | Closing project accounts | | | |
| | Portmortem and lessons learned | | | |
| | • Recognizing the team | | | |
| | Adjourning team | | | |

Table 14.1 The relationship of the project stages to the life cycle phases.

- *Quality requirements.* These include any high-level requirements that affect quality performance, such as operating environment, expected life, and so on. These requirements should also include reliability considerations.
- *Defining the scope*. Included in the scope are any limitations, boundaries, or authorities that must be followed. Documents closely related to the scope document or used as an alternative include: market requirements document, business requirements document, technical requirements or specifications document, project request, statement of work, project profile, work order, and contract.
- *Estimating resource requirements.* At this stage of the project, all that may be possible is a rough estimate of the resources required. This estimate may be limited to identifying the core project team and perhaps a percentage of their time, if they are not assigned full time.
- *Performing a cost-benefit analysis.* This was addressed in Chapter 10 and should be performed to assure the financial viability of conducting the project. This is particularly important when projects are simply handed down by management and no rigorous selection process is in place.
- *Obtaining the appropriate approvals.* These must be obtained prior to moving to the next stage and process. Approvals may be required from both internal and external clients and stakeholders.

In addition to the above, it is both beneficial and prudent to conduct a *stakeholder analysis*. Such an analysis will help the project leader understand the impact each stakeholder may have on the project and to develop plans to mitigate any negative reactions. Recall Figure 5.6, which depicts a rather sophisticated example of a stakeholder analysis, while Table 5.1 provides a similar, but simpler, version.

Planning. Planning, while time-consuming, is essential for keeping projects on track and moving them toward their specified goals. Consequently, many types of plans must be developed in detail. These include:

- *Communications plan*. A *communications plan* defines who will deliver, what will be communicated, to whom, how often, and by what means. Communications could include status reports, presentations, newsletters, and so on. Methods may be either formal or informal. In all cases, though, the communications plan must reflect the needs of the audience. Table 14.2 illustrates a potential form for a communications plan. Also, remember, the RACI matrix discussed in Chapter 3 identifies individuals who require consultation or must be informed.
- *Quality plan.* During the planning process, the quality requirements must be understood and developed in detail. The plan should include details on test plans, test equipment, calibration and metrology, supplier restrictions, personnel qualifications, training, certifications, and so on. Anything that could affect quality should be reflected in a plan.

| Table 14.2 Constructing a communications plan. | | | | | | |
|------------------------------------------------|--------------|---------|----------------|-----------|--|--|
| Who delivers | What message | To whom | By what method | How often | | |
| | | | | | | |
| | | | | | | |
| | | | | | | |
| | | | | | | |
| | | | | | | |

 Table 14.2
 Constructing a communications plan.

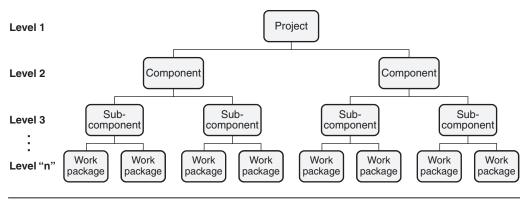


Figure 14.3 A generalized work breakdown structure.

- Work breakdown structure (WBS). A work breakdown structure is a hierarchical decomposition of the work content for a given project. The WBS permits the project manager to systematically break the work into smaller, more manageable sizes that can be planned, executed, and tracked more efficiently. Figure 14.3 represents a generalized work breakdown structure. The project is first decomposed into small modules as shown in level 2. This level is subsequently broken down further into even smaller modules of work as shown in level 3. This process continues down to the level "n." This level represents the smallest work contents that must be tracked in order to manage the project successfully. At level "n," the work packages (also called *tasks*) can be assigned to individuals for execution. Time, material, and expenses are collected at this level and can be summarized upward to any level desired. Thus, considerable thought must be given to the development of the WBS.
- *Project plan and schedule.* This plan can be developed in several ways depending on the specific needs of the project. For example, plans may be developed using Gantt charts, activity network diagrams, and PERT and CPM methodologies, all of which have been addressed in Chapters 13 and 17 of *The Certified Six Sigma Black Belt Handbook*.

Originally, these methodologies were quite distinct. However, the availability of low-cost, sophisticated project planning software has blurred their distinction considerably. In addition, many software packages simultaneously integrate resource planning.

As noted in the previous bullet, the WBS is developed to assign and track work. Consequently, the work packages should appear on the project schedule. However, some of the off-the-shelf software available permits the whole WBS to be entered, thus giving the project leader increased visibility into project performance.

- *Resource plan.* The resource plan should identify the need for funding as well as human capital. If specific skills are required, these should be identified, including when they will be needed and when they will be released. Remember, specific skills are limited and may be in demand by other projects. This information is critical, particularly when it comes to resolving resource conflicts.
- *Risk management plan. Risk* is the probability of not achieving a project goal or schedule, budget, or resource target because something did or didn't occur. Risk may be negative, in the form of threats, or positive, in the form of opportunities. Every project incurs some degree of risk. Risk can quickly send the balanced quadrant (Figure 14.2) out of balance. The questions the project leader must address are:
 - What is the risk factor? This must describe the risk event clearly and concisely in a manner similar to the way the potential failure mode or potential cause columns in FMEA documents are completed. Table 6.1 provided a useful starting point for identifying various types of risk.

Often, it is useful to categorize risks in a meaningful way. For example, risk might be categorized as: schedule, budget, technical, quality, personnel, and so on. Categorization can bring additional insight into the risk management process as it is possible that a risk strategy that applies to a particular risk event could apply to more than one event in the same category.

- What is the magnitude of the risk? This question addresses the issue of quantifying the probability of the risk event occurring. Assessing the probability of risk events occurring may be based on history of like projects, subject matter expert opinions, and last, but not least, guesswork.
- How the can risk be managed? Generally, there are four ways that risk can be managed. They are: avoidance, transfer, monitoring, and mitigation. Avoidance refers to the practice of eliminating the risk factor. For example, falling back on a tried-and-true software package rather than introduce a new, flashier, but relatively unproven version. Transfer refers to moving the risk to another party or individual and allowing them to assume the risk responsibility. For example, contracting a construction job to an outside contractor

rather than have it completed by in-house personnel. Another example, though simple, would be buying insurance. *Monitoring* refers to continuing to observe low risks where the cost of mitigation or avoidance is too high. The risk is monitored until the severity level is no longer acceptable. *Mitigation* refers to minimizing the impact of the risk. For example, splitting a contract across multiple suppliers rather than one supplier. Although this may introduce additional risks of other forms, it limits the impact of the failure of a single supplier.

The above four ways address managing risk before events occur. Once a risk event occurs, some action must be taken. This action is described by a contingency plan. Let's consider an organization with multiple suppliers of the same part. If a supplier should fail, the contingency plan might be to immediately move the work to the remaining suppliers, if their capacity or schedule permits. If their capacity or schedule does not permit this, perhaps the work is moved to a backup supplier who is not already performing the work. Consider the impact of the March 2011 Japanese earthquake on paint supplies for several large automobile makers; a lack of a contingency plan affected production.

Risk management plans can be documented in many different ways. Certainly, the FMEA document is one such form. Recall Figure 6.1. Although Minitab Inc. originally prepared this example for use with a Lean Six Sigma project, the format is equally applicable for any project type.

Executing. Many organizations like to start here because they want to demonstrate that action is taking place. (I once had an executive tell me, "We're not a planning organization, we're a doing organization" in response to my request for time to establish a project plan.) Skipping the first two processes (which is equivalent to skipping the first two stages) will surely send most projects into a death spiral from which there is usually no recovery. However, even within the "executing" process, there is a "preparation" subprocess.

This process includes several key activities aimed at ensuring the project runs smoothly:

- Assigning roles and tasks. This must be done with the utmost specificity and clarity. In addition, each team member must understand how their roles relate to the other team members. The RACI matrix discussed in Chapter 3 helps provide the necessary clarity.
- *Establishing team norms*. This establishes the operating rules and environment for the team. Team norms include: communication protocols, treatment of other team members, the process for making decisions, the process for resolving conflicts, and, in some cases, consequences for unacceptable behavior. Team norms should be well thought-out in advance, with full participation of the team. Team members joining the team late should be apprised immediately of the norms.

• *Establishing tracking systems.* Some organizations have project management systems that must be loaded with data to formally establish the project and open the necessary cost accounts. Other organizations leave it up to the discretion of the project leader, who may use tracking systems built entirely of spreadsheets.

Also, this activity includes familiarizing team members with the work breakdown structure, reporting due dates, expenditure guidelines, and so on.

• Announcing the project to the organization. As trivial as this may sound, it is quite important. Making the organization aware minimizes the opportunity for conflicts and places it in a position of authority for requesting resources. In many organizations, there are formal channels for making such announcements.

At this point, the proverbial rubber meets the road and the team is ready to begin making results happen. To remain on track and progress toward the established goals on schedule, the following activities must take place:

- *Tasks must be accomplished.* Simple enough. Plan the work. Work the plan.
- *Quality must be assured.* The work performed must conform to the project quality plan requirements and any appropriate standards or guidelines. Frequently, verification and validation tasks must be performed.
- *The team must be managed.* Recall that teams pass through multiple growth stages (forming, storming, norming, performing, adjourning, and recognition) and, on occasion, regress to prior stages only to move forward once more. It is the job of the project leader to manage the team through these stages as quickly and as painlessly as possible. If needed, the project leader may elect to bring in an experienced facilitator.

In addition to the above tasks, the project leader will have to review or have reviewed results, reassign tasks, and manage resources.

• *The team must be developed*. Although the team goes through the series of growth stages mentioned previously, there will be times when team members or the team as a whole requires special training or development. Sometimes this is predictable. For example, consider a software project that requires special skills in a new programming language. Of course, the software engineer assigned to the team does not possess the necessary skills. Fortunately, a schedule was developed that allowed the engineer to secure the proper training just prior to its need. In other cases, development needs may not be foreseeable. Consider the situation where the team becomes stuck in the storming stage. The project leader determines that the team as a whole requires team member training. Such training will permit the

members to work in a more unified and cooperative manner so that they can progress to the other growth stages.

• *Information must be shared.* Sharing information means communicating. This means acting on the communications plan previously developed. Also, it means that teams members are the first to receive team and project-related information from their project leader and each other. Team members should never be blindsided by their leader or each other. It only acts to create discord and demoralize the team.

Controlling (and Monitoring). This process involves comparing actual performance to plan. Recall that the planning process produced numerous plans that require ongoing tracking. As a result, information must be collected routinely and measurements generated so that plan-versus-actual comparisons can be made, variances computed, and corrective actions taken to remedy unfavorable variances or other relevant project metrics.

In addition, the project leader must be on constant guard against scope creep. Almost assuredly this will occur, particularly if the project is not strong. When it becomes necessary to adjust the scope for whatever reason, the project leader must work to introduce the scope change smoothly and efficiently into the project as well as all appropriate plans.

The *planning* and *executing* along with the *controlling* processes must work together in an iterative manner as depicted in Table 14.1. Consequently, the project may alternate between the second and third stages depending on the stability of the plans and the execution processes. Perturbations and unforeseen circumstances can significantly lengthen the duration of these phases, causing cost and schedule overruns and the loss of team members. This may create a very disturbing effect on the project team, causing further performance degradation.

Closing. Closing a project sounds like a trivial activity. However, in many organizations, projects never officially close. Instead, they seem to quietly dissolve for a wide variety of reasons. For example: the project sponsor or champion loses interest, takes on a new position, or leaves the organization; funding dries up; team members become unavailable due to transfers or departures; and so on. In some cases, team members may be caught in a limbo state, no longer able to return to their home organizations and now without a job.

For the above reasons and more, it is important that a project be closed out formally. The *closing* process focuses on:

- Demonstrating goals as having been achieved
- Ensuring all appropriate approvals have been obtained from internal and external clients and stakeholders
- Closing all project accounts, particularly those where time, material, and expenses can be charged
- · Conducting a postmortem and gathering lessons learned
- Finding new assignments for team members, if necessary

- Recognizing the team members in a fair and equitable manner
- Adjourning the team

Ideally, the lessons learned from the postmortem are captured for the benefit of the entire organization, not just a walk down memory lane for the team. Progressive organizations formally capture lessons learned in some form of a knowledge management system. Project leaders and other members of the organization have the ability to access the system. Hopefully, learning is transferring to future projects.

Managing Multiple Projects

Rarely do project leaders have the luxury of managing one project at a time. However, if the same tools, formats, forms, and templates are used to manage multiple projects, the ability to manage across projects becomes significantly less painful. Furthermore, it is easier to summarize project resource requirements to higher organizational levels beyond the project leader.

Consider the benefit of using the same template or plan format to identify resources for each project. The ability to quickly identify gaps and conflicts will become easier almost immediately. Conflicts can be resolved first among project leaders or escalated, if necessary, for prioritization.

Of course, this idea can be extended across all the project management processes. Simply put, standardization is key to managing multiple projects.

MEASUREMENT

Support and review the development of an overall measurement methodology to record the progress and ongoing status of projects and their overall impact on the organization. (Evaluate)

Body of Knowledge III.B.2

Every project requires some measure of progress and, in many cases, multiple forms of measurement must be used to gauge the overall health of the project. Project measures might be used to answer basic questions like:

- Is the project on track to finish on time?
- Is the project expending funds within budget?

Measures provide the project leader with information to monitor and communicate the progress of the project and to take corrective actions, when necessary.

Let's consider some of the common forms of measures used on most projects at the task (work package) level:

- *Planned value (PV).* This is the approved budget for work scheduled for completion by a specific date. It is also known as the *budgeted cost of work scheduled* (BCWS).
- *Earned value (EV).* This is the approved budget for the work actually completed by a specific date. It is also known as the *budgeted cost of work performed* (BCWP).
- *Actual value (AV).* This is the actual cost incurred for work completed by a specific date. It is also known as the *actual cost of work performed* (ACWP).
- *Budget at completion (BAC).* This is the total planned value for a given task at the end of the project.
- *Estimate at completion (EAC).* This can be simply an estimate or it can be calculated as:
 - 1. Assumption: cost performance for the remainder of the project will revert to what was originally planned:

EAC = Approved budget for the entire task – Cost variance for work performed to-date on task = BAC + AC - EV

2. Assumption: cost performance for the remainder of the project will continue as it has for the work performed to date:

$$EAC = \frac{BAC}{Cumulative CPI}$$

• *Estimate to complete (ETC).* This can be simply an estimate or it can be calculated as:

$$ETC = BAC - AC$$
 to-date

• *Cost variance (CV).* The cost variance is the difference between the amount budgeted for the work performed and the actual cost of the work performed. It is calculated as:

$$CV = EV - AC$$

If the CV is positive, the project is under budget. If the CV is zero, the project is on budget. If the CV is negative, the project is over budget and corrective action must be taken.

• *Schedule variance (SV).* The schedule variance is the difference between the amount budgeted for the work performed and the planned cost of the work performed. It is calculated as:

$$SV = EV - PC$$

If the SV is positive, the project is ahead of schedule. If the SV is zero, the project is on schedule. If the SV is negative, the project is behind schedule and corrective action must be taken.

• *Cost performance index (CPI)*. This is a dimensionless index used to measure the project's cost efficiency. It is calculated as:

$$CPI = \frac{EV}{AC} = \frac{BCWP}{ACWP}$$

If the CPI is greater than 1, the project is under budget. If the CPI is equal to 1, the project is on budget. If the CPI is less than 1, the project is over budget and corrective action must be taken.

• *Schedule performance index (SPI).* This is a dimensionless index used to measure the project's schedule efficiency:

$$SPI = \frac{EV}{PV} = \frac{BCWP}{BCWS}$$

If the SPI is greater than 1, the project is ahead of schedule. If the SPI is equal to 1, the project is on schedule. If the SPI is less than 1, the project is behind schedule and corrective action must be taken.

• *Run rates.* This is simply a projection estimate of future performance based on the assumption that the remainder of the project will continue as it has for the work performed to date. These estimates may be 12-month, year-end, or project-end projections. For example, consider an 18-month project. The first month's results are in. These figures would be multiplied by 18 to obtain the run rate for the entire project duration. Run rates can be highly misleading since they project in a linear manner whereas most cost curves tend to follow a shallow "S" as shown in Figure 14.4.

Figure 14.4 also illustrates the various cost curves associated with a project and many of the measures just discussed. The assessment date in this context is usually the end of a reporting period such as a month. In this particular figure, the project is one month behind schedule, but under budget.

In addition to the above set of measures, the project leader may need to include measures of quality. For example, if the project involves the production of a component, the project leader might want to consider including a defect rate or cycle time. In general, the performance against any plan created should be measured. This includes intermediate goals and milestones.

MONITORING

Apply appropriate monitoring and control methodologies to ensure that consistent methods are used in tracking tasks and milestones. (Apply)

Body of Knowledge III.B.3

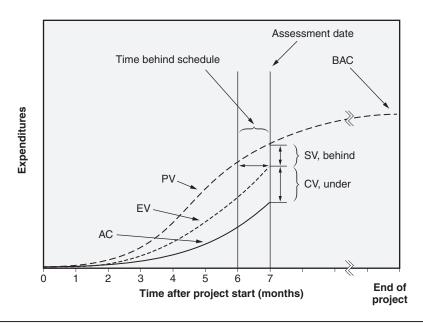


Figure 14.4 Various performance indicators—a graphical approach. *Source:* Adapted from Portny (2010).

Recall that *controlling and monitoring* was discussed previously as one of the five project management processes. Briefly, *monitoring* is about measuring the results or actuals from the *execution* process, comparing them to the plans from the *planning* process, and taking corrective action on undesirable differences.

Due to the overarching and comparative nature of the *controlling and monitoring* process, the project will iterate between the *planning* and *executing* processes. This is demonstrated in Table 14.1.

PROJECT STATUS COMMUNICATION

Develop and maintain communication techniques that will keep critical stakeholders and communities apprised of project status, results, and accountability. (Create)

Body of Knowledge III.B.4

Ongoing communication throughout the duration of a project is essential. Recall that two of the outputs from the planning process were a communications plan and a RACI matrix. An example of the format for a communications plan was depicted in Table 14.2, and a RACI was shown in Table 3.1. These documents help direct the flow of communication both within and outside of the team.

Communication Within the Team

Communication within the team should take place on a regular basis. All communication within the team should be open, honest, and with no fear of retribution. In general, team members should be aware of any project problems or issues before anyone outside of the team. No team member should ever be blindsided because project information bypassed him or her. Also, team members need to be apprised of their own performance as a team member. Too often, this valuable activity is ignored either because it is uncomfortable for the project leader or human capital resources are limited and the project leader is reluctant to alienate a scarce resource.

Many teams operating in today's large organizations may never come faceto-face or, for that matter, ever see one another. Fortunately, technology facilitates communication. However, careful planning for communication is still required. Accommodations for extreme time zone differences might be necessary, and technologies may be new to some sites and localities. Therefore, it is important to test any form of technical communication before operating it in a live environment.

Communication Outside the Team

As with communication within the team, communication outside the team should also take place on a regular basis. Such communications typically include status reports and presentations. Status reports should be brief. Generally, a bullet format is sufficient when grouped under a series of headings. Typical headings include:

- Accomplishments since last report
- Problems, issues, or concerns
- Actions being taken
- 30-day plans

A recipient of a status report in the above format might conclude that the project leader: is indeed making progress; has incurred problems, but has recognized them (some leaders don't); has taken mitigating action (therefore, I do not need to intervene); and is proceeding forward once more. The recipient reads the status report and completes it with a feeling of confidence in the project leader. Poorly written status reports invite criticism, and occasionally, unwarranted scrutiny and intrusion.

Presentations are another form commonly used to communicate outside the team and are usually delivered to higher levels of the organization. The project leader may request a spot on the sponsoring executive's staff meeting agenda or may be invited to attend.

While the above bullet format is acceptable, more detail is required. Remember, the presentation will be interactive and usually face-to-face. Therefore, messages need to be clear and crisp. There is no room for ambiguity. Also, the presenter should expect interruptions, and questions out of order from, and not related to, the intended delivery sequence. Hence, the presenter's motto is: "Be prepared!" As with poorly written status reports, a poorly delivered presentation can be a disaster that makes recovery difficult and often brings unwanted help.

SUPPLY/DEMAND MANAGEMENT

Generate accurate project supply/ demand projections, associated resource requirements analysis, and mitigate any issues. (Create)

Body of Knowledge III.B.5

Keeping the Pipeline Full

Two important aspects of supply and demand management are the ability to keep the pipeline full and the development of trained resources to sustain the Lean Six Sigma deployment initiative.

Chapters 5 and 6 addressed the processes for project identification, qualification, and selection. Also discussed was the idea of holding project identification workshops. Once developed, these processes can be triggered at any time or based on predetermined triggers (for example, the number of projects remaining in the pipeline). Hence, they can be used to keep the pipeline "full." Subsequently, the project prioritization and assignment processes follow when appropriate and assign available resources.

Predicting Resources

The one question that remains and is troublesome for many organizations is how many resources do you train to meet project demands. The answer is organizationally dependent and represents a balance among the following key factors:

- Strategies for the Lean Six Sigma deployment
- Attrition and churn out of both the Lean Six Sigma department and the overall organization
- Inflow to the Lean Six Sigma department from both inside and outside the organization
- Inflow of Green Belts from both inside and outside the organization

The above is not a complete list, but is meant only to establish a line of thinking. There may be many other factors that are important in your organization that should be added to the list.

Recall that strategies for the Lean Six Sigma deployment were the subject of Chapter 2. A bottom line of at least one of these strategies will inevitably be a resource plan via an operational plan that identifies the requirements for belts by type and time frame. Of course, at initial deployment, the plan will be subject to start-up fluctuations and hopefully will reach a reasonable state of predictability over time. The reasonableness of the predictability can be enhanced when one of the key themes of Chapter 1 is followed: horizontal and vertical planning and integration.

CORRECTIVE ACTION

Facilitate corrective actions and responses to customers about the corrective action and its impact. (Apply)

Body of Knowledge III.B.6

Generally, corrective action is required when performance to plan is lacking. Corrective action may take different forms depending on the root cause of the problem. Tools for determining root cause are discussed in *The Certified Six Sigma Black Belt Handbook*.

One of the basic rules of quality is to contain the problem. Once the problem is contained, corrective action can be taken to bring the project back on track, and preventive action taken to prevent problem recurrence.

If a project were to fall behind schedule, and additional resources were not available or the ability to schedule tasks in parallel was not sufficient to make up lost time, the project leader has several alternatives. But they may not be acceptable to all stakeholders. These include:

- Accepting more risk in the project at a later time where such risk provides the opportunity to shorten the schedule
- Accept the schedule as late and manage to the remainder of the planned schedule
- Reschedule the remainder of the plan using the current due date
- Reschedule the remainder of the plan without current constraints and attempt to sell it to the stakeholders

The above list is certainly not all-inclusive. Consider it a starting point.

Clearly, some stakeholders will object to some of the proposed recovery plans. However, responsibility for providing a logical and convincing argument regarding the most appropriate recovery plan lies with the project leader. Unfortunately, there are situations in which there are no-win scenarios. This usually occurs when project due dates have been established before initial budget and resource estimates have been made or project goals have been established. Such no-win scenarios are common in the aerospace industry or when dealing with the department of defense. In these situations, most contractors must begin work on a contract before the contract is signed, technical requirements completed, and/or funding released by the government. Consequently, such projects, more often than not, result in budget overruns and late deliveries.

Chapter 15

Project Management Infrastructure

GOVERNANCE METHODS AND TOOLS

Develop governance documents, tracking tools, and other methodologies that will support project success. (Create)

Body of Knowledge III.C.1

In Chapter 3, governance was discussed in the context of a Lean Six Sigma deployment. In this chapter, governance will be addressed in a more general sense.

The purpose of governance is to provide a consistent approach for managing a project. As such, it may include the following:

- *Governance structure*. Governance structures vary widely. However, many have elements similar to those shown in Figure 15.1. Notice the inclusion of the *project steering committee*. This committee generally comprises mid-level management and provides operational support and guidance to project managers. Figure 15.2 provides an expanded view of "project management." Generally, when multiple projects are involved or when multiple projects support a common objective, a program manager may be named to provide oversight to this group of projects.
- *Roles, responsibilities, and accountabilities.* Each of these is defined in detail by team or committee and, in some cases, by individual (for example, project manager). Frequently, a RACI is included to provide clarification. See Table 3.1 for an example of a RACI matrix. Also, it is important to keep in mind that there should be single points of accountability with sufficient authority to make decisions. This ties in with the next bullet.
- *Decision-making authority.* This establishes boundaries regarding who can make what type of decisions or up to what threshold (that is, dollar limit). For example, the project manager is authorized to sign a purchase order up to and including \$25,000. Purchase orders above

that amount require the signature of the chair of the project steering committee.

- *Status reporting.* This specifies the frequency and types of reports required as well as the recipients of each report.
- *Project organization structure.* Some organizations require all project teams to use the same project structure while others permit the project manager flexibility to adapt the team structure to fit the project needs.

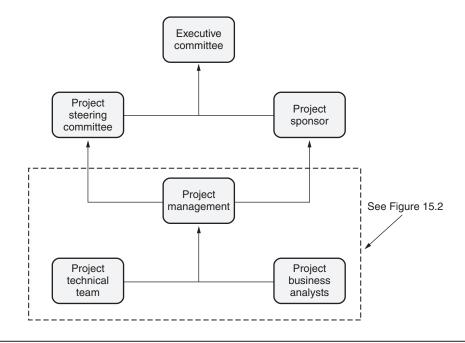


Figure 15.1 A generalized project management governance structure.

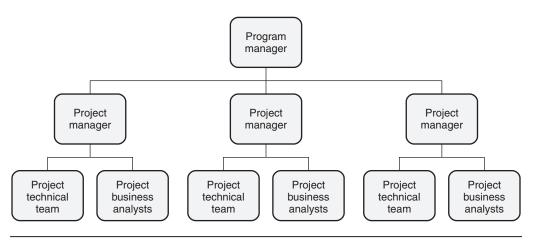


Figure 15.2 An expanded view of the project management governance structure.

- *Conflict resolution or escalation practices.* This item is occasionally included in the governance plan. It is useful in that it allows for early resolution of issues before they manifest into major problems that can significantly affect or stall a project.
- *Tools and systems to be used.* Many organizations use project management software packages. Use of these systems is often mandatory since they are expensive and senior leaders see their use as beneficial. Generally, these packages include all of the necessary templates to execute a project from start to completion. Examples of such templates include:
 - Risk analysis
 - Project plan
 - Meeting agenda
 - List of project assumptions and ground rules
 - Project charter
 - Issue log
 - Question log
 - Status report
 - Change request forms
 - Change request log
 - Project stage report
- *Communication protocols and practices.* Although this item relates to status reporting, addressed previously, it is more than just that. Organizations like to publicize to their entire organization the progress of their projects, particularly when they are moving forward on plan. There may be specific publication frequencies and media for doing so, which this item covers.
- *Progress reviews*. Progress reviews are generally tiered. The project technical team and the business analysts review their progress with the project manager. Project managers will review their progress with the project steering committee. The project steering committee, in turn, will review the projects with the executive committee. It is important that the frequency and, in some cases, the format of the presentations conducted at these reviews be specified.
- *Progress measurements.* Chapter 14, section III.B.2, outlined numerous project performance measurements. Generally, each organization has its own preferences and will specify those that will be reported upward. However, it should be clear that this does not preclude the project manager from using additional measurements to manage the project. Keep in mind this important distinction. What is used for

reporting purposes might be different from what is used for managing purposes.

• *Policies and procedures.* Specific policies and procedures may affect projects. For example, projects dealing with head count reduction, customers, or suppliers may be highly governed since these areas can affect the business greatly.

In short, governance provides structure and legitimizes the rigor needed to ensure successful project outcomes. (Notice that many of the items in the above list were identified in Chapter 3 associated with Lean Six Sigma deployments.)

Although "governance" does not ensure project success, its presence creates a working environment that enhances the probability of success.

PERFORMANCE MEASUREMENT

Design a system for measuring project and portfolio performance. (Create)

Body of Knowledge III.C.2

Together, Chapters 5 and 6 addressed the issue of project identification, qualification, selection, prioritization, and pipeline management.

Chapter 9, section I.I.1, addressed a set of financial measures that might be used to select and prioritize projects. Briefly, the measures discussed included:

- Net present value (NPV)
- Return on investment (ROI)
- Payback period
- Internal rate of return (IRR)
- Cost–benefit ratio

Chapter 14, section III.B.2, addressed the area of project performance measurement. Briefly, the measurements discussed included:

- Planned value (PV)
- Earned value (EV)
- Actual value (AV)
- Budget at completion (BAC)
- Estimate at completion (EAC)
- Estimate to complete (ETC)

- Cost variance (CV)
- Schedule variance (SV)
- Cost performance index (CPI)
- Schedule performance index (SPI)
- Run rates

Collectively, this subject matter in conjunction with current estimated project savings provides a holistic perspective and the needed components to formulate a project and portfolio performance measurement and assessment system.

How might such a system work? Consider the following as a starting point for designing your own project and portfolio performance measurement system:

- The performance measurements described above are collected on projects both active and inactive. Inactive projects are those in the pipeline that have not been assigned or have been active and were postponed. Note that financial measures associated with projects in the pipeline may change over time as they are subject to constant internal and external influences. Therefore, these measurements need to be updated on a regular basis. Organizations may update the financial measures for active projects and tend to forget about those that are inactive or haven't been assigned.
- The project performance measurements described above are updated on a regular basis as part of the overall project management process.
- The project prioritization criteria are updated or, at least, reviewed for update on a regular basis. Durations of projects in the pipeline can be quite long depending on the effects of changing priorities. Criteria for initially selecting and prioritizing projects at the outset may no longer apply. Market influences, economic conditions, political environment, and so on, may have drastically changed. Consider the impact that natural disasters (for example, Japan's earthquake and tsunami) and political unrest in the Middle East have had on economies globally.
- Project reviews are conducted on a regular basis. It was mentioned previously that such reviews are often tiered. The project technical team and the business analysts review their progress with the project manager. Project managers review their projects with the project steering committee and the project steering committee reviews the projects with the executive committee. Figure 15.2 introduces the role of the program manager. If multiple program managers are involved, an additional set of reviews may be inserted between the project managers and the project steering committee. As we saw in Chapter 5, section I.E.4, reviews can add value and prevent projects from failing. However, too many reviews can delay decision making and stall projects. This is one reason the governance plan should be very specific regarding decision-making authority.

All of the information necessary to evaluate projects is now current and available to the authority responsible for managing portfolio performance. At this point, any of the following events can take place:

- Pipeline projects can be prioritized again to reflect the current criteria and needs.
- Active projects can be canceled. This would trigger the project closure process and allow resources to be allocated elsewhere. See Figure 5.1.
- Active projects can be provided with additional support.
- Pipeline projects can be canceled.
- Projects can be postponed and sent back to the pipeline.
- Postponed projects can be removed from the pipeline and restarted.
- New projects can be assigned out of the pipeline. This would trigger the project assignment process. See Figure 5.1.

The above process should be conducted on a regular basis or as conditions warrant. Such conditions might include a midcourse shift in strategy or a major influx of new projects into the pipeline.

At this point, it should be readily apparent that measuring and managing project and portfolio performance is an ongoing process. Organizations that fail to do this systematically and with rigor often find that their pipeline runs dry or their projects are not advancing the accomplishment of their strategies. Consequently, goals and objectives are not being met.

Chapter 16 Project Financial Tools

BUDGETS AND FORECASTS

Assess and explain budget implications, forecasting, measurement, monitoring, risk analysis, and prioritization for portfolio level projects. (Evaluate)

Body of Knowledge III.D.1

In Chapters 5 and 6, we addressed the issues of project selection and prioritization, while in Chapter 14 the concept of project management was discussed. Project management, of course, covered project budgeting, measurement, monitoring, and risk analysis. Therefore, these topics will not be discussed in this chapter.

However, none of the preceding discussions addressed the implications of budgets, forecasting, measurement, monitoring, and risk analysis on the prioritization of portfolio-level projects.

Budgets

Budgeting for a Lean Six Sigma project can be difficult since the path from start to completion is unknown. In Lean Six Sigma, we follow the data. Consequently, the path we follow could be circuitous. Fortunately, there are several items we may be able to budget for directly, or alternately, establish a reserve budget. For example, we have an assembled team, with an estimated percentage contribution to the project per week, and estimated project duration. Therefore, we can budget labor dollars for the project. If we expect to conduct experimental designs, these can be budgeted as well, both in terms of labor and material. Likewise, a pilot can be budgeted. Also, don't forget about implementation training.

In instances where doubt exists, it may be possible to establish a *project reserve budget*. Such budgets may be set as a percentage (for example, 10%, 15%, 20%) of the actual dollar budget. These budgets are typically set aside as contingency dollars to help mitigate project risk.

In addition to budgeting dollars, we may also budget hours relative to the project schedule. Deviation from the planned schedule provides us with a schedule variance. This measurement is also used to assess the project performance.

Forecasting, Measurements, and Monitoring

As with all budget processes, forecasting is eventually required. Chapter 14, section III.B.2, discussed the common measurements used for forecasting, while Chapter 14, section III.B.3, covered monitoring. Since these topics have been addressed elsewhere, they will not be covered here.

The program manager (or quality council) or whoever is accountable for overseeing the portfolio of projects will request periodic budget forecasts for all active projects. Ideally, this individual will be monitoring the budget reports for all projects. Thus, he or she will have a full financial perspective of each project, including expenditures and updated project benefits. This is important as we enter the next section.

Project Prioritization

Recall the methodology discussed in Chapter 5 for prioritizing projects in the pipeline. Remember, specific criteria were required. The organization has great flexibility in setting the criteria as long as they are meaningful and relevant to the decision-making process. The criteria could just as well have been budget requirements and project risk.

The program manager with the project budget plan, actuals, and forecast financials along with a risk analysis now has the ability to prioritize active projects. Projects that are over budget or with expected benefits less than originally estimated may be postponed or eliminated and resources allocated to other projects in need. Alternately, the decision may be made to intervene and provide the team leader with additional support. For example, the Master Black Belt may take a stronger or more active role. Additional team members may be added or others replaced. Any number of decisions can now be made.

These hard financial data combined with the soft selection criteria (for example, alignment with strategy, urgency to business, project duration) brings reality and practicality to the prioritization process. Adding financial information to the prioritization process can have dramatic effects on the project rankings. This is something to be aware of, as you may have to adjust the expectations of senior leaders and project sponsors accordingly.

COSTING CONCEPTS

Define the concepts of hard and soft dollars and use cost of poor quality tools, activitybased costing, and other methods to assess and prioritize portfolios. (Apply)

Body of Knowledge III.D.2

Defining Hard versus Soft Dollars

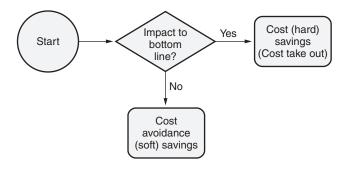
The discussion of what constitutes hard and soft dollar savings is a long-standing debate. It would seem to be a simple issue, but so many organizations have defined them differently over the years that confusion abounds. Ashenbaum (2006) noted that, "Generally speaking, cost savings are understood as tangible bottom-line reductions resulting in saved money that could be removed from budgets or reinvested back into the business. There also needs to be a prior baseline or standard cost for the purchased product or service so that these savings could be measured against a prior time period's spend." (Note: In the context in which Ashenbaum offered this definition, cost savings are synonymous with hard dollar savings.) Based on this definition, let's extract the key characteristics of *cost (hard) dollar savings*:

- A prior baseline of spending must be established. This is typically considered twelve months.
- The dollars must have been planned and in the budget. This goes along with the baseline requirement above.
- The cost savings must affect the bottom line (that is, the profit and loss statement or balance sheet).

Unexpectedly, this definition also settles the question regarding whether spent or reinvested cost savings were ever savings at all. Some organizations tend to empty the savings pot as soon as it's filled. Then they wonder or even complain about the lack of savings from the Lean Six Sigma strategic initiative. Once a project books its hard savings, these savings exist regardless of what the organization chooses to do with them thereafter.

So, what are soft dollar savings? *Soft dollar savings* are *cost avoidance savings* and are, by exclusion, everything that is not hard dollar savings. This concept is illustrated in Figure 16.1. This figure makes the simple point that if it isn't hard dollar savings, then it is soft dollar savings. However, Ashenbaum offers some additional clarity on the subject:

• Avoidance is a cost reduction that does not lower the cost of products/services when compared against historical results, but rather minimizes or avoids entirely the negative impact to the bottom line that a price increase would have caused.



- When there is an increase in output-capacity without increasing resource expenditure, in general, the cost avoidance savings are the amount that would have been spent to handle the increased volume/output.
- Avoidances include process improvements that do not immediately reduce costs or assets but provide benefits through improved process efficiency, employee productivity, customer satisfaction, improved competitiveness, etc. Over time, cost avoidance often becomes cost savings.

If we consider Ashenbaum's clarifications of soft dollars, we can expand Figure 16.1, resulting in Figure 16.2. Notice that "cost avoidance" now depends on whether it has a budget impact or not. Let's consider the following examples:

- *Cost avoidance (Budget impacting).* This type of cost avoidance eliminates or reduces items in the budget marked for future spending. For example, three engineers are budgeted for hire in the fourth quarter. The cost of the three engineers is not in the baseline, nor has any spending for these engineers occurred. Eliminating these planned expenses has an impact on the future budget and is thus cost avoidance.
- *Cost avoidance (Non-budget impacting).* This type of cost avoidance results from productivity or efficiencies gained in a process (that is, reduction of non-value-added activities) without a head count reduction. For example, the process cycle time that involved two workers was reduced by ten percent. Assuming the ten percent amounts to two hours per worker per week, the two workers save four hours per week. These four hours are allocated to other tasks.

Another way to impact the bottom line is through revenue or top-line growth. How might a Lean Six Sigma project affect such growth? Consider a project that

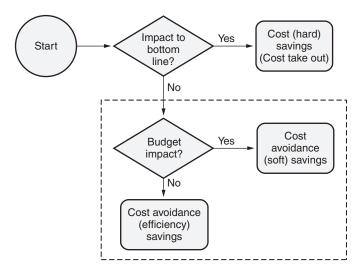


Figure 16.2 Expanding the definition of cost avoidance to include efficiency savings.

identifies and removes a production bottleneck or constraint. As a direct consequence of removing that constraint, the organization is able to accept new customer orders (something it wasn't able to do previously) or work off backlogged orders. In this situation, revenue growth is achieved, and the reason is the constraint removal as an outcome of the project. It would seem that "revenue growth" would be a desirable category to which to attribute the dollar impact of Lean Six Sigma project dollars. This is illustrated in Figure 16.3.

Some organizations further refine cost savings into the third category of working capital/cash flow. Projects falling into this category might improve internal invoicing processes that result in lower account receivables, or reduce inventory requirements from increased process efficiency and reduced claims. Working capital/cash flow is reflected in Figure 16.4.

In summary, two major categories of savings have been identified, each of which can be further refined:

- Hard savings (Cost savings)
 - Cost take out
 - Revenue growth
 - Working capital/cash flow
- Soft savings (Cost avoidance)
 - Cost avoidance (Budget impacting)
 - Cost avoidance (Non-budget impacting)

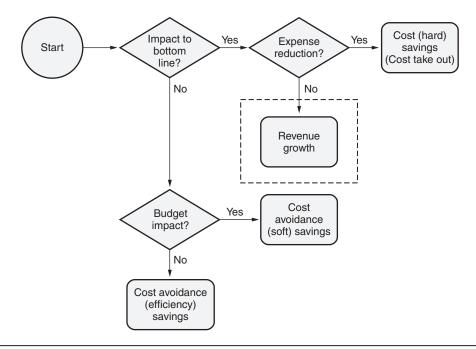


Figure 16.3 Expanding the definition of cost savings to include revenue growth.

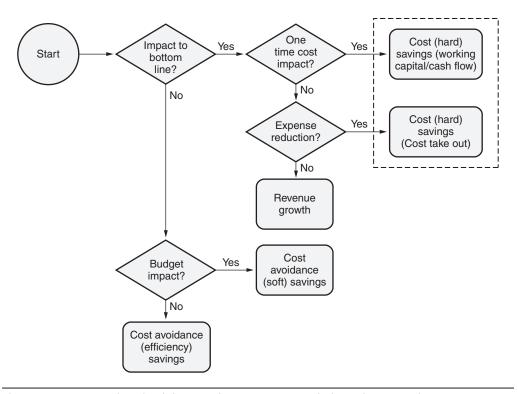


Figure 16.4 Expanding the definition of cost savings to include working capital.

At one time or another, several of the forerunners in Lean Six Sigma, such as AlliedSignal (later Honeywell International) and GE, favored the categories of savings similar to those above.

False Savings

One of the most troubling and abused areas of savings is the cost avoidance (non-budget impacting). This area of savings is typically generated from productivity or efficiency gains. Many organizations will save a small slice of time in an employee workday, multiply that by the number of workers performing the same job and extend that over a year. Voilà! Enormous savings are generated and booked. This approach is common in many transaction-based organizations such as large banks and call centers where a few seconds may be saved on a transaction or calling up a computer screen with customer data. These transactions may be performed hundreds of thousands of times per day across multiple individuals.

On the surface, this approach seems fine and is generally accepted. However, the fallacy lies with the ability to aggregate these miniscule time slices into a sufficiently meaningful size that will accommodate other work. If this can't be done, the "time saved" will simply be absorbed into adjacent work activities. Then they are not even paper savings. Furthermore, the ability to measure such time savings might be questionable. Consider trying to measure five or ten seconds saved from a two- to three-minute transaction. In many cases, we are dealing at the noise

level. Yet, these types of savings are reported all the time. Although they may be categorized as cost avoidance due to productivity or efficiency gains, are they really that if the gains can not actually be used or even measured reliably?

Identifying Project Savings

Table 16.1 provides a starting point for categorizing improvement in business metrics to the various categories of savings at the outset of a project. However, at the beginning of a project our knowledge of potential savings is limited. Some categories of savings by business metric may hold throughout the project duration with only minor increases or decreases. Others that were initially projected may never materialize. Still others that were unforeseen may surface near the project's end and yield healthy returns.

It is important to remember that any given project may have a wide variety of savings. Therefore, it is crucial to involve the financial representative to ensure that potential savings are not overlooked and to provide or enhance the credibility of the savings.

Multi-Year Project Savings

The recognized time frame for calculating a project's savings is twelve months. Consequently, we annualize a project's savings unless those savings had a onetime impact. Some organizations may argue that some projects legitimately save dollars over multiple years. However, savings in years two and beyond are generally built into budgets. Hence, they would reflect as a zero change, as the previous twelve months serves as the baseline.

Methods for Determining Cost Baselines

Chapter 9 addressed the details of four cost allocation models. For convenience, they are repeated here:

- *Undifferentiated costs.* This method considers those costs that are known or assumed to be associated with producing the product or rendering the service. There is no categorization by cause, type of product, or service.
- *Categorized costs (cost of quality).* Such costs may be aligned to the cost of quality categories: appraisal, prevention, and failure.
- *Traditional cost model.* Costs allocated by a percentage of direct labor dollars, or some other arbitrary factor such as amount of space occupied, number of personnel in a work unit, and so on.
- *Activity-based costing (ABC)*. The costs of materials and services are allocated by the activity or process (cost drivers) by which they are consumed.

Each of the above models provides a method for establishing baseline performance costs for processes.

| Business metric | Savings categories | | | | | | | |
|-------------------------------------------------------------------------------------|--------------------------------------|------------------------------------|-------------------------------------|-------------------------------|---------------------------------------|--|--|--|
| | | Cost (hard) savings | Cost (soft) avoidance | | | | | |
| | Cost take out (bottom line) | Revenue growth (top line) | Working capital/ cash flow | Cost avoidance (budget) | Cost avoidance (non- budget) | | | |
| Labor | | | | | | | | |
| The amount of labor required to perform a process decreases | | | | | | | | |
| Shift premium decreases or is eliminated | | | | | | | | |
| Overtime decreases | | | | | | | | |
| Less rework occurs | | | | | | | | |
| Material | | | | | | | | |
| Less material is consumed (for example, solvents, office supplies, and so on) | | | | | | | | |
| Less raw materials are purchased | | | | | | | | |
| Less material is stocked in inventory | | | | | | | | |
| Shelf life is extended/ expirations are reduced | | | | | | | | |
| Reduced lead time in acquiring parts | | | | | | | | |
| Reduced price of raw materials | | | | | | | | |
| Increased inventory turns | | | | | | | | |
| Inventory carrying costs are reduced | | | | | | | | |
| Capacity | | | | | | | | |
| Machine cycle time decreases | | | | | | | | |
| Available capacity increases and can be used | | | | | | | | |
| Overhead | | | | | | | | |
| Travel expenses are reduced or eliminated | | | | | | | | |
| Reduction of floor space | | | | | | | | |

Table 16.1 Examples of savings by category

Continued

Table 16.1 Continued.

| Business metric | Savings categories | | | | | | | |
|--------------------------------------------------------------|--------------------------------------|------------------------------------|-------------------------------------|-------------------------------|---------------------------------------|--|--|--|
| | | Cost (hard) savings | | | (soft) lance | | | |
| | Cost take out (bottom line) | Revenue growth (top line) | Working capital/ cash flow | Cost avoidance (budget) | Cost avoidance (non- budget) | | | |
| Improvement in building and grounds | | | | | | | | |
| Reduction in expenses and supplies | | | | | | | | |
| Fewer utilities are used (for example, electric, gas, water) | | | | | | | | |
| Equipment | | | | | | | | |
| Capital requirements are reduced or eliminated | | | | | | | | |
| General and miscellaneous | | | | | · | | | |
| Less scrap is produced | | | | | | | | |
| Customer penalties are decreased or eliminated | | | | | | | | |
| Planned future costs are reduced or eliminated | | | | | | | | |
| Less office supplies | | | | | | | | |
| Customer | | | | | | | | |
| Improvement in customer satisfaction | | | | | | | | |
| Improvement in voice of the customer | | | | | | | | |
| Reduced warranty claims | | | | | | | | |
| Increased OEM acceptance rates | | | | | | | | |
| Process | | | • | | • | | | |
| Increased yields | | | | | | | | |
| Improvement in voice of the process | | | | | | | | |
| Revenue growth | | | | | | | | |
| Increased capacity | | | | | | | | |
| Removal of process bottlenecks | | | | | | | | |
| | | | | | | | | |

Assessing and Prioritizing Portfolios

The previous discussions in this section have helped us understand how:

- Accurate cost baselines can be established to derive project savings
- Project savings can be categorized in a meaningful way

In particular, the program manager will be interested in the category of savings. Recall Figure 16.4 for the moment. The five categories of savings shown in the figure permit the program manager to identify projects with savings categories that might meet the needs of the organization at any given time. For example, an organization might be emphasizing working capital. With this knowledge, the program manager (and associated team) is now able to add this new criterion to the prioritization process addressed in an earlier chapter. Similarly, the same is true for any other savings category. (Note: With some additional thought we can easily link our saving categories to the various costs of quality categories [for example, external failure costs].) Of course, this may have a dramatic affect on how the projects are re-ranked in the pipeline. Remember, as stated previously, the prioritization process is dynamic. In practice, this means that some projects in the pipeline may never be assigned for execution. This is foreseeable, and project sponsors and senior leaders should be apprised of it early on.

Part IV

Training Design and Delivery

| Chapter 17 | Training Needs Analysis |
|------------|-----------------------------------|
| Chapter 18 | Training Plans |
| Chapter 19 | Training Materials and Curriculum |
| | Development |
| Chapter 20 | Training Effectiveness Evaluation |

Chapter 17 Training Needs Analysis

Assess the current level of knowledge and skills in each target group in relation to the skills and abilities that are needed. Conduct a gap analysis to determine the training needs for each target group. (Evaluate)

Body of Knowledge IV.A

Successful organizations understand and acknowledge the importance their human assets play in achieving their success. However, the degree of success depends on the appropriate level and relevance of their human assets' education and training. Remember, an organization's strategies are executed through its people. If the current education and training of the people can not support the strategies, the organization must act on this gap via the training needs analysis.

HUMAN RESOURCE STRATEGIC PLAN

A key outcome of the strategic planning process should be the human resource strategic plan. One aspect of the human resource strategic plan should focus on employee knowledge, skills, and attitudes (KSAs), which can be operationalized through an organization's:

- Permanent hiring practices
- Contract hiring practices
- Outsourcing practices
- Training practices

The human resources department should analyze the organization's strategic plan to determine the following:

• What KSAs are required of our human assets in support of our organization's strategies, and how does that compare with current capabilities?

- Do we have those KSAs today? If not, then what? For example, should the organization hire to obtain the KSAs, contract for the KSAs, outsource for the KSAs, or train for the KSAs?
- Do those who possess the KSAs perform their jobs with competency? If not, then what? This becomes a matter of job fit, motivation, and, of course, the subjective and rather elusive area of job performance appraisal.
- What is required to close the gap? A plan should be developed to close the gap. Furthermore, every plan should include the initial introduction of the KSAs as well as KSA refreshment and enhancement. Too often, the latter is forgotten. Organizations typically fail to recognize that KSAs become stale, forgotten, or obsolete if not used or maintained.

The above questions, among others that should be considered in the human resource strategic plan, will be addressed in the "Strategic Analysis Phase" subsection below.

A critical tool for developing the human resource strategic plan is the training needs analysis. The details of how a training needs analysis is accomplished will be addressed in the remainder of this chapter.

THE IMPORTANCE OF A TRAINING NEEDS ANALYSIS

Organizations plan for training in different ways. Many organizations circulate a training catalog, which serves as a "wish list" of training offerings that employees can choose from to fulfill training and education requirements. Other organizations allow employees to spend a specified dollar amount on training from internal or external offerings of the employee's choice. Still other organizations place importance on conducting a thorough training needs analysis before training development or expenditures are approved.

A *training need* is essentially a performance gap between what people know or are able to do and what they should know or be able to do to perform their work competently. Therefore, a *training needs analysis* is a diagnostic method to identify the gap between current performance and desired performance. A *needs analysis* is also known as a *needs assessment* or *training requirements analysis*.

A needs analysis uses a variety of tools and techniques to identify the training needs for a specific population or target group. Through a systematic process, relevant data are collected and used to formulate a training plan for employees. In addition to identifying training needs for individual employees, a needs analysis also investigates how such training can support the organization's strategies.

A training needs analysis effort produces facts about the target audience, training needs, learning styles, and so on. However, a good needs analysis does more than merely gather information. It makes a case for training. To that end, a training needs analysis must be well executed to garner top-down support.

Furthermore, a needs analysis should differentiate training wants from training needs. A *training want* may surface when an employee's proficiency is low in certain job tasks or behaviors. However, those tasks and behaviors may not necessarily be important to work performance outcomes. For example, consider a customer service employee who wants to learn more about an engineering topic because of a personal interest.

A *training need* exists when an employee's proficiency is low in certain job tasks or behaviors that are important to work performance outcomes. For example, consider the situation where new piece of equipment is to be installed and an employee needs to know how to operate it.

The frequency of training needs analysis varies according to the size of the organization, the industry, competitive factors, the rate of technology change, churn and attrition rates, hiring practices, and so on. A training needs analysis may be conducted annually or as needed. It might be conducted on the same cycle as strategic planning. Whatever the mandate, a training needs analysis should begin with top management support and take into account how the potential training outcomes support the organization's strategic plan.

DEFINING THE EXTENT AND NATURE OF THE JOB

Before any type of training needs or performance gap analysis can be conducted, it is critical that the organization be aware of the extent and nature of the jobs associated with each target group involved. To achieve this awareness, the following three aspects of each job must be considered in detail:

- Job analysis. Pearn and Kandola (1993) describe a job analysis as a "systematic procedure for obtaining detailed information about a job, task, or role that will be performed or is currently being performed." However, a more traditional definition describes a *job analysis* as a process for collecting information about duties, responsibilities, skills, and outcomes. Also, a description of the work environment may be included if it is unique or extraordinary in some manner. An example of such an environment might be a manufacturing clean room.
- *Task analysis.* The purpose of the *task analysis* is to identify the specific tasks required to perform a particular job. Often this is accomplished by direct observation of the individual performing the job. It is important that all tasks be identified regardless of whether or not they are observable at the time the task analysis is being performed. An example of a task might be "prepare minutes for all management meetings."
- *Skills analysis.* The purpose of the *skills analysis* is to identify the specific skills required to perform the specific tasks identified by the task analysis. In many instances, it is likely that multiple skills will be required to perform a specific task. Examples of skills associated with the task example might be "ability to listen effectively" and "ability to write clearly." Once skills have been identified, the real issue becomes how to measure them effectively. This will be discussed in a later chapter.

Once completed, the above three aspects of each job will be used to draft a job description and a job specification. *Job descriptions*, according to Rae (1997) "are statements of the outline of the whole job and show the duties and responsibilities involved in the job." The job description also "details the skills, knowledge, and attitudes which are required by the individual in order to carry out the duties involved in the job." A subset of the job description, the *job specification*, according to Rae (1997), defines "what the job holder should do and be capable of doing." Bee and Bee (1994) add that the job specification also includes carrying out the job to a prescribed standard. The greater the detail of the job specification, the easier it will be to determine performance gaps.

Many organizations lack current job descriptions, or job descriptions at all, making it extremely difficult, if not impossible, to conduct a meaningful training needs analysis.

PURPOSES OF AND TYPES OF TRAINING

Let's consider the various types of training an organization might require of its employees. Seven categories quickly surface:

- *Mandated*. Such training may be required by customers, regulatory agencies, or senior leaders. It may be directed at all employees of an organization or a targeted subset. For example, many to most organizations require all employees to take sexual harassment training. The opportunity to forgo mandated training is generally not an option. Mandated training is usually conducted as a legal risk mitigation strategy and almost never cost justified.
- *Recurring.* Recurring training is training that must be repeated on a regular basis. A commonly used time frame is yearly. Like mandated training, recurring training is usually conducted as a legal risk mitigation strategy. Sexual harassment training also is conducted on a recurring (that is, yearly) basis at many organizations.
- *New employee.* As new employees join an organization, they are indoctrinated and often receive an initial burst of training. Typically, this training includes much of the mandated training as well as other training designed to orient new employees to the organization.
- *Work systems.* This training addresses changes or additions to existing systems, processes, procedures, methods, equipment, or tools.
- *Maintenance*. Such training is used to keep current knowledge and skills fresh. Such training is frequently overlooked.
- *Improve existing skills*. Such training may be soft skills (for example, effective listening, conflict management) and technical skills (statistical process control, C⁺⁺ programming, and so on). Training directed at improving existing skills may stem from individual performance appraisals or the "wish list" approach mentioned earlier. Ideally, a training needs analysis will surface this need. The difference

between "maintenance" and "improve existing skills" training is quite subtle. "Maintenance" training recognizes the need to keep skills fresh and up-to-date before the fact, while "improve existing skills" training recognizes that deterioration in skills has occurred after the fact. This is analogous to preventive and corrective action.

• *Create new skills.* As mentioned previously, the organization's strategic plan will generate the human resource plan. This, in turn, will initiate a training needs analysis activity, which will identify new skills required to move the organization forward toward achieving its strategies.

It should be apparent that the above seven categories of training are not necessarily mutually exclusive. The fact that they overlap should be evident by the examples. In some cases, the categories of training are actually education. However, for the purposes of this discussion, we will not make such a distinction.

TOOLS AND TECHNIQUES FOR CONDUCTING A TRAINING NEEDS ANALYSIS

Organizational training needs stem from the strategic planning process, while individual training needs stem from both strategic planning and individual performance. Different processes exist for conducting training needs analysis. However, for the purposes of this discussion, the process given in Figure 17.1 will be followed.

Strategic Analysis Phase

The purpose of the strategic analysis phase is to develop a broad view of the training needed. A logical flow that systematically provides answers to key questions provides an important foundation for the remainder of the analysis.

During this phase, it is important to identify strategic issues such as:

- Organizational requirements and business challenges for training
- Mission and philosophy for training
- The vision and goals for providing training

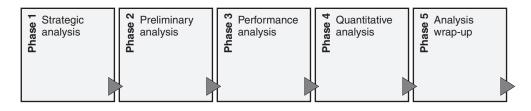


Figure 17.1 The training needs analysis process.

• Overall training budget allocations and specific availability for new training initiatives

A structured, step-by-step planning process is often used. It is important to begin with an analysis project plan. The analysis project plan should include what the desired state of performance should be, the purpose of the analysis, the scope, the target audience, and possible sampling procedures. Also, it is advisable to form a steering committee for the analysis and involve subject matter experts in the planning process.

Preliminary Analysis Phase

The preliminary analysis phase provides important background information and qualitative input that helps shape subsequent research activity. Investigative analysis activities might include:

- *Review of relevant material.* This includes the examination of records, reports, documentation, job descriptions, current training, performance reports, performance appraisals, process audit reports, critical incident reports, safety incident reports, commercially available training, and competitive training. The output is a matrix showing the records accessed, the needs uncovered, and the magnitude of the need.
- *Research*. Research addresses the evaluation of trade association data, industry data, and competitors' data.
- *Interviews.* Several types of interviews may be conducted. These include personal and telephone interviews with key executives and managers in the organization, subject matter experts, individuals who represent a cross-section of the target population (for example, representative sample of supervisors, job incumbents, and new trainees), as well as customers of the organization. This includes both internal and external customers. However, keep interviews in perspective. Interviews are generally opinions and opinions are not data.
- *Focus groups*. Focus groups work well when the discussion is carefully planned, with approximately seven to ten participants in a session. The questions should be well structured, and an experienced facilitator is frequently used. The purpose is to obtain relevant perceptions about the organization, customer requirements, training needs, and so on. The output is a summarization of the needs surfaced and the tasks, processes, and work unit where the needs originate. Like interviews, keep focus groups in perspective as well.
- *Qualitative data analysis techniques.* These techniques can be used to analyze nonquantitative data such as data expressed in words, phrases, and paragraphs.

• *Data collection instruments.* These would include any tool or technique to collect data for analysis. Examples include tests, questionnaires, surveys, checklists, simulations, games, and self-completing assessments.

Preliminary analysis methods do not necessarily require an individual with needs analysis experience or an extensive research background. However, a knowledgeable individual should plan or oversee the activity.

Performance Analysis Phase

The purpose of the performance analysis phase is to determine the current state of workplace performance. During the performance analysis phase, individual performance characteristics are reviewed. One reason to conduct performance analysis is to determine whether the performance gaps really do relate to knowledge and skill gaps that training can address, or whether the gaps relate to motivation or organizational barriers such as process or system failure or lack of adequate resources. This important distinction should be recognized.

Various techniques, including field observations and simulated experiences, may be used to assess performance. Assessment through simulation or assessment centers generates information that can not be easily obtained through observation. Common techniques are role-playing, problem solving and decision-making activities, and self-assessments.

Generally, the work analysis methods discussed previously (that is, job analysis, task analysis, and skill analysis) will be conducted during this phase.

Quantitative Analysis Phase

Quantitative analysis can help explain the causal relationships between training and individual or organizational performance and results. Collecting good data and analyzing them carefully will help reveal root causes of performance gaps. Quantitative analysis uses survey instruments to quantify training needs and related information, such as training priorities and delivery preferences.

Survey instruments are typically based on the results of the preliminary and performance data-gathering activities, and solicit the opinions of employees, supervisors, and other stakeholders. For example, a survey can verify or crosscheck how many people share similar opinions or perceptions about training needs. Again, remember that extreme caution is warranted, though, when soliciting opinions and perceptions, as they are not facts.

Survey instruments typically take the form of a written questionnaire that may be administered in a variety of ways including through the mail, electronically, in face-to-face interviews, or telephone interviews. Depending on the target audience, multiple methods of administration may be necessary.

All employees may be surveyed, or a representative sample. The decision is based on the size of the organization and the resources available to administer the survey. Surveys can be an effective data-gathering tool. However, valid surveys are difficult to design, administer, and interpret. Statistical analysis is required, and appropriate expertise and resources are critical.

On a simplified level, the array of needs analysis tools and techniques determine training needs on an individual level. Tools and techniques are used to gather information from:

- Review of related materials
- Management
- Employees targeted for training
- Supervisors and others who are familiar with job performance and performance requirements

Understanding how the various training needs analysis tools and techniques are categorized is secondary to choosing the appropriate ones to use during the research process. The purpose of the needs analysis, the resources available, time constraints, and so forth, will ultimately influence which ones are best.

Analysis Wrap-Up Phase

Typically, a combination of tools and techniques is used. Appropriate data analysis is required. Team meetings and debriefing sessions with top management are incorporated as milestones to review progress and findings throughout the process. A final report or presentation of the results signifies the end of the process.

Near the end of the process, some attention must be paid to training cost justification. The ingredients used in calculating the estimated costs and benefits of a training program vary by industry and training purpose. However, key factors to consider include:

- Identify the desired performance outcomes
- Define units that can be used to measure the successful outcomes
- Assign dollar values to the units
- Compare the current performance output value to the projected performance output value after training
- Subtract the training costs from the value difference after training

In the end, an effective training needs analysis will:

- Yield reliable and valid qualitative and quantitative data
- Establish consensus between management and personnel
- Define the appropriate intervention and training strategies, including content, priorities, delivery methods, and funding
- Consider the training costs and return on investment

- Form a sound basis for conclusions and training recommendations
- Provide information for formulating training evaluation measures
- Reduce overall risks moving forward

In addition to training needs, a thorough needs analysis often reveals factors not related to training such as ineffective organizational policies, procedures, and practices; process and system issues; organizational issues; practices related to rewards and recognition; and individual job fit. While not directly related to training, these items are important to consider because they can create roadblocks to successful training implementation.

Chapter 18 Training Plans

Design training plans to close the knowledge and skills gaps. Refine the plans based on the number of people needing to be trained in a particular technique or skill, and whether multi-disciplinary or multi-level competency training is appropriate. (Create)

Body of Knowledge IV.B

ALIGNING TRAINING TO STRATEGY

If training is to be positioned as supporting the needs of the organization, senior leaders must be involved with and stand behind the training plans. In Chapter 12 we introduced the concept of action plans. Technically, training plans are action plans that support the strategic plans of the organization. If the plans do not have strategic outcomes, then the organization should not be supporting the training. Therefore, to encourage the success of their training programs, organizations must align their training needs to their strategic needs as illustrated in Figure 18.1.

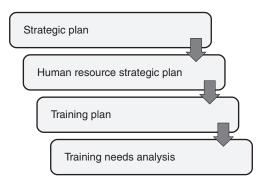


Figure 18.1 Aligning training with strategy.

THE IMPORTANCE OF TRAINING AND EDUCATION

Generally, organizations of all sizes in all sectors strive to be world-class or best in class. Countless organizations around the world embark on the quest for high performance regardless of industry or business sector. Training and education enable an organization's workforce to develop and grow to its fullest potential. As organizations strive to prosper and achieve high performance, training and education provide the capability crucial for success. That is why billions of dollars are spent each year on training and education of employees.

The terms "training" and "education" are often used interchangeably. However, *education* focuses on broadening an individual's knowledge base and expands thinking processes. Further, it helps employees understand concepts and accept increased job responsibilities, and prepares them for future jobs and leadership roles. Simply, education helps individuals learn how to think.

Training is considered a subset of education that focuses on increasing proficiency in a skill. Therefore, training typically refers to skill-based instruction and addresses the specific skills employees need to perform a current job or task. Such skills can be highly diverse. For example, they might include: learning how to apply quality tools, implementing customer service principles, running a new piece of equipment, or facilitating a team. Training may take place in either a formal or informal setting such as a classroom or on-the-job training. Training is appropriate when skills are lacking, but also when the individual is willing to learn. A primary attribute of training is that it produces measureable performance outcomes, whereas education alone does not. Education can be measured only by the amount of knowledge gained or retained. Table 18.1 provides a comparative analysis between training and education.

Both training and education activities are ongoing for many organizations. In fact, such organizations have set a strategic objective to become a "learning organization."

THE IMPORTANCE OF TRAINING PLANS

When skills training is required, it is crucial to have a training plan. Every organization faces important decisions about the wide variety of organizational needs that might be supported, including new hire orientation; technical and process training; quality training; certification training; team, interpersonal, and communication skills training; diversity training; and management and leadership programs. Consequently, training dollars represent a significant budget item.

In spite of its organizational importance, training expenditures are often vulnerable to cost-reduction pressures. When senior leaders do not view training as a strategic, cost-justified investment, business downturns can result in training budget cutbacks. Therefore, employee training should reinforce strategic plans with direct line of sight.

Training plans provide the road map for meeting critical training requirements. Without a plan, important organizational training requirements may go unnoticed, may be poorly addressed, or not addressed at all. In fact, many organizations with strategic plans that have training implications still leave such training to happenstance.

| Characteristic | Training | Education Helps an individual learn how to think; broadens an individual's knowledge base | | |
|--------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|--|
| Description | Typically skill-based instruction | | | |
| Focus | Primarily addresses the skills employees need to improve upon to perform a current job or task: appropriate when skills are lacking and the individual has a willingness to learn | Primarily addresses the knowledge employees need to improve performance for a future job or to accept increased job responsibilities; often focuses on general needs related to achievement of strategic goals | | |
| Setting | Formal or informal | Generally formal | | |
| Examples | Waste-reduction methods, troubleshooting a process or a service, mechanics of a web-based technology, team building | Understanding change management philosophies, motivation principles, or the impact of e-commerce | | |
| Outcome example | As a result of lean training, a department was able to document a manufacturing process cycle time reduction of 15% | As a result of a three-day workshop on technology trends, managers formed a task force to study potential future impact and make proactive recommendations to senior leaders | | |

 Table 18.1
 A comparative analysis between training and education.

Source: Adapted from *Certified Manager of Quality/Organizational Excellence: ASQ's Foundations in Quality Learning Series.* 2005. Milwaukee: ASQ.

In large organizations, various business units, sites, functional groups, and departments may have individual training plans. All training plans, though, should be a result of a strategic business planning process and should provide a mechanism for ensuring that training and education build the organization's capabilities as well as enable employees to a make a positive contribution to the organization's success.

TRAINING PLANS ARE NOT A PANACEA

Often, poor performance is attributed to a skill or knowledge deficiency that can be corrected by training. Nonetheless, many times the root cause of poor performance is not skill related at all. However, training can not remedy a situation caused by poor performance or other root causes that are not skill related. For example, to expect that training can fix operating system problems is unrealistic. Consider the following situations in which training would not be beneficial:

- Increased defects in a manufacturing environment caused by outdated or inadequate equipment
- Unsatisfactory help desk customer support resulting from poor morale and compensation issues

Mistakes caused by employees working overtime on a sustained basis

Frequently, poor performance can be remedied by changing resources, providing more objective feedback, or by adding or removing behavioral consequences.

Therefore, it is very useful to conduct a needs analysis to uncover the true causes of performance gaps and to select an appropriate rectifying approach. In some cases, a change to a system or process may be necessary, and not a resource.

COMPONENTS OF AN EFFECTIVE TRAINING PLAN

Bee and Bee (1994) suggest that a viable training plan should comprise three critical components:

- *Statement of policy.* This constitutes a statement of direction for the time period covered by the training plan. For example, there may be a specific focus of training such as qualification or certification training, leadership or first-line supervisory training, interpersonal skills training versus professional or technical skills training, and so on. In addition, this part of the plan might include the following:
 - Challenges and opportunities facing the organization. Recall the SWOT analysis discussed in Chapter 1.
 - Strategies and tactics for dealing with the challenges.
 - Required and desired competencies to achieve organizational goals.
 - The training system the organization needs and how it will evolve to keep pace with the learning needs. This system should reflect the needs of the various target groups. It should not be cast as a one-size-fits-all system.
 - Training's strategic role in helping the organization develop required competencies.
 - Training's strategic goals and objectives. This should address how training supports the human resources strategic plan, which in turn supports the organization's strategic plan.
- *Training budget*. This would include all costs associated with developing and executing the training plans. Also, it is important to specify how training is paid. Some organizations charge it as a general overhead expense while others charge directly to the organization whose employees are to receive the training. This latter method is sometimes known as "re-charging." Each method has its own advantages and disadvantages, as suggested by Newby (1992). The overhead method tends to focus on a longer-term view of training, but also tends "to encourage the inertia of running a standard menu of courses from year to year with little attempt to review training provision against business needs." By contrast, Newby believes

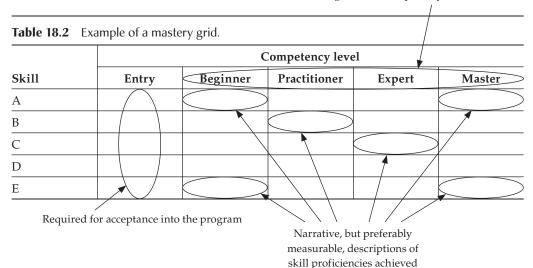
the re-charging method encourages "line management interest on selecting the right people for the right training as they are being charged for the training need."

- *Operational plan.* This covers all aspects of timing and non-financial resources. For example:
 - Identify the need
 - State objectives and outcomes
 - Determine content and sequence of instruction, if designed internally, or select from qualified external suppliers
 - Identify the target groups
 - Develop schedules for all aspects of training such as development, delivery, evaluation, and so on
 - Specify resource requirements such as instructors, facilities, equipment, materials
 - Establish measurements
 - Select training delivery methods, if delivered by internal instructor as assessed, or select from qualified external suppliers
 - Train instructors
 - Deliver training
 - Evaluate outcomes
 - Modify training as needed
 - Conduct lessons learned
 - Add documentation to the organization's knowledge database

Remember, executing the training plan should be treated as any other project. That is, it should be managed properly using the appropriate project management tools and techniques. Therefore, planning for training should closely resemble the planning done for any other project.

APPLYING THE TRAINING PLAN

In simplest terms, one of the key deliverables of a training plan will be a matrix similar to that shown in Table 18.2. This table illustrates for each identified skill an associated level of competency. The "entry" level competency is the minimal level of competence required to be accepted into the training program, curriculum, or course. For example, in the Lean Six Sigma context, an entry-level skill might be basic math or algebra. Both are prerequisites to Lean Six Sigma training. Also, the nomenclature used to describe the competency levels in the table is not overly important. These are simply an ordinal categorization that indicates an increase in skill level as one moves from left to right across the table. Many



Terminology is subject to change, but reflects increasing levels of competency

| | | | Target | Competency level | | | | | |
|------------|--------|-------|------------|------------------|----------|--------------|--------|--------|--|
| Curriculum | Course | Skill | group | Entry | Beginner | Practitioner | Expert | Master | |
| Ι | 101 | A | 1,6 | | 1, 6 | | | | |
| III | 102 | A | 3 | | | 3 | | | |
| Ι | 103 | С | 1 | | | 1 | | | |
| I, II | 201 | D | 1, 2, 4 | | | 1, 2, 4 | | | |
| I, II, III | 202 | E | 1, 2, 3, 4 | | | 1, 2, 3, 4 | | | |
| I, II | 203 | F | 1, 2, 4 | | 1, 2, 4 | | | | |
| II | 301 | G | 4,5 | | | 4,5 | | | |
| II, III | 302 | Н | 3, 4, 5 | | | 3, 4, 5 | | | |
| II | 303 | Ι | 4 | | | | 4 | | |
| IV | 401 | J | 4 | | | | | 4 | |

Table 18.3 Example of a training plan in matrix summary form.

different categorizations are in use today. Find one that works for your organization and stick with it.

As organizations mature and their training plans are refined, Table 18.2 will likely evolve into a more comprehensive matrix similar to that shown in Table 18.3. Let's briefly interpret this table. There are four curriculums, ten courses, nine skills, and six target groups. Target group 1 is required to take all of curriculum 1 while target groups 2, 3, 4, and 6 are required to take specific courses from curriculum 1. Similarly, target group 4 is required to take all of curriculum

2 while target groups 2, 3, and 5 are required to take specific courses from curriculum 2.

Notice that two courses are dedicated to skill A. These are courses 101 and 102. However, they are directed at different target groups and are intended to allow each target group to achieve a different level of competency. This is an example of multilevel competency training across different target groups. However, suppose target group 1 required skill A at higher levels. Multiple courses could easily be established to accommodate this requirement. For example, 101 could be renamed 101A, which allows target group 1 to achieve competency level designated as "beginner." Additional courses could be added, such as 101B for "practitioner," 101C for "expert," and so on, all for target group 1. Of course, this requires course 102 to be changed to 101B for consistency purposes.

Curriculum 3 has been carved out of curriculums 1 and 2 by aggregating courses 102, 202, and 302 into a single curriculum for target group 3. If courses are sufficiently modular in design, curriculums can be designed and built quickly to accommodate changing needs of the organizations.

Table 18.4 is an expanded version of Table 18.3 that permits more analysis and sorting in this format. For example, Table 18.5 is Table 18.4 sorted by curriculum, target group, and course. This provides a summary of which target groups are taking which curriculums. Similarly, Table 18.6 is sorted by target group, course, and skill. This particular sort provides insight into the training demand placed on a specific target group. Likewise, Table 18.7 is sorted by skill, target group, and course, thus giving an overall indication of course and skill demand. Of course, other sorts and analyses are possible as well.

If the target groups are extended by the number of individuals in those target groups, and the class size is taken into consideration, the number of classes and instructors can be determined. Subsequently, a schedule can be determined, facilities acquired, funding allocated, and so on. In some instances, specific target groups may be critical to service, production, or manufacturing. When this occurs, only limited numbers from these target groups can be absent from their job at any given time. Therefore, this must be taken into consideration when developing the training schedule.

MULTIDISCIPLINARY TRAINING

As stated throughout this chapter, training plans should be developed based on the strategic needs of the organization. In some cases, an organization may find that a single discipline may not be capable of meeting its needs. When this occurs, the organization may need to train individuals beyond their original academic discipline. For example, a chemical engineer is singled out to receive an extensive amount of financial training. Perhaps the engineer is seen as a future leader of the organization. As such, he or she must be proficient in understanding and interpreting financial data, analyses, and reports, among other things. This individual may even be sent back to school to receive an MBA.

Although the above example reflects a single individual, it may be applicable to target groups as a whole, particularly in high-tech organizations with fastchanging technologies.

| | | | Target | Competency level | | | | | | |
|------------|--------|-------|--------|------------------|----------|--------------|--------|--------|--|--|
| Curriculum | Course | Skill | group | Entry | Beginner | Practitioner | Expert | Master | | |
| Ι | 101 | А | 1 | | 1 | | | | | |
| Ι | 101 | А | 6 | | 6 | | | | | |
| III | 102 | А | 3 | | | 3 | | | | |
| Ι | 103 | С | 1 | | | 1 | | | | |
| Ι | 201 | D | 1 | | | 1 | | | | |
| Ι | 201 | D | 2 | | | 2 | | | | |
| Ι | 201 | D | 4 | | | 4 | | | | |
| II | 201 | D | 1 | | | 1 | | | | |
| II | 201 | D | 2 | | | 2 | | | | |
| II | 201 | D | 4 | | | 4 | | | | |
| Ι | 202 | Е | 1 | | | 1 | | | | |
| Ι | 202 | Е | 2 | | | 2 | | | | |
| Ι | 202 | Е | 3 | | | 3 | | | | |
| Ι | 202 | Е | 4 | | | 4 | | | | |
| II | 202 | E | 1 | | | 1 | | | | |
| II | 202 | E | 2 | | | 2 | | | | |
| II | 202 | Е | 3 | | | 3 | | | | |
| II | 202 | Е | 4 | | | 4 | | | | |
| III | 202 | Е | 1 | | | 1 | | | | |
| III | 202 | E | 2 | | | 2 | | | | |
| III | 202 | Е | 3 | | | 3 | | | | |
| III | 202 | Е | 4 | | | 4 | | | | |
| Ι | 203 | F | 1 | | 1 | | | | | |
| Ι | 203 | F | 2 | | 2 | | | | | |
| Ι | 203 | F | 4 | | 4 | | | | | |
| II | 203 | F | 1 | | 1 | | | | | |
| II | 203 | F | 2 | | 2 | | | | | |
| II | 203 | F | 4 | | 4 | | | | | |
| II | 301 | G | 4 | | | 4 | | | | |
| II | 301 | G | 5 | | | 5 | | | | |
| II | 302 | Н | 3 | | | 3 | | | | |
| II | 302 | Н | 4 | | | 4 | | | | |
| II | 302 | Н | 5 | | | 5 | | | | |
| III | 302 | Н | 3 | | | 3 | | | | |
| III | 302 | Н | 4 | | | 4 | | | | |
| III | 302 | Н | 5 | | | 5 | | | | |
| II | 303 | Ι | 4 | | | | 4 | | | |
| IV | 401 | J | 4 | | | | | 4 | | |

 Table 18.4
 Example of a training plan in expanded format.

| | | | Targat | Competency level | | | | | |
|------------|--------|-------|-----------------|------------------|----------|--------------|--------|-------|--|
| Curriculum | Course | Skill | Target group | Entry | Beginner | Practitioner | Expert | Maste | |
| Ι | 101 | А | 1 | | 1 | | | | |
| Ι | 103 | С | 1 | | | 1 | | | |
| Ι | 201 | D | 1 | | | 1 | | | |
| Ι | 202 | Е | 1 | | | 1 | | | |
| Ι | 203 | F | 1 | | 1 | | | | |
| Ι | 201 | D | 2 | | | 2 | | | |
| Ι | 202 | Е | 2 | | | 2 | | | |
| Ι | 203 | F | 2 | | 2 | | | | |
| Ι | 202 | Е | 3 | | | 3 | | | |
| Ι | 201 | D | 4 | | | 4 | | | |
| Ι | 202 | Е | 4 | | | 4 | | | |
| Ι | 203 | F | 4 | | 4 | | | | |
| Ι | 101 | А | 6 | | 6 | | | | |
| II | 201 | D | 1 | | | 1 | | | |
| II | 202 | Е | 1 | | | 1 | | | |
| II | 203 | F | 1 | | 1 | | | | |
| II | 201 | D | 2 | | | 2 | | | |
| II | 202 | Е | 2 | | | 2 | | | |
| II | 203 | F | 2 | | 2 | | | | |
| II | 202 | Е | 3 | | | 3 | | | |
| II | 302 | Н | 3 | | | 3 | | | |
| II | 201 | D | 4 | | | 4 | | | |
| II | 202 | Е | 4 | | | 4 | | | |
| II | 203 | F | 4 | | 4 | | | | |
| II | 301 | G | 4 | | | 4 | | | |
| II | 302 | Н | 4 | | | 4 | | | |
| II | 303 | Ι | 4 | | | | 4 | | |
| II | 301 | G | 5 | | | 5 | | | |
| II | 302 | Н | 5 | | | 5 | | | |
| III | 202 | Е | 1 | | | 1 | | | |
| III | 202 | Е | 2 | | | 2 | | | |
| III | 102 | А | 3 | | | 3 | | | |
| III | 202 | Е | 3 | | | 3 | | | |
| III | 302 | Н | 3 | | | 3 | | | |
| III | 202 | Е | 4 | | | 4 | | | |
| III | 302 | Н | 4 | | | 4 | | | |
| III | 302 | Н | 5 | | | 5 | | | |
| IV | 401 | J | 4 | | | | | 4 | |

Table 18.5 Training plan sort: curriculum, target group, and course.

| | | | Target | Competency level | | | | | |
|------------|--------|-------|--------|------------------|----------|--------------|--------|--------|--|
| Curriculum | Course | Skill | group | Entry | Beginner | Practitioner | Expert | Master | |
| Ι | 101 | А | 1 | | 1 | | | | |
| Ι | 103 | С | 1 | | | 1 | | | |
| Ι | 201 | D | 1 | | | 1 | | | |
| II | 201 | D | 1 | | | 1 | | | |
| Ι | 202 | Е | 1 | | | 1 | | | |
| II | 202 | Е | 1 | | | 1 | | | |
| III | 202 | Е | 1 | | | 1 | | | |
| Ι | 203 | F | 1 | | 1 | | | | |
| II | 203 | F | 1 | | 1 | | | | |
| Ι | 201 | D | 2 | | | 2 | | | |
| II | 201 | D | 2 | | | 2 | | | |
| Ι | 202 | Е | 2 | | | 2 | | | |
| II | 202 | Е | 2 | | | 2 | | | |
| III | 202 | E | 2 | | | 2 | | | |
| Ι | 203 | F | 2 | | 2 | | | | |
| II | 203 | F | 2 | | 2 | | | | |
| III | 102 | А | 3 | | | 3 | | | |
| Ι | 202 | Е | 3 | | | 3 | | | |
| II | 202 | Е | 3 | | | 3 | | | |
| III | 202 | Е | 3 | | | 3 | | | |
| II | 302 | Н | 3 | | | 3 | | | |
| III | 302 | Н | 3 | | | 3 | | | |
| Ι | 201 | D | 4 | | | 4 | | | |
| II | 201 | D | 4 | | | 4 | | | |
| Ι | 202 | Е | 4 | | | 4 | | | |
| II | 202 | Е | 4 | | | 4 | | | |
| III | 202 | Е | 4 | | | 4 | | | |
| Ι | 203 | F | 4 | | 4 | | | | |
| II | 203 | F | 4 | | 4 | | | | |
| II | 301 | G | 4 | | | 4 | | | |
| II | 302 | Н | 4 | | | 4 | | | |
| III | 302 | Н | 4 | | | 4 | | | |
| II | 303 | Ι | 4 | | | | 4 | | |
| IV | 401 | J | 4 | | | | | 4 | |
| II | 301 | G | 5 | | | 5 | | | |
| II | 302 | Н | 5 | | | 5 | | | |
| III | 302 | Н | 5 | | | 5 | | | |
| Ι | 101 | А | 6 | | 6 | | | | |

 Table 18.6
 Training plan sort: target group, course, and skill.

| | | | Target | Competency level | | | | | |
|------------|--------|-------|--------|------------------|----------|--------------|--------|--------|--|
| Curriculum | Course | Skill | group | Entry | Beginner | Practitioner | Expert | Master | |
| Ι | 101 | А | 1 | | 1 | | | | |
| III | 102 | А | 3 | | | 3 | | | |
| Ι | 101 | А | 6 | | 6 | | | | |
| Ι | 103 | С | 1 | | | 1 | | | |
| Ι | 201 | D | 1 | | | 1 | | | |
| II | 201 | D | 1 | | | 1 | | | |
| Ι | 201 | D | 2 | | | 2 | | | |
| II | 201 | D | 2 | | | 2 | | | |
| Ι | 201 | D | 4 | | | 4 | | | |
| II | 201 | D | 4 | | | 4 | | | |
| Ι | 202 | Е | 1 | | | 1 | | | |
| II | 202 | Е | 1 | | | 1 | | | |
| III | 202 | Е | 1 | | | 1 | | | |
| Ι | 202 | Е | 2 | | | 2 | | | |
| II | 202 | Е | 2 | | | 2 | | | |
| III | 202 | Е | 2 | | | 2 | | | |
| Ι | 202 | Е | 3 | | | 3 | | | |
| II | 202 | Е | 3 | | | 3 | | | |
| III | 202 | Е | 3 | | | 3 | | | |
| Ι | 202 | Е | 4 | | | 4 | | | |
| II | 202 | Е | 4 | | | 4 | | | |
| III | 202 | Е | 4 | | | 4 | | | |
| Ι | 203 | F | 1 | | 1 | | | | |
| II | 203 | F | 1 | | 1 | | | | |
| Ι | 203 | F | 2 | | 2 | | | | |
| II | 203 | F | 2 | | 2 | | | | |
| Ι | 203 | F | 4 | | 4 | | | | |
| II | 203 | F | 4 | | 4 | | | | |
| II | 301 | G | 4 | | | 4 | | | |
| II | 301 | G | 5 | | | 5 | | | |
| II | 302 | Н | 3 | | | 3 | | | |
| III | 302 | Н | 3 | | | 3 | | | |
| II | 302 | Н | 4 | | | 4 | | | |
| III | 302 | Н | 4 | | | 4 | | | |
| II | 302 | Н | 5 | | | 5 | | | |
| III | 302 | Н | 5 | | | 5 | | | |
| II | 303 | Ι | 4 | | | | 4 | | |
| IV | 401 | J | 4 | | | | | 4 | |

Table 18.7 Training plan sort: skill, target group, and course.

Chapter 19

Training Materials and Curriculum Development

ADULT LEARNING THEORY

Evaluate and select training materials and resources that adhere to adult learning theories. (Analyze)

Body of Knowledge IV.C.1

In Chapter 18, *training* was defined as a subset of education that focuses on increasing proficiency in a skill. It is a "push" strategy that pushes knowledge, skills, and attitudes toward employees to enhance their job performance. By contrast, learning is a "pull" strategy by which the employee pulls knowledge, skills, and attitudes to enhance their job performance.

Hergenhahn and Olson (1997) state that *"learning* refers to a change in behavior potentiality, and *performance* refers to the translation of this potentiality into behavior." Rothwell (2008) states this more simply as: *"learning gives individu*als the potential to get results. But performance is the actual realization of that potential."

Assumptions About Adult Learners

The following assumptions about adult learners are helpful in understanding how to design curricula, deliver material, and more effectively transfer knowledge:

- They learn for different reasons:
 - To build social networks
 - To meet expectations such as those that might be imposed by those in authority
 - To advance in their careers
 - To seek stimulation
 - To help others

- To learn for the sake of learning

At the outset of the training, the instructor should consider addressing the above topics with the learners to help gain a feel for the individual motivations for participating in the learning activity.

- They prefer to focus on learning that centers on key ideas, principles, and experiences rather than on general learning topics.
- They prefer to learn in an environment that is both physically and psychologically comfortable.
- They retain only about eight percent of material when training is conducted off the job. This percentage can be increased by providing the learning with memorable instruction, appealing to their physical senses, relating new concepts to existing knowledge and understanding, or helping them solve problems faced on the job.
- They face barriers to learning such as scheduling difficulties, money, management support, and so on. The best way to learn about them is to simply ask.

Table 19.1 provides a summary of training preferences of adult learners as seen by Tolbize (2008). Preferences are further categorized by hard and soft skills. Notice

| Generation | Time frame | Skills | Training preferences |
|--------------|------------|-------------|-------------------------------------------------------------------------------------------------------------------------|
| Baby boomers | 1946–1964 | Soft skills | On the job; discussion groups; peer instruction and feedback; live classroom instruction; and one-on-one coaching |
| | | Hard skills | On the job; live classroom instruction; one-on- one coaching; workbooks and manuals; and books and reading |
| Gen X | 1965–1979 | Soft skills | On the job; discussion groups; peer instruction and feedback; one-on-one coaching; and assessment and feedback |
| | | Hard skills | On the job; live classroom instruction; one-on- one coaching; workbooks and manuals; and books and reading |
| Gen Y | 1980–1999 | Soft skills | On the job; discussion groups; peer instruction and feedback; one-on-one coaching; and assessment and feedback |
| | | Hard skills | On the job; live classroom instruction; one-on- one coaching; workbooks and manuals; and books and reading |

| Table 19.1 | Folbize's training preferences of adult learners by generation | |
|------------|-----------------------------------------------------------------|---|
| | foldize s training preferences of addit rearriers by generation | • |

Source: Adapted from Tolbize (2008).

| | 01 | 70 |
|--------------|------------|-------------------------------------------------------|
| Generation | Time frame | Training preferences |
| Baby boomers | 1946–1964 | Transfer learning to jobs |
| | | • Formal classrooms |
| | | • Mixed reception on computers and Internet use |
| Gen X | 1965–1979 | • Personal interaction such as coaching and mentoring |
| | | • Interactions with manager and peers |
| | | • e-learning |
| Gen Y | 1980–1999 | • Virtual learning |
| | | Threaded discussions |
| | | • Social media |

 Table 19.2
 Carnes's training preferences of adult learners by generation.

Source: Compiled from Carnes (2011).

that all generation types cite similar learning preferences for hard skills. Carnes (2011) offers slightly different insights in Table 19.2.

Characteristics That Affect Adult Learners

Trainers need to be aware of any characteristics of adult learners that might impede their ability to learn or the trainer's ability to transfer knowledge and skills. Several key characteristics to be mindful of include: hearing, seeing, memory, and specific learning disabilities. All of these just mentioned may be agerelated in nature. In addition, specific learning disabilities can affect anyone of any age. For example, consider dyslexia as one type of learning disability.

Of course, one key aspect regarding adult learners, and maybe the most obvious, is generational differences. Consider Table 19.3, which depicts the various generations currently in the workforce. This table offers some strategies and tactics when dealing with different generations. Note the marked differences between the generations.

Additional Factors Affecting Adult Learners

Two additional factors are important when training adult learners: language and culture.

Language is an important consideration, particularly when training material and the delivery are not in the learner's native language. Although English is recognized worldwide as the language of business, it would be generally expected to find that those who are receiving training are more comfortable using their primary language. Unfortunately, translation from English is not always possible,

| Generation | Timeframe | Dealing with generational differences | |
|------------------------|-----------|------------------------------------------------------------------------------------------------------------------|--|
| Baby boomers 1946–1964 | | • Explain why they should care about the training and what is in it for them | |
| | | • Clarify how the organization benefits from their application of the training to their jobs | |
| | | Incorporate their experiences into the training and engage them to share their knowledge | |
| | | • Be aware of time limitations | |
| Gen X | 1965–1979 | • Clarify how others benefit from their training and what is in it for them | |
| | | Engage others when challenged | |
| | | Provide them with a way to vent by engaging in the discussion | |
| Gen Y | 1980–1999 | • Clarify how training can help them meet their personal goals | |
| | | • Appeal to the group since individuals are greatly influenced by their peers | |
| | | • Use technology | |

Table 19.3 Training strategies for dealing with generational differences.

Source: Compiled and adapted from Rothwell (2008).

particularly when the audience is highly geographically diverse, possessing many different native tongues. In these cases, it might be possible to stratify the audiences by language, if possible, and work in smaller groups.

The other factor to consider is culture. It is especially important given that many organizations are global, encompassing multiple cultures. Culture must be considered when designing and delivering training material as well as when selecting instructors.

Hofstede (2008) identifies five dimensions of culture and characterizes their impact on training:

- *Power distance.* The power distance represents the "extent to which the less powerful members of institutions and organizations expect and accept that power is distributed unequally." It is scaled as "small" or "large." Hofstede identifies the implications of this dimension on training in Table 19.4.
- *Uncertainty avoidance.* Uncertainty avoidance is the "extent to which members of a culture feel threatened by ambiguous and unknown situations." It is scaled as "weak" or "strong." Hofstede identifies the implications of this dimension on training in Table 19.5.

| Power distance | | |
|-----------------------------------------------------|-------------------------------------------------|--|
| Small | Large | |
| Teachers treat students as equals | Students are dependent on teachers | |
| Students treat teachers as equals | Students treat teachers with respect | |
| Student-centered education | Teacher-centered education | |
| Students initiate some communication in class | Teachers initiate all communication in class | |
| Teachers are experts who transfer impersonal truths | Teachers are gurus who transfer personal wisdom | |

 Table 19.4
 Implications of the "power distance" dimension on training.

Source: Adapted from Hofstede (2008).

| Table 19.5 | Implications of the | "uncertaint | y avoidance" | dimension on | training. |
|------------|---------------------|-------------|--------------|--------------|-----------|
|------------|---------------------|-------------|--------------|--------------|-----------|

| Uncertainty Avoidance | | |
|-------------------------------------------|-----------------------------------------------|--|
| Weak | Strong | |
| Students want good discussions | Students want to know right answers | |
| Teachers may say, "I don't know" | Teachers are supposed to have all the answers | |
| Emotions should be controlled everywhere | Emotions can be expressed in class | |
| Tolerance exists for differences in class | Pressure exists among students to conform | |
| Teachers involve parents | Teachers inform parents | |

Source: Adapted from Hofstede (2008).

- *Relationship with others.* This comprises two sub-dimensions: individualism and collectivism. *Individualism* in a society means the ties between individuals are loose and that everyone is expected to look after oneself and immediate family. By contrast, *collectivism* in a society means that individuals are members of strong in-groups from birth onward. Hofstede identifies the implications of this dimension on training in Table 19.6.
- *Emotional gender roles.* A masculine society is one in which both emotional gender roles are distinct. The male is expected to be assertive, tough, and focused on material success, while the female is expected to focus on the quality of life. The male culture is the dominant culture. A feminine society is one in which the emotional gender roles overlap. Both the male and female are expected to be modest, tender, and focused on the quality of life. Hofstede identifies the implications of this dimension on training in Table 19.7.

| Relationship with others | | | |
|------------------------------------------------------|-------------------------------------------------------------------|--|--|
| Individualist | Collectivist | | |
| Purpose of education is learning how to learn | Purpose of education is learning how to do | | |
| Students' individual initiative is encouraged | Students' individual initiative is discouraged | | |
| Students are expected to speak up in class | Students only speak up in class when sanctioned by the group | | |
| Students associate according to interests | Students associate according to in-groups | | |
| Diplomas increase economic worth and/or self-respect | Diplomas provide entry to higher-status group/sometimes bought | | |

 Table 19.6
 Implications of the "relationship with others" dimension on training.

Source: Adapted from Hofstede (2008).

Table 19.7 Implications of the "emotional gender roles" dimension on training.

| Emotional gender roles | | |
|---------------------------------------------|------------------------------------------|--|
| Masculine | Feminine | |
| Brilliant teachers are admired | Friendly teachers are liked | |
| Best student is the norm | Average student is the norm | |
| Competition exists in class | Overambition is unpopular | |
| Praise for a good student | Praise for a weak student | |
| Students overrate their own performance | Students underrate their own performance | |
| Competitive sports belong to the curriculum | Competitive sports are extracurricular | |
| Failing in school is a disaster | Failing in school is a minor incident | |

Source: Adapted from Hofstede (2008).

• *Time horizon.* The time horizon is scaled as a "short-term orientation" or "long-term orientation." The "short-term orientation is the extent to which members of a society take their guidance from the past and try to fulfill their present needs and desires" while the "long-term orientation is the extent to which members of a society adapt themselves to reach a desirable future." Hofstede identifies the implications of this dimension on training in Table 19.8.

In addition to the above, Hofstede analyzed the five dimensions in the following manner and associates them with various countries around the world:

• *Power distance versus uncertainty avoidance.* Hofstede's findings are shown in Table 19.9.

| Time horizon | | | |
|----------------------------------------------------------------------|--|--|--|
| Long-term | | | |
| Students attribute success to effort and failure to a lack of effort | | | |
| Studying hard is the norm | | | |
| High performance in mathematics | | | |
| Talent for applied, concrete sciences | | | |
| Children learn to save | | | |
| | | | |

Table 19.8 Implications of the "time horizon" dimension on training.

Source: Adapted from Hofstede (2008).

 Table 19.9
 Hofstede's quadrant analysis of "power distance" versus "uncertainty avoidance" dimensions.

| | | Power distance | | |
|--------------------------|--------|-----------------------------------------------|-----------------|--|
| | | Small | Large | |
| Uncertainty avoidance | Weak | Anglo countries | • China | |
| | | United States | • India | |
| | | Netherlands | | |
| | | Nordic countries | | |
| | Strong | Baltic states | • Czechia | |
| | | German-speaking countries | • Japan | |
| | | • Hungary | • Korea | |
| | | • Israel | Latin countries | |
| | | | • Poland | |
| | | | • Russia | |

Source: Adapted from Hofstede (2008).

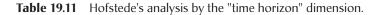
- *Emotional gender roles versus relationships with others.* Hofstede's findings are shown in Table 19.10.
- *Time horizon across the short-term to long-term orientation spectrum.* Hofstede's findings are shown in Table 19.11.

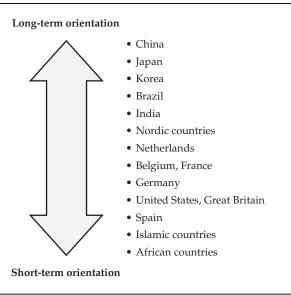
Although an organization may (but in many cases is unlikely to) have one organizational culture, there may be many different national cultures present within it. Collectively, Tables 19.4 to 19.8 provide an instructor with some insight into how to navigate training in differing cultures.

| | | Emotional gender roles | |
|--------------|---------------|--------------------------------------|---------------------------|
| | | Feminine | Masculine |
| Relationship | | • Bulgaria | • Arab world |
| with others | | • Chile | • China |
| | | • Costa Rica | • Greece |
| | Collectivist | • Korea | • Japan |
| | | • Portugal | • Mexico |
| | | • Russia | • Venezuela |
| | | • Thailand | |
| | | Baltic countries | Anglo countries |
| | | • France | • Czechia |
| | | Netherlands | German-speaking countries |
| | Individualist | Nordic countries | • Hungary |
| | | • Spain | • Italy |
| | | | • Poland |
| | | | United States |

| Table 19.10 | Hofstede's quadrant analysis of "emotional gender roles" versus "relationship with |
|-------------|------------------------------------------------------------------------------------|
| | others" dimensions. |

Source: Adapted from Hofstede (2008).





Source: Adapted from Hofstede (2008).

The Intelligences and Adult Learning

Gardner (1983) proposed that there are multiple types of intelligence and that a person will learn better if the learning experience enables them to apply their own strengths. These intelligences include:

- Verbal or linguistic. Skilled in the use of words
- Logical or mathematical. Skilled in the analysis of concepts
- Visual or spatial. Skilled with manipulating symbolic relationships
- Musical. Skilled with sound
- Bodily or kinesthetic. Skilled in physical touch or movement
- Interpersonal. Skilled in relationships with others
- Intrapersonal. Skilled in self-analysis and development

Later, Gardner proposed the following two additional intelligences:

- *Naturalistic.* The relationship and sensitivity to nature and a person's place in it
- Existentialist. The ability to reflect on the meaning of life

Others have added Daniel Goleman's *emotional intelligence* to the list. Emotional intelligence is the ability to perceive, control, and evaluate emotions.

Differences in skill levels in each of the original seven intelligences affect how well one will learn various subjects in different environments. Most training has traditionally focused on the first two intelligences, while accelerated learning techniques often integrate others in order to take advantage of different learning styles and how each fits with a particular aspect of the topic.

Linksman (1996) considers three of Gardner's intelligences from another viewpoint. Instead of classifying them as intelligences, she reclassifies them as learning styles. Further, she dissects the "bodily or kinesthetic" into two learning styles: tactile learners and kinesthetic learners. Thus, Linksman identifies four learning styles as follows:

- Visual learners. These individuals learn by seeing.
- Auditory learners. These individuals learn by hearing.
- *Tactile learners.* These individuals learn by touching, sensation, and connecting what they learn to their sense of touch or emotions.
- *Kinesthetic learners.* These individuals learn by moving large muscle groups through activities, game playing, exercises, and so on.

Further, each learning style is paired with a brain hemisphere preference (that is, right- or left-brained). Linksman's theory does not preclude individuals from using both brain hemispheres or preferring multiple learning styles. Thus, the possibility of a combination learner is addressed. Linksman provides extensive recommendations for dealing with each type of learning style and brain preference. Details can be found in her book *How to Learn Anything Quickly*.

Theories of Learning

Rothwell (2008) notes "every theory of learning and every individual philosophy of teaching influences what trainers or learning professionals do on a daily basis. Important practical ideas may be obtained by contemplating various theories of learning. Hence, theories of learning affect the practice of training and the learner's practice of learning."

There is no right or wrong theory of learning. Each has its own advantages, disadvantages, and implications to both instructors and learners.

With the above in mind, we will address the following categories of theories of learning:

• *Functionalistic*. See Table 19.12.

| Theory Leading thinkers | | Theory highlights | | | |
|-------------------------|---------------|-----------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------|--|--|
| Functionalistic | Edward Lee | Learning occurred by trial and error | | | |
| | Thorndike | • Law of readiness: | | | |
| | | 1. People will find it satisfying to act when ready | | | |
| | | People will find it irritating to not act when ready | | | |
| | | 3. People will find it irritating when forced to act when unwilling | | | |
| | | • Law of effect: | | | |
| | | 1. Rewards will increase learning | | | |
| | | 2. Punishment will not effect learning | | | |
| | | • The transfer of learning is more likely when the environment where the learning occurred is similar to where it will be applied | | | |
| | B. F. Skinner | • Two kinds of behavior: operant (self-initiated) and respondent (affected by stimuli) | | | |
| | | • People do not have free will | | | |
| | | | Control behavior by controlling the reinforcements | | |
| | | • Behaviors that are reinforced through rewards are repeated | | | |
| | | Behaviors that are ignored are extinguished | | | |
| | | • Punishment reduces the likelihood of recurrence, but does not extinguish | | | |

 Table 19.12
 Theory highlights of the *functionalistic* theories of learning.

Source: Compiled from Rothwell (2008).

| Theory | Leading thinkers | Theory highlights | | |
|------------------|----------------------|---------------------------------------------------------------------------------------------------------------------------------|--|--|
| Associationistic | Ivan Pavlov | Famous salivating dog and bell experiment | | |
| | | • Unconditioned stimulus associated with an unconditioned response | | |
| | | Conditioned stimulus associated with a conditioned response | | |
| | Edwin Ray Guthrie | • Law of contiguity: people repeat the same behaviors that worked for them in previous similar situations | | |
| | | Practice improves performance | | |
| | | • Unlearning: to unlearn a behavior the cue that created the behavior must be replaced by a different cue | | |
| | William Estes | • Developed stimulus sampling theory of learning | | |
| | | • Transfer of learning takes place when the learning situation is similar to other situations the learner can associate with it | | |
| | | • Similar to Thorndike's approach | | |

| Table 19.13 | Theory highlights | of the ass | ociationistic | theories o | f learning. |
|-------------|-------------------|------------|---------------|------------|-------------|
| Tuble 17110 | incor, inginging | or the uss | ociumonistic | theories e | n ieurinig. |

- Associationistic. See Table 19.13.
- Cognitive. See Table 19.14.
- Constructivist. See Table 19.15.
- *Neurophysiological*. See Table 19.16.
- Other. See Table 19.17.

Because each category of theory is extensive in its own right, only highlights of theories have been shown. Readers interested in details should pursue more-detailed sources dedicated to these topics.

In addition to those identified above, one particular learning theory stands out. Kolb (1975) proposed an experiential learning model as follows:

- 1. *Concrete experience (Hands-on stage)*. This is also known as the "Do" stage.
- 2. *Reflective observation (Abstract thinking stage).* This is also known as the "Observe" stage.
- 3. *Abstract conceptualization (Abstract thinking stage).* This is also known as the "Think" stage.

| Theory | Leading thinkers | Theory highlights | |
|-----------|------------------|-------------------------------------------------------------------------------|--|
| Cognitive | Edward Tolman | Learn through viewing the "bigger picture" | |
| | | • Rejected the stimulus-response approach | |
| | | • Focused on looking at pattern | |
| | | Distinguished between: | |
| | | Learning—potential to perform | |
| | | - Performing—manifestation of that ability | |
| | | • People do not apply learning without a reason to do so | |
| | | • Defined latent learning as choosing to not apply what has been learned | |
| | Albert Bandura | Observational or social learning | |
| | | • Watch others and imitate them (has implications for coaching and mentoring) | |
| | | Observers must be exposed to both good and bad examples | |
| | | • People remember their observations in both images and words | |
| | Donald Norman | Information processing view of learning | |
| | | – Input (information) | |
| | | – Process (learn) | |
| | | - Action (apply) | |

Table 19.14 Theory highlights of the *cognitive* theories of learning.

4. *Active experimentation (Hands-on stage).* This is also known as the "Plan" stage.

Figure 19.1 depicts this model. (Notice the similarity to the plan–do–check–act cycle.) When the knowledge is applied, another experience is created and the cycle continues. The cycle is often viewed as an increasing spiral. The model is useful not only for designing training programs, but also understanding how people react to certain types of training since most people tend to prefer a particular stage of the learning cycle (that is, hands-on and abstract thinking stages).

Before an organization can select the appropriate methods and tools to use in quality training, it must first understand how adults learn. In general, adults exert more control over their lives than do younger people, and this desire for control extends to the learning process as well.

| Theory | Leading thinkers | Theory highlights |
|----------------|------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Constructivist | Jean Piaget | Based on "accommodation" and "assimilation" |
| | | Accommodation is the process of reframing the external world to fit new experiences. This accommodates learning from experiences. |
| | | Assimilation is the incorporation of new experiences into an existing framework of the external world. Experiences may be changed to fit into the framework. |
| | | • Learners are more responsible for learning than teachers |
| | | Learners are motivated to learn when they believe they can be successful. |
| | | • Instructors are facilitators who guide and stimu- late learners |
| | | Instructors are not subject matter experts |

| earning. |
|----------|
| 28 |

| Table 19.16 | Theory highlights of the <i>neurophysiological</i> theories of learning. |
|-------------|--------------------------------------------------------------------------|
|-------------|--------------------------------------------------------------------------|

| Theory | Leading thinkers | Theory highlights |
|--------------------|------------------|------------------------------------------------|
| Neurophysiological | Donald Hebb | Focus is placed on brain anatomy and chemistry |
| | | • Two types of learning: |
| | | Stimulation and response |
| | | – Cognitive |
| | | Adult learning involves creative thinking |
| | | Input requires the external environment |

Source: Compiled from Rothwell (2008).

Knowles (1980) identifies six factors that influence adult learning:

- 1. Need to know. Adults need to know the reason for learning something.
- 2. *Foundation*. Experience (including mistakes) provides the basis for learning activities.
- 3. *Self-concept.* Adults need to be responsible for their decisions on education, including involvement in the planning and evaluation of their instruction.

| Theory | Leading thinkers | Theory highlights | |
|------------------------|------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|
| Anchored instruction | John Bransford | • Learning occurs through case studies, role playing, activities, exercises, games, and problem-solving exercises. | |
| Cognitive load | John Sweller | • Provide people with a schema to help them remember and relate to topics. Avoid memorization. | |
| Conversation | Gordon Pask | • People learn through conversations, and the best way to learn something is to teach it. | |
| Experiential | Carl Rogers | • Experience facilitates learning. | |
| Functional context | Tom Sticht | • Relate what is to be learned to what the learned does. Methods include deriving instructional materials from work materials. | |
| Minimalist learning | John Carroll | • Minimize the amount of learning material provided and immediately immerse the learning in self-contained learning projects likely to be faced in the real world. | |
| Situated learning | Jean Lave | • Learning should occur in the situation or environment in which it will be applied. | |
| Subsumption | David Ausubel | • Individuals can master massive amounts of material in short periods of time. | |

Table 19.17 Theory highlights of other various theories of learning.

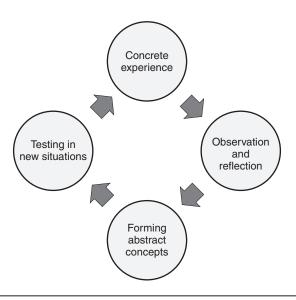


Figure 19.1 Kolb's experiential learning model.

- 4. *Readiness.* Adults are most interested in learning subjects that have immediate relevance to their work and/or personal lives.
- 5. *Orientation*. Adult learning is problem-centered rather than content-oriented.
- 6. *Motivation*. Adults respond better to internal versus external motivators.

Numerous theories, methods, principles, and ideas have been presented regarding adult learning. As mentioned previously, there is no right or wrong approach; there are only considerations that should be kept in mind when dealing with adult learners.

Evaluating and Selecting Training Materials and Resources

Although some organizations may do well at conducting a training needs analysis and developing a training plan, most organizations invoke minimal effort when it comes to evaluating and selecting training materials and resources. This occurs for one or more of the following reasons:

- They simply don't know how.
- It was overlooked and wasn't planned into the budget.
- It takes time and effort, and after all, it's the employee's responsibility to learn.
- If purchased: they assume that the high cost of training materials makes them high caliber.
- If developed in-house: we only do good work, so why do we need to evaluate it.
- They may use existing materials with some modifications due to cost constraints.

Evaluating and selecting training materials and resources does take considerable effort. However, it is important to keep in mind that the training we speak of is directly derived from the organization's strategic plan. It is being put in place to help the organization achieve its goals and objectives. To shortcut the process now is to do the organization a grave injustice.

To help organizations more effectively evaluate and select training materials and resources, the following checklist has been developed:

- Language
 - Is the language at the appropriate level for the target group?
 - Is the spelling correct?
 - Is the grammar correct?

- Are idioms, slang, or excessive jargon used?
- Accuracy
 - Is the material technically accurate?
 - Is the material current?
 - Is the math correct?
- Format
 - Is the material consistent and uniform in design and presentation format?
 - Is the format easy to follow?
 - Is the material too dense?
 - Is there a mixture of words, illustrations, and graphics?
 - Is the material sufficiently modularized both in content and in time?
- Progression
 - Is an adequate foundation presented?
 - Does the material flow logically?
- Infrastructure
 - Does the material stand alone?
 - Are supplementary materials required?
 - Is an instructor required and, if so, how many?
- Content
 - Is the content unambiguous?
 - Is the material presented without bias?
 - Does the content support the learning objectives and outcomes?
 - Is the material consistent with other material that either exists or is under consideration?
- Development and outcomes
 - Does the material address different learning styles?
 - Does the material address different generational preferences?
 - Will the material meet the objectives established by the training plan?
 - Will the material permit the target group to achieve the desired level of competency?
 - What target groups are suitable for the training materials?

- Do the training materials require solution manuals and, if so, are they provided?
- Responsibilities and ownership
 - Who is responsible for updating the material (if the material is purchased)?
 - How often is the material updated (assuming the material is purchased and the supplier is responsible for updates)?
 - How are errors in the materials handled (assuming the material is purchased and the supplier is responsible for updates)?
 - Who owns the training materials (if delivered by a third party)?
- Delivery
 - Do instructors require certification?
 - Are the instructors knowledgeable in the subject matter?
 - Is the material available for delivery in different mediums (that is, classroom, electronic, and so on)?
 - Is the supplier accredited (assuming a third-party trainer and accreditation is required)?
 - Do the training materials require the training to take place in a special environment or require special facilities, tools, audiovisual equipment, computers, or software?
 - How much notification is required, if delivered by a third party?
- Evaluations
 - Who evaluates the training materials and resources, and what are their qualifications for doing so?
 - How are students evaluated against training objectives?
 - How are the instructors evaluated?
 - Has the material been tested previously and, if so, what were the results?
- Duration
 - What is the course duration?
 - Is the course contiguous, periodic, or ad hoc (for example, three days in a row, the first week of every month for four months, or as time permits, respectively)?

Recall the weighted criteria approach discussed in Chapter 6 that was used to select and prioritize projects. This approach can be easily adapted to evaluating and selecting training material as well. Further, it would provide a more objective

basis to the evaluation process that would facilitate meaningful discussions among the evaluators. Evaluators would now have a quantitative basis for justifying their decisions.

Consider the above checklist to be a starting point. Evaluating and selecting training materials and resources can take a significant amount of time and effort. Consequently, it should be incorporated into the training plan schedule and budget. Also, remember that not everyone is capable of evaluating training material. This little tidbit was included in the checklist and should not be overlooked. "Who is qualified to evaluate the training material?" should be addressed early on and not left to whomever is available.

INTEGRATION

Ensure that the training harmonizes and leverages other tools and approaches being used and that it is aligned with the organization's strategic objectives and culture. (Evaluate)

Body of Knowledge IV.C.2

It should be clear that training is derived from the strategic plan since it has been the basic premise of the last three chapters. This concept is updated and depicted in Figure 19.2. However, too often organizations design training without regard to other tools or approaches that are ongoing within or planned by the organization. For example, consider an organization that has adopted ISO 9000, the balanced

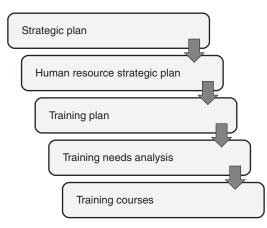


Figure 19.2 Creating the line of sight from course to strategy.

scorecard, and lean. Currently, each of these quality approaches is being taught to the organization independently of the others. More specifically, each curriculum makes no mention of the others. Essentially, each is treated as if the others do not exist. Now, suppose the organization wishes to introduce Six Sigma. If the past is any indication, it will likely create a Six Sigma curriculum alongside the others, with no thought given to how the curricula might be integrated or leveraged off of one another to create a more effective training program that might accelerate achieving the organization's goals and objectives.

The above is unfortunate because it is the integration of the tools and approaches where true value and benefit is found. Kubiak (2003) states, "The various quality approaches are not independent of one another but can work in a mutually supporting and integrated manner. It is not necessary to abandon one for another." He goes on to say that, "It is not necessary, or productive, to leap from fad to fad as so many organizations have done. Take the time to gain a deeper understanding of each approach. When thoroughly understood and implemented properly, each approach brings a unique set of perspectives and insights for driving organizational excellence."

TRAINING DELIVERY

Monitor and measure training to ensure that it is delivered effectively and efficiently by qualified individuals. (Apply)

Body of Knowledge IV.C.3

Some of the material on training delivery is extracted from Westcott (2006).

Methods of Training Delivery

Traditionally, training delivery methods have included live instructors, printed handouts, and media aids (for example, whiteboards, chalkboards, flip charts, overhead projection of slides and transparencies, or computerized projection). Lectures, discussions, role-plays, simple paper-based games, case studies, workbooks, videotape, audiotape, and self-directed learning with printed materials are often used. Lectures and role-playing date back to classical Greece, more than 2000 years ago. Lectures were the primary method of instruction, but creative instructors now use many other tools to enhance learning.

Self-Directed Learning (SDL)

Increasingly, newer technologies for delivering *self-directed learning (SDL)*, also called *learner-controlled instruction* (LCI), are becoming available. Working without an instructor and at their own pace, adult learners build toward achieving the

desired competency level in the needed knowledge or skill. Self-directed learning works well when there is only one person to train at any given time, when the learners are geographically distant, when training needs to occur at multiple sites simultaneously, when consistency of skill execution is a necessity, and when refresher training is needed.

The trend toward SDL reinforces the trend toward the use of technologybased training, as new technologies both stimulate and enable direct interaction between the individual and the material to be learned. Interactive CD-ROMs are a currently popular method for delivery of certain types of standardized training (for example, computer application training).

Note: ASQ's Foundations in Quality Learning Series is an example of SDL technology, combining both printed text and interactive questioning.

Lectures and Presentations

A lecture is a one-dimensional, oral transmission of information to students by an instructor, either in person or via electronic media. It is the most frequently used manner of conveying material to students and often is used in combination with other tools. Lectures are one type of presentation, but presentations can also include other oral, visual, and multimedia formats.

Lectures and presentations offer the advantage of being relatively easy to develop, present, and revise. However, it is usually difficult to master knowledge and skills by this means. Also, this delivery method results in low retention rates. Figure 19.3 provides the approximate retention rates associated with a learner's degree of participation for varying delivery methods.

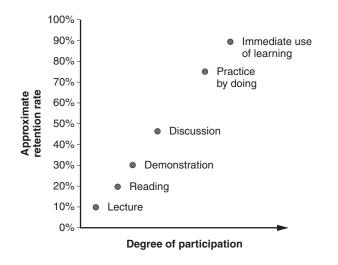


Figure 19.3 Retention rates based on delivery method and degree of participation. *Source: Certified Manager of Quality/Organizational Excellence: ASQ's Foundations in Quality Learning Series.* 2005. Milwaukee: ASQ Quality Press.

Discussion Format

The discussion format allows participants freedom to present views, opinions, and ideas in an unrestrained environment. Roundtables and panel discussions are variations. These methods normally work best with a knowledgeable and experienced discussion leader. The Socratic method of repeated questioning has been successfully used to elicit and share the accumulated knowledge and experience of course members.

However, the discussion format requires a skilled facilitator to execute it successfully. Also, some participants may be reluctant to speak up in this environment.

Experiential Training

The focus of this form of training is on experiencing the effects typically encountered in a real-life situation. The experiential approach employs many types of structured experiences (for example, games, simulations, role-plays) to facilitate learning. Typically, participants assume roles within a stated scenario designed to surface one or more learning points. They then "play out" the scenario to a conclusion under the eye of a skilled facilitator. No outcome is prescribed, but a well-designed scenario will tend to produce a predictable range of responses highlighting the training content. Experiential training is used more frequently with soft skill training.

Experiential training permits faster learning than on the job, and allows for mistakes to occur without penalties. However, it requires a great deal of planning and can be costly to provide.

Coaching

Used on a one-to-one basis, or for a small group or team, and conducted at the workplace, *coaching* is a teaching, learning, counseling relationship designed to develop job-related knowledge and skills and improve performance. It requires a continuous flow of instructions, comments, explanations, and suggestions from coach to employee, with the coach demonstrating, listening, questioning, relating learner's experiences, assisting, motivating, encouraging, and rewarding performance. The coaching option is used primarily to teach complex skills and operations—when the number of trainees is small and the training is needed infrequently—and is a follow-on to other forms of employee training. It has the advantage of being flexible and adaptable. Its disadvantage is that its success depends on the competence of the coach.

Coaching can be applied to work groups or teams, in which it closely resembles the sports team coaching model. Finnerty (1996) outlines the following steps for effective coaching:

- Define performance goals
- Identify necessary resources for success
- Observe and analyze current performance
- · Set expectations for performance improvement

- Plan a coaching schedule
- Meet with the individual or team to get commitment to goals, demonstrate the desired behavior, and establish boundaries
- Give feedback on practice and performance
- Follow up to maintain goals

Case Studies

Many different kinds of learning exercises fall under the category of "case studies." Such exercises illustrate the application of study content and show how different approaches can be used to solve problems. Case studies can serve to stimulate intense discussions and idea sharing.

Case studies can be developed in-house or purchased from a variety of sources. However, a form of mini case study, the *critical incident*, may often be of greater value than a larger, more inclusive case study. It is carefully crafted, usually presented in a succinct written form, and is designed to engender learning about how to respond to a specific event, situation, or condition. The critical incident is frequently derived from real situations within the organization for which the training is designed. A critical incident might consist of just a few sentences describing a specific type of event or condition without reference to names or locations. Critical incidents are usually "sprinkled" throughout the training to illustrate specific learning points. The advantages of critical incidents are:

- They are easier to prepare than a full-blown case study.
- Participants can usually identify with the situation since it is their organization.
- Discussion is more targeted, with less confusion generated than with the multifaceted case study.
- They take less discussion time and get to the learning points quickly.

Concerns relating to critical incidents are:

- Data used to compile critical incidents must be collected prior to course design, necessitating a plan and time to do so.
- Critical incident should not be confused with anecdotal material that is used extemporaneously to illustrate a point and/or to inject humor.
- Care must be taken to not target, purposely or accidentally, any person or work unit in the company. Data must be sanitized so as to appear anonymous.

Workbooks

Workbooks contain a collection of discussion topics, exercises, extra readings, and other reference material. They may be used to supplement other materials provided during classroom presentations. Workbooks can serve several purposes:

- As a learning tool: actively used during the presentation to encourage note taking (fill-in-the-blanks), as a self-test, as a supplement to material presented in class, as a more job-specific interpretation of material presented in a generic textbook, and as a reference to be used during class testing.
- To demonstrate knowledge acquired or skills obtained at each stage of the training: specific notes, research data, or checklists compiled by participants and signed-off by the instructor.
- As a guide to be used by the trained person back on the job (that is, the guide might contain workmanship standards and other reference material).

Workbooks can combine some or all of the features in the previous list. They can range from a binder for filing class handouts and making notes to a more formal reference manual.

However, workbooks do have some drawbacks. They require self-motivated participants to use them effectively, poorly designed materials may be boring, well-designed materials may be expensive, and participants may become "stuck" and lose interest.

Instructional Games, Simulations, and Role-Playing

Experience has shown that adult learners prefer to practice rather than just listen to lectures and that they prefer to interact with other learners when in a group learning environment.

Instructional games are activities designed to augment other training methods in order to focus the participants' learning toward specific training outcomes. One example: "In-box" exercises usually present each participant with a variety of papers (for example, reports, charts, tables) that resemble what a manager may find in an in-box. The participant analyzes the documents and takes what they consider appropriate action for each. Responses are either discussed in class or evaluated off-line by the instructor, with instructional notes affixed to the documents.

Simulations resemble instructional games but also include correspondence between some aspect of the game and reality. Role-playing consists of spontaneously performing in an assigned scenario.

Remote Learning

Correspondence schools and open learning programs at educational institutions are types of remote learning. Students at all levels of postsecondary enrollment can earn degrees in many fields without ever stepping into a classroom. This selfdirected learning style consists of an instructor mailing (that is, through traditional or electronic mail) a course syllabus and a list of assignments to students. Students complete the required work and send it to the instructor, who then returns the graded assignment. Remote learning is especially popular with nontraditional students who would not be able to attend classes in person. This type of learning requires a tremendous amount of self-discipline.

Distance Learning

One popular form of distance learning is where students in more than one location are scheduled and linked together electronically (for example, by cable or satellite TV) to enable one instructor to simultaneously teach students in multiple locations. The video provides face-to-face contact between the students and the instructor. Audio interaction between student and instructor and between students may be included.

Computer-Based Instructional Techniques

Although much training continues to be conducted using the traditional techniques of lecture, discussion, recitation, and on-the-job training, technology-based approaches are rapidly overtaking the traditional methods. Computers, electronic communications, and the Internet have created dramatic new opportunities to provide learning. New products and techniques are appearing nearly every day.

Examples of existing computer-related technology used include:

- Instructional modules embedded within computer software (for example, help screens, "wizards," pop-ups)
- Learning from "mistake-proofing" software (for example, spelling and grammar checkers, templates)
- Websites (for example, offering technical knowledge, testing, articles, links to other sites)
- Internet access:
 - Online equipment instructional manuals
 - Webinars (interactive education and training modules)
 - Articles, research papers, books
 - Forums, "chat rooms," and "blogs"
- Intranets (in-organization Internet-like networks providing training and education)

Computer-based instructional techniques have the added advantages of being able to be widely distributed with ease, updated with little difficulty, and offered with more activity options such as simulations and games. However, learners must be motivated to use this method of delivery, and not everyone adapts well to it.

Job Aids

Job aids are virtually any type of media that can either substitute for formal training and/or provide reinforcement or reference after training. Often, providing a job aid can be the simplest and most cost-effective way to assist workers in doing their jobs. A few types of job aids are:

1. Procedure manual

- 2. Laminated checklist hung at a workstation
- 3. Message on a computer screen, a buzzer, or other physical or aural interruption warning of a missing step or incorrectly performed function
- 4. Nested (that is, drop-down) computer help screens taking the learner to increasingly greater levels of detail
- 5. Instructions printed on the back of a form giving detailed instructions for completing the form
- 6. Pertinent information for each critical step described on a shop work order or traveler
- 7. Tool mounting board that has a clearly marked place for each type of tool
- 8. Diagnostic decision tree for solving a problem
- 9. Audio device accessed by the learner wearing earphones that guides the learner through each task step at the sequence and pace expected upon achieving the required level of skill competency

Rossett and Gautier-Downes (1991) cite eight situations in which job aids are appropriate:

- 1. When the individual can not be expected to remember how to do a task because he or she is rarely required to do it
- 2. To offer support for individuals who confront lengthy, difficult, and information-intensive challenges
- 3. When the consequences of error are high
- 4. When performance relies on a large body of information
- 5. When performance is dependent on knowledge, procedures, or approaches that change frequently
- 6. When turnover is high and the task is simple
- 7. When employees are expected to act in an empowered way with emphasized or new standards in mind
- 8. When there is little time or few resources for training

Overall, jobs aids can be cost-effective, available when and where they are needed, easily kept current, and diminish the need for individuals to rely on memory. However, job aids are not a substitute for expertise or performance, and do not replace good judgment.

On-the-Job Training (OJT)

A significant amount of skills training occurs through on-the-job training (OJT). The learner is usually an employee who has been newly hired, transferred, or

promoted into a position and lacks the knowledge and skill to perform some specific job component. A more experienced employee (for example, peer or supervisor) is usually assigned to work with the trainee. Specific tasks are demonstrated and performed in the actual work environment to help the trainee gain the knowledge or skills previously lacking.

Johnson (1993) presents six principles of OJT:

- 1. Usually used for skills training.
- 2. Useful for individuals or small groups.
- 3. Without a consistent OJT program, there is a strong possibility that people will learn incorrect techniques from trial and error or their teammates.
- 4. TQM requires OJT in the workplace, whether it be shop floor, computer, or a sales call. (Note: Somewhat dated, but included for completeness.)
- 5. OJT must have a plan even though it might not be as formal as those used in the lecture method.
- 6. Teaching process:
 - a. Tell them.
 - b. Show them.
 - c. Have them show you.
 - d. Repeat until performance is satisfactory.

Like all training programs, OJT programs require a needs analysis, training objectives, clear statement of responsibilities, the procedures and work instructions for the specific job or task, and the basis for measuring job or task competency.

Because it is common OJT practice for an employee who is familiar with the job to train another employee, without objectives or a planned system inconsistencies can result in the way employees perform their duties. In addition, the lack of structure can cause bad habits to be included in the training. Another problem occurs when a relatively inexperienced employee is responsible for training others. The employee might know how to do the job, but, as anyone who has done training knows, it is much more difficult to teach someone to do a job than to do it yourself.

It has been estimated that up to 75 percent of all training is OJT. OJT provides instant feedback on the learner's achievements, and relates directly to the job or task where training is needed. The trainer should positively reinforce improvement through as many iterations as necessary for the trainee to achieve the desired level of competence. Hands-on training can be supplemented with photos of properly completed tasks at each stage of production and/or videos of workers properly performing the tasks to enhance the learning experience. All training, even OJT, should be planned to ensure completeness, consistency, and correctness of instruction.

On-the-job training is highly regarded as a learning method. It matches the day-to-day realities of the organization and can be delivered just-in-time. However,

it can be hampered by physical constraints of the job site, it may disrupt work flow, and can create potential safety issues.

Blended Learning

Although each learning method has been discussed separately, it must be made clear that these methods can be mixed in any manner necessary to accommodate the needs of the target group. This mixing is called *blended learning*.

Blended learning accelerates learning and job application, provides the course leader with more options, and costs considerably less than a formal classroom setting. However, it requires more coordination, and tracking the progress of the learner is more difficult.

Keys to Making Training More Effective

Design the training with consideration for learning styles and preferences:

- *Seeing.* Participants need to be able to actually see, or if not, to visualize what is being taught. Demonstrate, show an example, or "paint" a word-picture to illustrate.
- *Hearing.* Participants need to hear and understand what is being taught. Paraphrase a second time. Check with participants to determine if instruction is getting through and is understood as intended.
- *Feeling*. Participants need to touch and feel. As appropriate, allow participants to feel surfaces and shapes. Allow for participants to feel the effect of hot and cold, wind and water, and other forces of nature. Allow for experiencing inner feelings such as sadness, joy, anger, disappointment, enthusiasm, and other emotions.
- *Doing*. Provide hands-on opportunity for participants to try to replicate what is instructed.

When delivering training, the trainer should remember to keep it simple. When participants are so enthralled with the gadgets and gizmos used in the training that they forget the training message, the trainer has gone too far. A theme-park type of production is seldom needed, practical, or economical for presenting training. Additionally, when training delivery is totally dependent on the technology, training technology that is difficult to use can detract from learning or even shut down the training in the event of equipment malfunction or user error.

Training delivered by straight lecture, although a prevalent method, is often boring, and learning fades rapidly, if it sticks at all. Trainees might understand little of the message if they are absorbed in taking notes. For this reason, the use of job aids should be considered both to augment initial training and to provide reinforcement on the job.

Whenever possible, avoid training an individual in a skill or topic in which the learner already has competence. Doing so could cause the learner to become disengaged or distracted from the learning process. The learner could also become disruptive when bored (that is, upstaging the instructor, acting as a "know it all," disagreeing, discounting the instructor's words, starting an irrelevant discussion, annoying other participants, and so on). If part of the training is redundant for a learner, the instructor has a few choices:

- Appoint the pre-informed learner as a co-instructor for the segment where the learner could become bored. (This suggests an in-depth needs analysis, with participant interviews where their competencies and strengths are assessed, and potential co-instructors are invited to prepare to share in the instruction of a training module.)
- Divide the participants and design the training for "basic" and "advanced" levels.
- Allow the pre-informed participant to choose between remaining in the class (and participating constructively) or temporarily leaving until called back.

In skills training, the charisma of the trainer is less important than the ability of the trainer to transfer knowledge and skills to the participant that will, in turn, be demonstrated on the job. Training is not intended to be pure entertainment. This does not preclude the judicious use of humor and having fun learning.

Ensuring Efficient and Effective Delivery by Qualified Individuals

So far, there has been much attention given to the methods of training delivery, but little given to ensuring that the training is delivered efficiently and effectively by qualified individuals. If fact, many organizations never think about this. Instead, they make the subtle, implied assumption that subject matter experts must be able to train since they are the experts in the subject matter. Of course, this is nonsense.

Requiring individuals who are not qualified to train can have disastrous effects on an organization's ability to implement its training plan. For example, suppose an organization is deploying Lean Six Sigma and is in the process of conducting training. Further assume that the instructors are not qualified. This doesn't necessarily mean they don't understand the subject material. They may or may not. However, it does mean they are unable to transfer knowledge and skills efficiently and effectively. Consequently, trainees can become bored, morale can suffer, and the deployment initiative can develop a bad image.

Therefore, it is crucial that an organization ensures training is delivered by qualified individuals. With this in mind, the following checklist is offered for consideration:

- When training is conducted by in-house instructors
 - Dry-run a novice instructor's training delivery with an audience of more experienced instructors.
 - Require instructors to alternate back and forth between performing the job and providing training for the job.

- Provide instructors with refresher courses to ensure their knowledge and skills are up-to-date.
- Require a more experienced instructor to audit a less experienced instructor's class on a random basis and provide feedback.
- Pair experienced and novice instructors during training.
- Involve potential instructors in the material design and development.
- Require novice instructors to audit an experienced instructor's class (for example, a Black Belt audits a Master Black Belt giving instruction to a class of Green Belts).
- Require instructors to be certified (if available) in the field for which they provide training.
- Provide mentoring to instructors, integrate it with class feedback, and devise an improvement plan.
- Administer qualification tests on a specified periodic basis.
- When training is outsourced
 - Obtain references from other clients.
 - Request to sit in on training given by instructors who will deliver it to your organization.
 - Request a mock class be delivered in-house by would-be instructors to experienced belts for evaluation of their efficiency and effectiveness.
 - Review the curricula vitae of potential instructors to assess their experience and knowledge levels.

Ensuring that an organization's training is delivered efficiently and effectively is absolutely critical to ensuring that strategies are achieved. Don't skip this important step. An ineffective instructor can derail a strategic initiative with little effort. The organizational grapevine will do the rest.

The entire first section of this chapter dealt with the subject of adult learning. While it did not specifically isolate Lean Six Sigma practitioners, it should be readily apparent that the concepts, tools, and techniques apply to practitioners and nonpractitioners alike. Consequently, as you teach, design, or redesign your next Lean Six Sigma module or class, give consideration to the important ideas presented in that subsection. In fact, you may want to consider reviewing your existing courses to determine how much consideration you already give to these ideas, perhaps without recognizing it. For example, you might already be integrating individual or team exercises, using videos and workbooks, lecturing, encouraging self-study, offering computer-based or Internet-based training, issuing case studies, applying simulation techniques or games, and so on. As you can see, you may be already appealing to the different learning styles and generational differences.

Chapter 20

Training Effectiveness Evaluation

Develop an evaluation plan to assess and verify the acquisition of required knowledge and skills. (Create)

Body of Knowledge IV.D

One of the biggest questions asked regarding training effectiveness evaluation is, "Why do it?" If the flow in Figure 19.2 was maintained and the necessary communications with senior and middle leadership were executed, then this question becomes moot. Otherwise, it is management's responsibility and duty to ensure they are using the organization's resources prudently to advance its strategies to achieve its goals and objectives. If this cause-and-effect relationship can not be answered through a well-designed training evaluation plan, the only answer is, "We don't know." Try selling that to shareholders and stakeholders.

TERMINOLOGY

The source for the following definitions is the Manpower Services Commission (MSC) *"Glossary of Training Terms"*:

- *Internal validation.* "A series of tests and assessments designed to ascertain whether a training programme has achieved the behavioural objectives specified."
- *External validation.* "A series of tests and assessments designed to ascertain whether the behavioural objectives of an internally valid training programme were realistically based on an accurate initial identification of training needs in relation to the criteria of effectiveness adopted by the organization."
- Assessment of training effectiveness. "A general term for the process of ascertaining whether training is efficient or effective in achieving prescribed objectives. It covers both evaluation and validation."

VALIDATION AND EVALUATION MODELS

Although many validation and evaluation models have been proffered over the last several decades, there are three widely accepted and recognized models. They are from:

- P. B. Warr, M. Bird, and N. Rackham (1970)
- A. C. Hamblin (1974)
- D. L. Kirkpatrick (1976)

Warr et al. (1970) identifies four categories of evaluation:

- Context. This involves reviewing the following:
 - Training need identification
 - Performance problems to be overcome
 - Changes in operational performance at the intermediate stage
 - Immediate objectives and their achievement
- Input. This evaluates the training event itself.
- *Reaction.* This evaluates the "reaction" of the learners both during and after the training event.
- Outcome. This comprises four substages:
 - Definition of the training objectives
 - Construction of the evaluation instruments
 - Use of the instruments
 - Review of the results

Notice that Warr's model includes a focus on a training plan whereas Hamblin's and Kirkpatrick's models do not.

Hamblin's model (1974) is similar to Kirkpatrick's model below. However, Hamblin describes his evaluation approach in terms of five levels:

- *Level 1: Reaction.* This evaluation level is carried out during, immediately after, and some time after the event. It focuses on the reactions of the learners to a range of factors.
- *Level 2: Learning.* The evaluation level is carried out before and after the event. It measures the change in knowledge, skills, and attitudes.
- *Level 3: Job behavior.* The evaluation level is carried out before and after the event. Its purpose is to determine whether performance has changed due to the event.
- *Level 4: Functioning.* This evaluation level is carried out some time after the event. It is intended to quantify the effect of the event on the learner's home organization and the overall organization. Ideally, this

quantification is reflected in monetary terms and presented in the form of a cost-benefit analysis.

• *Level 5: Ultimate value.* The evaluation level is carried out quite some time after the training. The idea behind this level is to understand the extent to which the event has affected the "ultimate" profitability and/or survival of the organization.

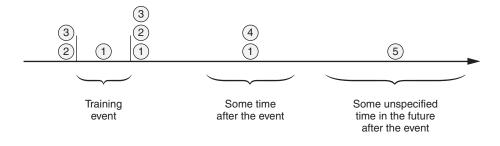
Notice that the points in time associated with the level four and five evaluations are vague and unspecified. Figure 20.1 attempts to place Hamblin's training evaluation timeline into perspective.

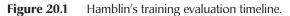
Kirkpatrick's (1976) model is probably the most well-known training evaluation model, and the simplest. His model is based on four evaluation levels:

- *Reaction*. This level focuses on measuring the learner's reaction to the training process, feelings about the structure of the training, its content, and the methods used. This level is closely aligned to Hamblin's level 1.
- *Learning.* This level focuses on the learning that was achieved within the event. Again, this level is closely aligned to Hamblin's level 2.
- *Behavioral.* This level focuses on the change in results of the learning in terms of behavior and performance. This aligns to Hamblin's level 3 and some aspects of level 2.
- *Results*. This level focuses on effects of change on the organization in terms of improvement. This relates to Hamblin's level 4.

The above three models exhibit some similarities and a few differences. However, they all have one thing in common that is essential for each model to work successfully. Simply, each model requires training to be evaluated or measured against the objectives set out for the training in the training plan. This might occur at multiple levels:

- Individual
- Target group





- Course
- Curriculum
- Organization

Also, the reader might recall the discussion on job specifications in Chapter 17. Job specifications, if written to the appropriate level of detail, will serve as objectives. Phillips and Stone (2002) suggest writing objectives at five levels:

- *Level 1: Reaction/satisfaction.* The focus of the objectives at this level is to define the reaction and satisfaction of the participants with the training.
- *Level 2: Learning.* The focus of the objectives at this level is to define the knowledge and skills acquired by the participants.
- *Level 3: Application/implementation.* The focus of the objectives at this level is to define the behavioral change that takes place in the workplace as a result of the training.
- *Level 4: Business impact.* The focus of the objectives at this level is to define the business measures that will be affected directly by the training.
- *Level 5: ROI.* The focus of the objectives at this level is on specific return on investment. Recall from the discussion on developing training plans that a cost–benefit analysis should be performed. Level 5 attempts to determine how the actual ROI fared against the predicted ROI.

The bottom line is quite simple: without preestablished objectives, an effective evaluation is impossible.

MAGER'S LEARNING OBJECTIVE PRINCIPLE

Mager (1962) defined a learning objective principle as, "A description of a performance you want learners to be able to exhibit before you consider them competent." Although Mager developed eleven components to describe a learning objective, the most important three are summarized in Table 20.1.

In addition to the components described in Table 20.1, Mager suggested that "performance time" might be considered a learning objective as well. Although it is included in the example associated with the "criteria" component in Table 20.1, it is presented as a "condition." Mager believes that "performance time" might be considered another component in that it is important for measuring a learner's level of competence. Consider a "beginner" who can perform a task in 15 minutes. Perhaps the next level of competence requires the learner to perform the same task in 10 minutes, the next level (for example, expert) requires 7 minutes, and the final level of performance (for example, master) requires 4 minutes. In this context, "performance time" is less a "condition" and more of specific type of "criterion."

| Component | Criteria | Example | |
|-------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------|--|
| Performance | • Contain an active (action) verb and the object of the verb | Be able to construct confidence intervals | |
| | Describe the behavior of the student, not the instructor | | |
| | • Be observable during the training event | | |
| Conditions | • It must describe the situation under which the performance occurs, limitations imposed on the performer, or non-implicit resources available to the performer | Give a narrative describing a customer history, a product description booklet, a price list, and 15 minutes to prepare | |
| Criteria | • A criterion is defined as a standard on which a judgment or decision can be based | • <u>Given an unprogrammed</u> <u>DVR</u> , the purchaser will be able to program five events <i>without reference to the manual</i> | |
| | | Give a sales talk word for word within 6 minutes | |

 Table 20.1
 Important components of a learning objective.

Note: Bold words are "performance," italicized words are "criteria," and underlined words are "conditions."

Source: Compiled from Shapiro (1995).

TIPS FOR DEVELOPING MEASURES

Consider the following three tips as you developing your training measures:

- The performance stated in the objectives and the performance measured in the training should match.
- The performance conditions stated in the objectives and the performance conditions in the training should match.
- If a measurement can not be developed for an objective, the objective should be reviewed and, if necessary, restated.

Application of these tips helps strengthen the alignment of the training plan objectives with the training outcomes.

MEASUREMENT ISSUES

As with all processes and activities, certain measurement issues exist. With regard to training, two notable issues must be addressed:

• *Validity.* A valid measurement process is one that measures what it is designed to measure. The most commonly used method for training

programs is the content-validity approach. This is a highly subjective approach by which subject matter experts provide opinions regarding whether the process measures what is needed on the job.

• *Reliability*. In the training context, reliability refers to the consistency and stability of the measurement process over time. For example, if an individual's skills have not been enhanced between two designated time periods, a reliable measurement process would yield the same results for the same participant.

Both of these issues, as well as all other basic measurement systems analysis issues, must be kept in mind when evaluating the effectiveness of training.

DATA COLLECTION METHODS

Numerous methods exist for collecting data. Table 20.2 provides a convenient summary of these methods by "level." Technically, "operational data," the last method shown, is not an actual method. Instead, it is a source of data. It was included because level 5 requires hard data for use in calculating the training ROI. There are many sources of training-related hard data. Consider job specifications as a starting point. Though not containing the hard data itself, job descriptions might serve as an appropriate pointer to the root source.

| Data collection method | Level 1 | Level 2 | Level 3 | Level 4 | Level 5 |
|-----------------------------------------------|---------|---------|---------|---------|---------|
| Follow-up surveys | | | ~ | | |
| Follow-up questionnaires | ~ | | ~ | ~ | |
| Observations on the job | | | ~ | | |
| Follow-up interviews | ~ | | ~ | | |
| Follow-up focus groups | ~ | | ~ | | |
| Assignments related to the program | | | ~ | ~ | |
| Action planning/improvement plans | | | ~ | ~ | |
| Testing | | ~ | | | |
| Performance contracting | | | ~ | ~ | |
| Program follow-up session | | | ~ | ~ | |
| Performance monitoring | | | | ~ | |
| Operational data (for example, scrap, rework) | | | | | ~ |

 Table 20.2
 Suggested data collection methods for each of the five levels.

Source: Adapted and compiled from Phillips and Stone (2002).

ISOLATING THE EFFECTS OF TRAINING

Phillips and Stone (2002) enumerate multiple means by which they believe the effects of training can be isolated for use in levels 4 and 5 data collection. These include the use of: control groups, trend analysis, forecasting, participant estimates, supervisory estimates, management estimates, customer input, expert estimates, and subordinate input. Clearly, most of these methods rely on guesswork. The most scientific approach is the use of control groups. This approach is an experimental design that involves the establishment of two groups whose composition is as identical as possible. One group is trained, the other isn't. Performance is observed during the same time period. All the while, both groups are subjected to the same environmental factors and other influences so that the difference in performance between the groups can be attributed to training.

Part V

Mentoring Responsibilities

| Chapter 21 | Mentoring Champions, Change |
|------------|---------------------------------------|
| | Agents, and Executives |
| Chapter 22 | Mentoring Black Belts and Green Belts |
| Chapter 23 | Mentoring Non-belt Employees |
| | |

Chapter 21

Mentoring Champions, Change Agents, and Executives

PROJECT REVIEWS

Collaborate with executives and champions on reviewing projects, including timing, questions to ask, and setting expectations for project timing and completion. (Create)

Body of Knowledge V.A.1

Executive and project champions (or sponsors) should be actively engaged in the review of Lean Six Sigma projects. In Chapter 5 the concept of a set of routine hierarchal reviews was introduced. This set of reviews would include executive management, other senior leaders, middle management, and, of course, interested stakeholders, including champions.

On the other hand, champions must be engaged in a specific set of reviews related to the projects for which they are responsible. (Recall that the role of the champions was addressed in Chapter 12.) This set of reviews is known as *tollgates*. Tollgates first appeared in Figure 4.3, which demonstrated how projects flowed through the DMAIC process and the impact tollgates had on them.

Project Timing and Completion

The project charter was discussed fully in Chapter 5. Part of the discussion included the project plan. The project charter set forth a tentative schedule for the project timing. Recall that a critical estimate was the project completion date, which was set after a thorough discussion and subsequent agreement with the project champion. This information was needed to facilitate the project pipeline creation and portfolio management process. Executives need to be aware that project time and completion dates are not easily manipulated since they may be part of an overall schedule that is being managed closely along with critical resources.

Tollgate Reviews

Tollgate reviews are a necessary part of the DMAIC infrastructure. They exist as a check and balance to ensure that teams are ready to transition from one phase to the next in an orderly manner.

The *tollgate review* is a formal review process conducted by a champion who asks a series of focused questions aimed at ensuring that the team has performed diligently during this phase. The end result of a tollgate is a "go" or "no-go" decision. The "go" decision allows the team to move forward to the next phase. If it is in the last phase, the "go" decision" brings about project closure. If the decision is "no-go," the team must remain in the phase, retreat to an earlier phase (in some instances), or perhaps the project is terminated or suspended. The concept of "go/ no-go" decisions was illustrated in Figure 4.3.

Effective tollgate reviews are usually characterized by an objective evaluation of the work performed by the team during the phase and the willingness to identify and resolve problems during the reviews. Poor preparation, incomplete documentation, champions desiring to change the scope, and champions delegating replacements typically characterize poor tollgate reviews.

Many thoughts exist regarding the purpose of tollgates, who should attend them, what questions should be asked, and what are the exit criteria from the tollgate phase. Table 21.1 provides a summary of one viewpoint. The "questions to address" are minimal in the table, but they are critical. Notice that the first question asked is "Is the project consistent with the goals of the organization?" Project alignment to the strategy and subsequently the goals and objectives of the organization has been a recurring theme throughout this book. This question appears in every phase because of the highly dynamic nature of organizations. When the answer is "no," it is incumbent upon the champion to shut the project down and allow the project resources to be allocated elsewhere. However, notice there is something particularly unique about this question. Unlike the others in the table, this question should be both asked and answered by the champion for it is simply a rhetorical question posed to the team. It is the champion who is in the position to have direct access to the proper resources to be able to answer this question. The champion should come prepared at each and every tollgate to address this critical question.

Table 21.2 provides an additional set of questions the champion might ask during tollgate reviews. Of course, common to each phase are questions regarding budget and schedule. Also included is a question regarding what obstacles the team might have encountered and how the champion may be able to help. Recall that one job of the champion is the removal of organizational barriers the team might face.

Tollgate reviews, when executed well, provide real value to the Lean Six Sigma deployment. They keep projects focused and on track, and executives and champions engaged.

| Table 21.1 A | Table 21.1 A brief summary of the tollgate process. | ite process. | | | |
|--------------------------|--------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------|
| Component | Define | Measure | Analyze | Improve | Control |
| Purpose | Provide a compelling business case appropriately scoped, complete with SMART goals, and linked to a hoshin plan | Collect process performance data for primary and secondary metrics to gain insight and understanding into root causes and to establish performance baselines | Analyze and establish optimal performance settings for each X and verify root causes | Identify and implement process improvement solutions | Establish and deploy a control plan to ensure that gains in performance are maintained. |
| Participants required | Sponsor Process owner | SponsorProcess owner | SponsorProcess owner | Sponsor Process owner | Sponsor Process owner |
| | MBB/BB coach | MBB/BB coach | MBB/BB coach | MBB/BB coach | MBB/BB coach |
| | Deployment champion | Deployment champion | Deployment champion | Deployment champion | Deployment champion |
| | Project team | Project team | Project team | Project team | Project team |
| | Finance partner | | Finance partner | | Finance partner |
| Questions to address | • Is this project consistent with the goals of the organization? | • Is this project still consistent with the goals of the organization? | • Is this project still consistent with the goals of the organization? | • Is this project still consistent with the goals of the organization? | • Is this project still consistent with the goals of the organization? |
| | • Do we have the right level of engagement from stakeholders and business partners? | • Do we have the right level of engagement from stakeholders and business partners? | • Do we have the right level of engagement from stakeholders and business partners? | • Do we have the right level of engagement from stakeholders and business partners? | Have resources been allocated to move into replication? |

Part V.A.1

Continued

| Table 21.1 Continued. | ontinued. | | | | |
|----------------------------------------|------------------------------------------------------------------|------------------------------------------------------------------|------------------------------------------------------------------|------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------|
| Component | Define | Measure | Analyze | Improve | Control |
| Questions to address (continued) | • Have resources been allocated to move to the next phase? | • Have resources been allocated to move to the next phase? | • Have resources been allocated to move to the next phase? | • Have resources been allocated to move to the next phase? | • Is the replication schedule and plan appropriate? |
| | • Do conflicts exist with other projects or activities? | | What are the market and timing dependencies? | | What are the market and timing dependencies? |
| | | | | | Have responsibilities identified in the control plan been transferred to appropriate parties? |
| Exit criteria | Sponsor approval Finance approval | Sponsor approval | Sponsor approval Finance approval | Sponsor approval | Sponsor approval Finance approval |
| | Funding approval | | | | Hand-off to process owner |

Chapter 21: Mentoring Champions, Change Agents, and Executives 297

| Continu | |
|---------|--|
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| le 2 | |
| q | |

| Phase | Questions |
|---------|-------------------------------------------------------------------------------------------|
| Define | • Who are the stakeholders? |
| | • What are the primary and secondary measures? |
| | • Are you facing any barriers or obstacles I can help with? |
| | • Are we on schedule/budget? |
| Measure | • What is the process capability? |
| | • What data still remain to be collected? |
| | • What did the FMEA show? |
| | • What questions still remain to be answered about the process? |
| | • Are you facing any barriers or obstacles I can help with? |
| | • Are we on schedule/budget? |
| Analyze | • What are the critical X's? |
| | • What are the root causes? |
| | • Are you facing any barriers or obstacles I can help with? |
| | • Are we on schedule/budget? |
| Improve | • What is the process capability of the revised process? |
| 1 | • What criteria will be used to select the optimum solution from the set of solutions? |
| | • Will we run a pilot? |
| | • Are you facing any barriers or obstacles I can help with? |
| | • Are we on schedule/budget? |
| Control | • Is the process documented? |
| | • Is the control plan completed? |
| | • Has the process been transitioned back to the process owner? |
| | • What lessons have we learned? |
| | • Do we know where we might be able to replicate our new process across the organization? |
| | • Are you facing any barriers or obstacles I can help with? |
| | • Are we on schedule/budget? |

Table 21.2 Additional questions to be asked by the champion during tollgate reviews.

Source: Adapted and compiled from Phillips and Stone (2002).

PROJECT SIZING

Collaborate with executives and champions on sizing projects and selecting individuals and assignments for various projects. (Evaluate)

Body of Knowledge V.A.2

Lean Six Sigma projects sometimes suffer from a disagreement between the project team members regarding project boundaries. The process of defining scope, of course, can result in problems of extremes:

- Project definitions with scopes that are too broad may lead a team into a morass of connecting issues and associated problems beyond the team's resources. Example: "improve customer satisfaction" with a complex product or service.
- Project boundaries that are set too narrow could restrict teams from finding root causes. Consider, for example, that "Improve customer satisfaction by reducing variation in plating thickness" implies a restriction from looking at machining processes that may be the root cause of customer problems.
- The tendency is to err on the side of making the scope too broad rather than too narrow. Extra attention, effort, and time may be needed to ensure a proper scope. Don't shortcut the process.
- Several tools are available to assist in setting a project scope. These include:
 - *Pareto charts* to help in the prioritizing processes and sometimes in support of project justification
 - *Cause-and-effect diagrams* to broaden the thinking within set categories
 - *Affinity diagrams* to show linkages between the project and other projects or processes
 - Process maps to provide visualization and perhaps illuminate obvious project boundaries and relationships.

Briefly recall Table 4.1, which provides a more extensive list of tools commonly used in the *define* phase. Collectively, these tools help "belts" zero in on the scope and sometimes, even more importantly, what is out of scope.

Generally, there are two methods useful in defining project scopes. These are the:

- Dimensional method
- Cause-and-effect method

Dimensional Method

Though not labeled as such, this method was addressed in Chapter 5 within the discussion of the project charter. This approach identifies an initial eight possible dimensions for characterizing a project scope. Although stated in Chapter 5, they are repeated here for convenience.

- *Process*. Processes receive input from other processes and feed output to other processes. Tightly bound the process or subprocess associated with the project.
- *Demographics*. This would consider factors such as employee categorizations, gender, age, education, job level, and so on.
- *Relationships*. This would include such entities as suppliers, customers, contract personnel, and so on.
- *Organizational.* Which business units, divisions, sites, or departments are included?
- Systems. Which manual or computerized systems are included?
- *Geographical*. Which country or site is included? Note: In this example, a site was used in both the context of an organizational and geographical scope.
- Customer. Which segmentation or category will be considered?
- Combinations. Any combination of the aforementioned.

Note: This list is subject to expansion. Readers are encouraged to submit ideas to authors@asq.org.

The advantage of this method is that it communicates the scope in straightforward and easy-to-understand terms. For example, the project scope will entail process A, employee category B, in site C, who produce products for customer D. Unfortunately, this method does not focus in on process inputs as well as the functional method.

In addition to defining what is in scope, it is often useful to define what is out of scope. Although one would seem to define the other, experience has shown that what is out of scope is frequently overlooked or not understood fully unless explicitly stated. Many sponsors, team members, and other stakeholders often fail to make this important connection.

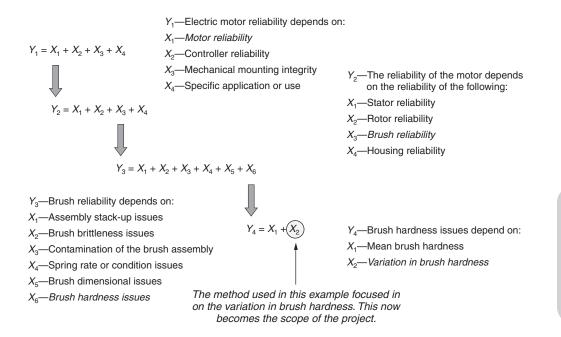
Unwieldy scopes are one of the most frequent reasons cited for the demise of projects. When the scope is too large to be completed within the project plan time frame, or additional resources are not available, there may be a reluctance to go back and re-scope the project. When it is too small, projected savings may be overstated. Remember, the project charter should be considered a living document and adjusted as knowledge occurs and learning takes place.

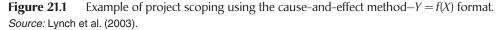
Functional Method

The functional method is rooted more in logic and functional relationships between input and output variables than the dimensional method. The intent is to isolate critical input variables. Other names for this method include: *causeand-effect method*, *process mapping decomposition*, *x-y diagrams*, Y = f(X), and *big Y exercises*. This method is best illustrated by an example provided by Lynch et al. (2003). The example deals with the issue of poor electric motor reliability. Figure 21.1 portrays the functional Y = f(X) breakdown. Each of the X's should be supported by operational data and may be supported by subject matter expert opinions. The breakdown continues, in this example, until the smallest meaningful part with continuous data is identified. Keep in mind that another "Y" from the brush reliability equation could have been identified as well. This would have yielded an additional, but concurrent project.

The disadvantage of using this approach lies with the difficulty of explaining it to senior leaders and other levels of management.

Figure 21.2 is the same as Figure 21.1 but uses a process mapping decomposition breakdown format to isolate critical input variables. This format may be more appealing to senior leaders and management, in general.





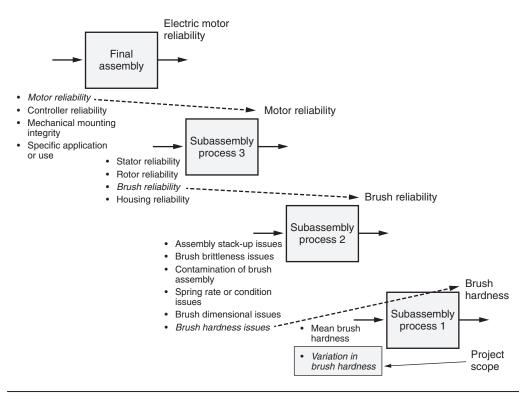


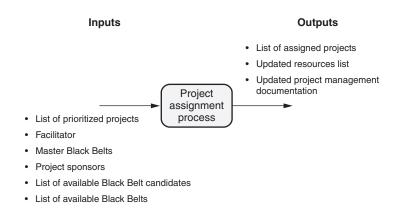
Figure 21.2 Example of project scoping using the cause-and-effect method–process mapping decomposition format.

Matching Projects and Individuals

There is a prevailing thought in some organizations that a Black Belt is a Black Belt, meaning they are essentially interchangeable. Of course, this is absurd and fails to recognize the background and experience of the Black Belt. The matching process should not be as simple as which project is next in line and which Black Belt is available. Such a process is a recipe for disaster. Considerable thought must be given to the project assignment process given in Figure 21.3. This figure takes us into step 5 in Figure 5.1, which dealt with managing the project flow.

Figure 21.3 illustrates all of the inputs and outputs associated with the project assignment process. Notice that both available Black Belts and available Black Belt candidates are inputs to the process. This accommodates the opportunity to assign easier projects to candidates as training projects.

Ramu (2007) suggests a set of weighting criteria that reflects the cycles of learning the Black Belt has completed. These criteria can be used to maximize the Black Belt's learning experience, maximize project success, help with Black Belt retention, and place Black Belts in a favorable position for future growth. Ramu's original criteria included a "strategic objectives link." This was deleted from Figure 21.4 because throughout this book the basic theme has been that all Lean Six Sigma projects will be linked to the organization's strategic plans.





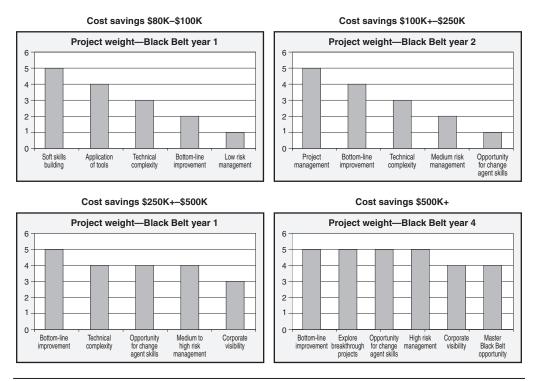


Figure 21.4 Ramu's criteria for assigning Black Belts to projects. *Source:* Adapted from Ramu (2007).

In addition to Ramu's criteria, there are opposing philosophies regarding whether Black Belts should be assigned to processes with which they are familiar. Those favoring familiarity believe the Black Belt will be able to communicate more effectively with the subject matter experts and expedite the completion of the project. Those favoring unfamiliarity with the process claim that greater improvement takes place because the Black Belt has fresh eyes, is not contaminated with organizational biases, and is in an objective position to ask unfavorable questions. Unfortunately, the latter situation occasionally results in the organization or team clamoring that the Black Belt "doesn't understand our processes." Such claims usually come from organizations with cultures more resistant to change and whose employees are entrenched in their existing processes.

COMMUNICATIONS

Coach executives and champions on the need for constancy of purpose and message, and the importance of using clear communication techniques and consistent messages. (Evaluate)

Body of Knowledge V.A.3

The first of W. Edwards Deming's fourteen points begins with "Create constancy of purpose." As a coach to executives and champions, the Master Black Belt must be forceful, yet tactful in delivering this message. Wavering, or otherwise inconsistent messaging, is the death knell for a Lean Six Sigma deployment. One approach to reducing the potential variation around "purpose and messaging" is by using a communications plan. Recall that the communications plan was described in Chapter 7. For convenience it will be repeated here.

Communications Plan

This plan would ensure a two-way flow of communication (that is, from employees to leadership and from leadership to employees). The plan should communicate information relevant to particular categories and groups of employees in a timely manner using multiple media as necessary. Messages should be clear, consistent with, and reinforcing of other messages. Time frames of events should be evident, and who is affected by what event or action and when should be clear. Everyone in a leadership role should be on the same page. Employees should understand why the change is required and how they are personally affected by the change. Tout successes.

An example of the format of a typical communications plan was depicted in Table 14.2.

The most effective way to implement a communications plan is by having it designed and developed jointly by the Lean Six Sigma deployment team and the organization's communications department. Do not expect executives and champions to do it.

Once the plan is developed, executives and champions, as well as other levels of management, should be briefed on the plan and their role in executing it. It should be made clear that the organization will be looking to them for guidance and confirmation regarding the Lean Six Sigma deployment. If they want a successful deployment, they must remain active, engaged, and visible to the organization. Executing the communications plan will help them in this regard.

FEEDBACK

Use constructive techniques to provide feedback to champions and executives. (Evaluate)

Body of Knowledge V.A.4

As might be expected, the occasional rogue executive, champion, or other representative of management appears and strays from the course set by the communications plan or other Lean Six Sigma deployment plans. When this occurs, the task will usually fall to the change agent, or in this case the Master Black Belt, to provide the necessary mentoring, guidance, and feedback to bring the individual back on course with the established set of actions, plans, communications, or even behaviors.

As mentioned above, this must be done forcefully, yet tactfully. To do this effectively, the Master Black Belt must employ the skills outlined in Chapter 11. These include:

- Choosing an effective leadership or interaction style
- Choosing an effective communication style
- Choosing an intervention style
- Dealing effectively with conflict and negotiation
- Influencing without authority

In addition to simply employing these skills when dealing with the rogue individual, the Master Black Belt might need to consider establishing a mentoring relationship with the individual, particularly if feedback alone is not sufficient.

Chapter 22

Mentoring Black Belts and Green Belts

INDIVIDUALS

Develop a career progression ladder for black belts and green belts. Assess their progress and provide constructive feedback to enable them to work effectively on team projects. Use coaching, mentoring, and intervention skills as needed, including canceling or reassigning projects if necessary. (Evaluate)

Body of Knowledge V.B.1

A critical success factor in every Lean Six Sigma deployment are the employees who serve as the Green Belts, Black Belts, or Master Black Belts. They are the individuals who work projects and get results. Therefore, it is important that the appropriate infrastructure be in place to help make them successful. Two aspects of that infrastructure are:

• *Career progression.* Career progression is a piece of infrastructure that is often overlooked or simply dismissed. Consequently, employees may feel that the organization is not rewarding their willingness to take a risk by stepping up and supporting the Lean Six Sigma deployment. In many cases, this is true.

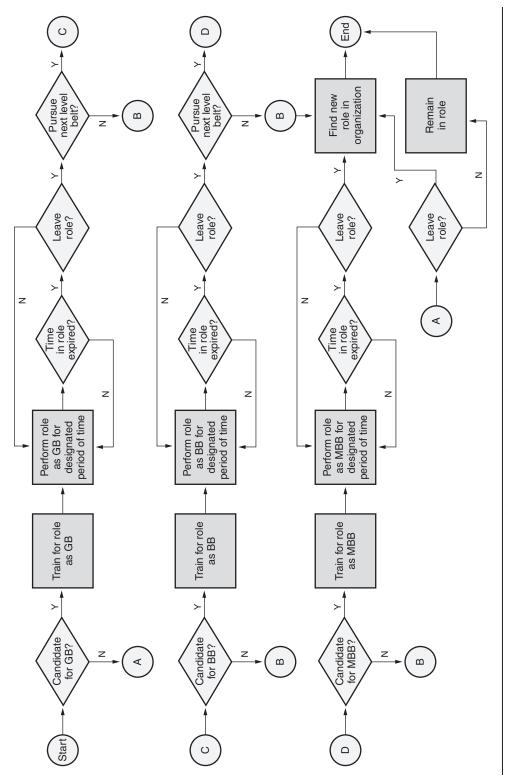
However, for those organizations that have decided to put career paths in place, two primary schools of thought have emerged. The first school of thought is that belts, typically Black Belts and Master Black Belts, will serve for a specified period of time in the Lean Six Sigma deployment department. This time frame commonly ranges from eighteen to twenty-four months. When their time in role has expired, the incumbent must move to another (that is, usually higher) position in the organization, but outside the Lean Six Sigma deployment department. The supporting logic is that by doing so, these individuals will use their Lean Six Sigma skills in their new jobs. Hence, this will help further ingrain Lean Six Sigma into the culture of the organization.

The second school of thought is really a hybrid. It allows individuals to continue to serve as belts in the Lean Six Sigma deployment department or seek employment elsewhere within the larger organization. The advantage of this approach is that it permits the die-hard Lean Six Sigma practitioner to continue to enjoy what he or she is doing. However, it limits opportunities for other individuals to become belts as well as the ability to drive Lean Six Sigma thinking into the DNA of the organization.

Figure 22.1 illustrates a potential career path for all belt levels. Some of the assumptions or ideas on which this model is built include:

- Green Belts stay in the Lean Six Sigma deployment organization working full-time for a specified duration. This is not typical, but was included for completeness purposes. (The entry path from "Start" to point "C" can be deleted if not needed.) Usually, Green Belts will enter the process at point "C."
- The belt has a choice to stay in the job or seek employment elsewhere in the organization.
- Candidates must be qualified before pursuing the next belt level.
 They are not automatically qualified for having served at their current belt level.
- Though not shown explicitly in the model, candidates may enter the career path at the Black Belt level without having served as a Green Belt.
- Individuals who are not qualified to serve as Green Belts may continue in their current jobs or seek employment elsewhere in the organization. This is an important concept and was mentioned in an earlier chapter. Not everyone is qualified to become a Lean Six Sigma practitioner, regardless of the level. Qualification criteria must be established at each belt level.
- This model does not address whether individuals joining the Lean Six Sigma department from outside the total organization (that is, new hires) who may be belt-level trained or certified, must go through the Lean Six Sigma training provided by the Lean Six Sigma deployment department. This issue is something all organizations must face regarding the training of new hires.

• *Coaching. Coaching* is a process by which a more experienced individual helps enhance the existing skills and capabilities that reside in a less experienced individual. Coaching is about listening, observing, and providing constructive, practical, and meaningful feedback. During training, coaching helps the trainee translate the theoretical learning into applied learning while also helping the trainee develop confidence in their newly developing knowledge and skills. Post training, coaches help projects stay on track and advance toward completion in a timely manner. Coaches provide guidance and direction on how to navigate organizational barriers, select and use the proper tools and techniques, prepare for tollgate reviews, discover solutions on their own, provide intervention where needed, and generally serve as a sounding board for project-related issues.





In some cases, the coach may find the individual is unsuited to work a specific project and consequently will have the project assigned to another belt or request the project champion have the project cancelled if it no longer appears viable.

Mentoring

While coaching focuses on the individual as it relates to Lean Six Sigma, *mentoring* focuses on the individual from the career perspective. Mentors are usually experienced individuals (not necessarily in Lean Six Sigma) who have in-depth knowledge about the organization as well as the individual (that is, mentee). Usually, they come from within the organization, though not necessarily the same department as their mentee. Their role is to help provide guidance, wisdom, and a possible road map to career advancement. Like the coach, the mentor also helps the individual navigate the organization. Recall Figure 22.1. It would be conceivable that both a coach and a mentor would support an individual belt concurrently. The coach would focus on the individual's role as a belt and the mentor would focus on the individual's overall career. Remember, the career progression ladder provided the opportunity for the belt to find a new role within the organization. The mentor would be of great help to the individual in finding a suitable role elsewhere.

TECHNICAL REVIEWS

Create guidelines and expectations for project reviews, and perform them in a timely manner. Assist project leaders in selecting appropriate content for presentation to management. (Create)

Body of Knowledge V.B.2

Coaches usually serve as the "go-to" person for helping belts prepare for project reviews, tollgate reviews, or other presentations. They can help the belt select the appropriate content and structure the presentation accordingly. One format that works well for tollgates and that can be adapted easily for other presentation needs follows:

- Executive summary
 - Project title
 - Project phase
 - Estimated dollar target
 - Estimated completion date
 - Stoplight status (that is, red, yellow, green)

- Main body
 - Project charter
 - Project team (note changes)
 - Identify tools used
 - Identify data collected
 - Identify results obtained
 - Identify conclusion made
 - Identify hard dollars and soft dollars by category
 - Identify potential problems, issues, or concerns
 - Updated project plan
 - Updated budget
- Open discussion
- Tollgate decision
- Previous tollgate presentation material (use if needed)

Inevitably, the team leader or presenter will be faced with questions from the previous phase, if this tollgate is not for the *define* phase. As such, it is good practice to have previously presented tollgate presentation material available for each tollgate so that questions can be answered immediately, without the need to "get back to" the individual.

Use of the above format or a similar one will allow team leaders to develop presentations in a timely manner and deliver them to senior leaders and other management individuals on short notice.

TEAM FACILITATION AND MEETING MANAGEMENT

Practice and teach meeting control, analyze team performance at various stages of team development, and support appropriate interventions for overcoming team challenges, including floundering, reviewing and diagnosing failing projects, etc. (Create)

Body of Knowledge V.B.3

Team Stages

Teams are said to go through several growth stages:

- *Forming.* In which members are struggling to understand the goal and its meaning for them individually
- *Storming.* In which members express their own opinions and ideas, often in disagreement with others
- *Norming.* In which members begin to understand the need to operate like a team rather than as a group of individuals
- *Performing.* In which the team members work together to reach their common goal

The above four stages have been considered traditional. However, in recent years two additional stages have been proposed. These are:

- *Adjourning*. In which a final team meeting is held, during which management decisions regarding the project are discussed and other loose ends are dealt with
- *Recognition*. In which acknowledgment of the team's contribution is made

In Figure 22.2, the solid horizontal arrows indicate the standard stages of team development. The dashed arrows show alternate paths taken by teams in some circumstances. Without good leadership, a team can backslide from norming or performing into any previous stage. If a team leader or facilitator observes signs that backsliding is occurring, the team leader should remind the group of the goals and agenda and the need to press forward. Team members who are accustomed to working with each other on similar projects may be able to skip the storming stage and possibly the norming stage. There have also been examples of teams that perpetually cycle through the forming, storming, and norming stages, never to reach the performing stage. Such teams tend to suffer a slow death and may never be officially adjourned.

Team Communication

Lack of adequate communication is one of the most frequently noted causes of team failure. Serious effort toward communication improvement should be made at each stage of team development. In some situations, such as large projects or those with wide geographical barriers, the development of a formal communications plan is necessary. This is particularly true in the case of virtual teams. With the

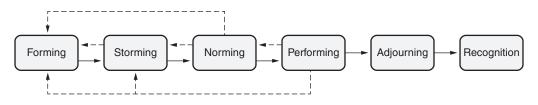


Figure 22.2 Team stages.

advent of technology that permits round-the-clock, worldwide contact, the use of virtual teams has become a reality. With virtual teams, team members may never see one another. This makes the communication process that much more difficult. If the communication process is conducted via audio only, team members have no ability to read body language. They are limited to verbal cues and voice intonations. Consequently, some team members may find it difficult to function as a virtual team member. Therefore, team communication must be carefully planned, monitored, and adjusted as necessary.

Initiating the Team

A clear statement of goals and objectives is critical. If team members have been selected to represent groups or supply technical knowledge, these roles should be announced at the initial or kickoff meeting. The initiation of the team should be in the form of a written document from an individual at the executive level of the organization. A separate document from each member's immediate supervisor is also very helpful in providing up-front recognition that the team member's participation is important for the success of the team.

The establishment of team norms is a helpful technique that is often initiated at the first team meeting. Team norms provide clear guidelines regarding what the team will and will not tolerate, and often define the consequences of violating the norms. Examples of team norms include:

- Being on time. A consequence of being late might be to put a dollar into a kitty that will be used at the team's discretion later.
- Holding one conversation.
- Demonstrating civility and courtesy to all members.
- Accomplishing assigned tasks on time.
- Providing advance notice of not being able to attend a meeting.
- Participating in each meeting.
- Following a prepared agenda.
- Pulling the team back on track when it strays from the agenda topic.

Norms are critical to helping a team function efficiently and effectively. In effect, norms help form the basis of the team culture.

During the Life of the Team

Announcements of team meetings, time, location, agenda, and such must be made as early as possible. Minutes or a summary of a meeting should be provided immediately after the session.

Following Project Completion

Final reports and follow-up memos regarding disposition of team proposals and such should be provided to each team member.

Team Performance Evaluation

At specified points in the life of a team, its progress should be evaluated. The measurement system and criteria should be agreed on in advance. The criteria against which the team is evaluated must relate to progress toward goals and objectives. It is common practice to compare team progress against the timeline set forth in the project schedule. Typical objective-oriented criteria include measurement against:

- Goals and objectives (to determine if the project is succeeding or failing)
- Schedule
- Budget

In addition to the above, it may be possible for the team coach to evaluate the team on other criteria, some of which are less objective:

- Attitudes
- Teamwork
- Attendance
- Following team norms
- Length in team stages

The coach will want to attend team meetings from time to time to make evaluations and observations. Above all, the coach will want to determine whether the team is making progress or is heading for a state of imminent failure. If failure is close or imminent, the coach should take immediate action to prevent it. This form of action is called *intervention* and was addressed in Chapter 11. An intervention might take the form of replacing the team leader, replacing team members, adding a professional facilitator to the team, providing additional training to specific members, or whatever the coach decides is appropriate. Regardless, the coach should discuss his or her evaluations and observations with the team frequently as well as follow up with additional evaluations and observations at later meetings to determine whether behavior changes have occurred.

Time Management

Time is perhaps the most critical resource for any team. Team meeting time must be treated as the rare and valuable commodity that it is. It is up to the team leader and the facilitator to make every minute count, although every team member has this responsibility as well. Some practices that have proved useful follow:

- Form an agenda committee that will generate the meeting agenda well in advance of the scheduled meeting. This group can be responsible for getting the resources called for by each agenda item. For smaller teams, the team leader often prepares the agenda.
- Use a Gantt chart timeline displaying milestones and dates, updating as needed.

- Publish meeting agendas with time limits on each item. Assign a timekeeper and stick to the schedule as closely as possible. Leave five to ten minutes as a buffer at the end of the session.
- Publish reminders of members' action assignments. In some cases, the team leader will want to review progress on these assignments between meetings.

Effective time and meeting management is something that requires training and practice to master. The team coach might want to consider having the whole team trained in these topics as part of an intervention.

Chapter 23

Mentoring Non-belt Employees

Develop information that will help nonbelt project participants to advance their understanding of Six Sigma and develop the necessary skills and knowledge to become green belts or black belts. (Create)

Body of Knowledge V.C

At the outset of most Lean Six Sigma deployments, organizations offer general awareness courses to their employees to explain "What this Lean Six Sigma stuff is all about." This training usually satisfies the needs of most employees, who never see themselves as becoming Lean Six Sigma belts.

Over and above the general awareness training, most, if not all, Lean Six Sigma deployment departments develop their own intranet site. These sites provide additional information on courses, criteria for becoming belts, expectations of belts, status of in-process projects, completed projects, lists of sponsors, and so on. On some sites, the actual training material is visible to all employees, while many organizations restrict its viewing to those employees who are belt candidates or who have completed training.

Chapter 22 discussed a career progression for belt-level employees. If the population of belt-level employees is not replenished as they move up or out into the organization, the Lean Six Sigma initiative will eventually grind to a halt. Therefore, it is imperative that the organization maintains an ongoing recruitment program for future belts.

Several ways were mentioned above. However, they were relatively passive. Other, more active ways include: holding brown-bag lunches, road shows (that is, presentations at department staff meetings), and targeted one-on-one meetings. These methods work well for the general population of employees.

In organizations where leaders are exceptionally committed and high profile, they may actively seek a coach/mentor relationship with a Black Belt or Master Black Belt to work their Green Belt or Black Belt training into their busy schedules. Other organizations have proactively established executive training programs to accomplish the same thing. Executives are paired with Black Belts and Master Black Belts (usually Master Black Belts). Training is conducted one-on-one and projects are still expected. In some instances, an executive may even be paired with a Green Belt partner.

In some cases, immature organizations have watered down the training or eliminated the project requirements so that executives can obtain their belts with less effort. This is unfortunate because it sends a message to the rest of the organization that perhaps leadership is not that serious about Lean Six Sigma. It comes across as, "Do as I say, not as I do!" This can result in low employee morale, frustration, and the eventual demise of the Lean Six Sigma initiative.

Part VI

Advanced Measurement Methods and Tools

| Measurement Systems Analysis (MSA) |
|---------------------------------------|
| Measuring and Modeling |
| Relationships Between Variables |
| Design of Experiments (DOE) |
| Automated Process Control (APC) |
| and Statistical Process Control (SPC) |
| |

Chapter 24

Measurement Systems Analysis (MSA)

PROPAGATION OF ERRORS

Use this technique to evaluate measurement systems and calculated values. (Evaluate)

Body of Knowledge VI.A.1

Propagation of errors, also known as *transmission of error,* has application in two major areas of Lean Six Sigma:

- *Response model analysis.* This type of analysis addresses how random noise in the *x_i* values affects the predicted value of *y*.
- *Tolerance stack-up analysis.* This type of analysis will address tolerance stack-up problems.

Response Model Analysis

In order to address the effects of random noise in the x_i values on the predicted values of y, we need a model that includes controllable and noise variables, and their interactions. The first-order model would be:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \gamma_1 z_1 + \delta_{11} x_1 z_1 + \delta_{21} x_2 z_1 + \varepsilon$$

where:

 x_1, x_2 = controllable factors (expressed in coded units)

 z_1 = noise factor (expressed in coded units)

 δ_{11} , δ_{21} = coefficients of the noise variables (at least one must be nonzero)

Further, noise variables have an expected value of zero and a variance, σ_z^2 . If more than one noise variable exists, zero covariances are assumed.

Given this information, we can now determine the following:

$$E_{z}(y) = \beta_{0} + \beta_{1}x_{1} + \beta_{2}x_{2} + \beta_{12}x_{1}x_{2}$$
$$V_{z}(y) = \sigma_{z}^{2} + (\gamma_{1} + \delta_{11}x_{1} + \delta_{21}x_{2})^{2} + \sigma^{2}$$

EXAMPLE 24.1

Mathews (2005) provides this example. A manufacturer of valves used in marine hydraulic steering controls built and analyzed an experiment to study hydraulic pressure leakback rate as a function of critical valve characteristics. The model that they obtained, after appropriate refinements, was:

$$y = 13 - 0.8x_1 + 2.0x_2 + 1.2x_1x_2$$

The standard error is $s_e = 0.4$. Control charts of x_1 and x_2 indicated that the normal manufacturing variation in those variables was $\hat{\sigma}_{s_1} = 0.2$ and $\hat{\sigma}_{s_2} = 0.3$. All quantities are in standardized (that is, coded) units. Determine the predicted response and variation induced in *y* by manufacturing variation in x_1 and x_2 when the nominal values for x_1 and x_2 are 1, -0.5, respectively.

The predicted value of *y* of the leak-back rate evaluated at the nominal values is given by:

$$y = 13 - (0.8)(1) + (2.0)(-0.5) + (1.2)(1)(-0.5) = 10.6$$

The values of the partial derivatives evaluated at the nominal points are:

$$\frac{\delta y}{\delta x_1}\Big|_{(1,-0.5)} = -0.8 + 1.2x_2\Big|_{(1,-0.5)} = -1.4$$
$$\frac{\delta y}{\delta x_2}\Big|_{(1,-0.5)} = 2.0 + 1.2x_1\Big|_{(1,-0.5)} = 3.2$$

The expected standard deviation of the leak-back rate due to the propagation of errors in x_1 and x_2 is:

$$\hat{\sigma}_{y}\Big|_{(1,-0.5)} = \sqrt{\hat{\sigma}_{\varepsilon}^{2} + \left(\left(\frac{\delta y}{\delta x_{1}}\Big|_{(1,-0.5)}\right)(\hat{\sigma}_{s_{1}})\right)^{2} + \left(\left(\frac{\delta y}{\delta x_{2}}\Big|_{(1,-0.5)}\right)(\hat{\sigma}_{s_{2}})\right)^{2}}$$
$$= \sqrt{(0.4)^{2} + \left((-1.4)(0.2)\right)^{2} + \left((3.2)(0.3)\right)^{2}}$$
$$= \sqrt{0.16 + 0.0784 + 0.9216}$$
$$= \sqrt{1.16}$$
$$= 1.08$$

Therefore, the predicted value of the leak-back rate at the nominal values is 10.6, and the manufacturing variation is expected to be $\hat{\sigma}_v = 1.08$.

Montgomery (2005b) provides a general form of the variance:

$$V_{z}[y(x,z)] = \sigma_{z}^{2} \sum_{i=1}^{r} \left(\frac{\delta h(x,z)}{\delta z_{i}}\right)^{2} + \sigma^{2}$$

where:

$$h(x,z) = \sum_{i=1}^{r} \gamma_{i} z_{i} + \sum_{i=1}^{\kappa} \sum_{j=1}^{r} \delta_{ij} x_{i} z_{j}$$

k = number of controllable variables

r = number of noise variables

Tolerance Stack-Up Analysis

With this type of analysis, we are interested in the variation in specifications of components and their cumulative effect. This is best illustrated in Example 24.2.

ATTRIBUTE (DISCRETE) MEASUREMENT SYSTEMS

Use various tools and methods (e.g., percent agreement, Kappa, Kendall, intra-class correlation coefficient (ICC) to analyze and interpret discrete measurement systems data. (Evaluate)

Body of Knowledge VI.A.2

Kappa Statistics and the Intra-class Correlation Coefficient

Two interesting statistical techniques can be used to help us gauge the effectiveness of an attribute (discrete) measurement system. These are the:

- Kappa statistics
- Intra-class correlation coefficient

Kappa Statistics. The kappa techniques are used under the following conditions or assumptions:

- There is the need to classify things in a nominal manner
- Some categories can be used more frequently than others
- Units must be independent
- Raters make their classifications independently of other raters
- The categories are mutually exclusive and collectively exhaustive

EXAMPLE 24.2

Montgomery (2005b) provides the basis for this example. There is a voltage drop across a resistor in a circuit. Let us assume that I (amperes) and R (resistance) are independent and normally distributed random variables with means equal to the nominal values of their specifications. Furthermore, the specifications are assumed equal to the natural tolerance limits. Finally, we will assume that V (voltage) is approximately normally distributed. The specification data are:

 $V = 100 \pm 2.0$ volts

 $I = 25 \pm 1.0$ amperes

 $R = 4 \pm 0.6$ ohms

After doing the appropriate mathematical manipulations, it can be determined that:

$$\sigma_V^2 \simeq \mu_R^2 \sigma_I^2 + \mu_I^2 \sigma_R^2$$

From above, we know that $\mu_R = 4$ and $\mu_I = 25$. Therefore, we must determine the values of σ_I and σ_R . Based on the assumptions that $I \sim N(25, \sigma_I^2)$, $R \sim N(4, \sigma_R^2)$, and that the specification limits are the natural tolerance limits, and knowing that Z = 3.00, we can write the following equations:

$$\frac{26-25}{\sigma_{l}} = 3.00$$
$$\frac{4.06-4}{\sigma_{R}} = 3.00$$

Solving each equation, we can determine that:

$$\sigma_{l} = 0.33$$
$$\sigma_{R} = 0.02$$

Note: Alternately, we know the C_p for both *I* and *R* is 1.0 since the assumption is that the specification limits are equal to the natural tolerance limits. We could have computed σ_I and σ_R from the process capability formula directly.

Therefore,

$$\sigma_V^2 \simeq \mu_R^2 \sigma_I^2 + \mu_I^2 \sigma_R^2$$

$$\simeq (4)^2 (0.33)^2 + (25)^2 (0.02)^2$$

$$\simeq (16)(0.1089) + (625)(0.0004)$$

$$\simeq 1.7424 + 0.25$$

$$\simeq 1.9924$$

Since $\sigma_v^2 = 1.9924$, then $\sigma_v = 1.41$.

Now we can determine the probability that the voltage is within specification. This is written as:

Continued

Continued

$$P(98 \le V \le 102) = P(V \le 102) - P(V \le 98)$$

= $\Phi\left(\frac{102 - 100}{1.41}\right) - \Phi\left(\frac{98 - 100}{1.41}\right)$
= $\Phi(1.42) - \Phi(-1.42)$
= $0.92219 - 0.07780$
= 0.84439

The C_p computes to:

$$C_p = \frac{102 - 98}{(6)(1.41)} = 0.47$$

From this example, we can see that the individual specifications are acceptable, but together they produce a component that will be in specification approximately 84% of the time. This issue results from a stack-up problem of the tolerances.

Futrell (1995) defines *kappa* (*K*) as "the proportion of agreement between raters after agreement by chance has been removed." The formula for kappa is:

$$K = \frac{P_{observed} - P_{chance}}{1 - P_{chance}}$$

When multiple raters are involved, we must be concerned with two types of kappa statistics: $K_{overall}$ and $K_{category}$. The overall kappa assesses the rater agreement across all of the categories, while the category kappa is used to compute individual kappa values for each category. This provides an indication regarding where each rater has trouble. These formulas are:

$$K_{overall} = 1 - \frac{nm^2 - \sum_{i=1}^n \sum_{j=1}^k x_{ij}^2}{nm(m-1)\sum_{j=1}^k \overline{p}_j \left(1 - \overline{p}_j\right)}$$
$$K_{category} = 1 - \frac{\sum_{i=1}^n x_{ij} \left(m - x_{ij}\right)}{nm(m-1)\overline{p}_j \left(1 - \overline{p}_j\right)}$$

where:

m = Number of raters n = Number of units k = Number of categories $\overline{p} = \frac{\text{Ratings within category}}{nm}$

Part VI.A.2

EXAMPLE 24.3

Consider the data shown in Table 24.1. We have five raters evaluate ten samples of carpet and classify them into five categories. Determine the kappa values. Applying the numbers to the formulas above, we obtain:

 $K_{overall} = 0.50$ $K_{gap \ too \ large} = 0.40$ $K_{gap \ too \ small} = 0.40$ $K_{seam \ frayed} = 0.15$ $K_{seam \ uneven} = 0.75$ $K_{seam \ perfect} = 0.69$

Although the kappa for "seam uneven" is above 0.70, overall the system requires improvement. This will likely be accomplished through additional training or clarification of operational definitions.

| Table 24.1 Data for for Example 24.3. | | | | | | | | |
|---------------------------------------|------------------|------------------|----------------|----------------|-----------------|-----------------------|--|--|
| Sample | Gap too large | Gap too small | Seam frayed | Seam uneven | Seam perfect | $\sum_{j=1}^5 x_{ij}$ | | |
| 1 | 0 | 0 | 1 | 0 | 4 | 17 | | |
| 2 | 2 | 0 | 1 | 0 | 2 | 9 | | |
| 3 | 3 | 0 | 0 | 2 | 0 | 13 | | |
| 4 | 0 | 0 | 0 | 0 | 5 | 25 | | |
| 5 | 0 | 2 | 3 | 0 | 0 | 13 | | |
| 6 | 4 | 0 | 0 | 0 | 1 | 17 | | |
| 7 | 0 | 4 | 1 | 0 | 0 | 17 | | |
| 8 | 0 | 0 | 0 | 5 | 0 | 25 | | |
| 9 | 0 | 0 | 0 | 0 | 5 | 25 | | |
| 10 | 3 | 2 | 0 | 0 | 0 | 13 | | |
| Totals | 12 | 8 | 6 | 7 | 17 | 174 | | |
| \overline{p} | 0.24 | 0.16 | 0.12 | 0.14 | 0.34 | \overline{p} | | |
| $1-\overline{p}$ | 0.76 | 0.84 | 0.88 | 0.86 | 0.66 | $1-\overline{p}$ | | |

| Table 24.1 | Data for | for | Example | 24.3. |
|------------|----------|-----|---------|-------|
|------------|----------|-----|---------|-------|

m = 5; n = 10, k = 5

The maximum value for kappa is 1.0 and the minimum is -1.0. When the kappa value is:

- 0.9 or above, the measurement system is considered excellent
- 0.7 or below, the measurement system requires attention
- 0.0, this indicates the agreement is the same as would be expected by chance

Intra-class Correlation Coefficient. The *intra-class correlation coefficient* (ICC) is used under the following conditions or assumptions:

- There is the need to classify things in order, rank, or scale
- Ranges are equally distributed, such as: -2, -1, 0, 1, 2
- There are consequences for misclassifying
- Units must be independent
- Raters make their classifications independently of other raters
- The categories are mutually exclusive and collectively exhaustive

Special note: Do not confuse the intra-class correlation coefficient with the interclass correlation coefficient, which measures the bivariate relationship between variables.

Actually, there are six forms of the intra-class correlation coefficient. The following definitions are important to understanding the different forms:

BMS = Between mean square

DF = Degrees of freedom

EMS = Error mean square

ICC = Intra-class correlation coefficient

JMS = Judges' mean square

WMS = Within mean square

k = number of judges

n = number of units

They are defined as follows:

1. Each unit is rated by a different set of judges who are randomly selected from a larger population of judges. Use the following to estimate the reliability of *each judge's ratings*:

 $ICC = \frac{BMS - WMS}{BMS + (k-1)WMS}$

2. Each unit is rated by a different set of judges who are randomly selected from a larger population of judges. Use the following to estimate the reliability of the *judges' averaged ratings*:

$$ICC = \frac{BMS - WMS}{BMS}$$

3. A random sample of judges is selected from a larger population of judges, and this set of judges rates all units. Use the following to estimate the reliability of *each judge's ratings*:

$$ICC = \frac{BMS - EMS}{BMS + (k-1)EMS + \frac{k(JMS - EMS)}{n}}$$

4. A random sample of judges is selected from a larger population of judges, and this set of judges rates all units. Use the following to estimate the reliability of the *judges' averaged ratings*:

$$ICC = \frac{BMS - EMS}{BMS + \frac{(JMS - EMS)}{n}}$$

5. All judges, who are the only judges of interest, rate all units. Use the following to estimate the reliability of *each judge's ratings*:

$$ICC = \frac{BMS - EMS}{BMS + (k-1)EMS}$$

6. All judges, who are the only judges of interest, rate all units. Use the following to estimate the reliability of the *judges' averaged ratings*:

$$ICC = \frac{BMS - EMS}{BMS}$$

Kendall's Coefficients

When dealing with *Kendall's coefficients,* two types are appropriate for Lean Six Sigma professionals to be familiar with. These are:

- *Kendall's coefficient of concordance.* This coefficient measures the association between appraisers and the association within appraisers as follows:
 - *Between appraisers.* The consistency of each appraiser's rating across trials. This coefficient expresses the degree of association among the ratings between appraisers. The hypotheses tested are:

 H_0 = There is no association between the appraisers' ratings

 H_1 = Ratings between appraisers are associated

EXAMPLE 24.4

Consider the data shown in Table 24.2. Three judges or raters will evaluate five samples of fabric and classify them on a scale from 1 to 9, with 9 being excellent. Determine the six ICC values.

Probably the quickest way to obtain the needed values is to conduct a quick twoway ANOVA in Minitab. This will allow us to quickly populate Table 24.3. Although the source table provided by Minitab did not provide all the information for Table 24.3, we were able to compute the additional information required from the source table Minitab provided. If we fill in the number using the above formulas, we obtain the following ICC values:

- 1. ICC = 0.78
- 2. ICC = 0.92
- 3. ICC = 0.78
- 4. ICC = 0.91
- 5. ICC = 0.74
- 6. ICC = 0.90

Futrell (1995) recommends using the same values for assessing the effectiveness of the measurement system as used with kappa. Notice that the ratings in cases 1, 3, and 5 are lower. This is the situation where we looked at the reliability of each judge's ratings. In all cases where it was averaged, the system performed substantially better.

| Sample | Judge 1 | Judge 2 | Judge 3 |
|--------|---------|---------|---------|
| 1 | 5 | 7 | 7 |
| 2 | 4 | 3 | 2 |
| 3 | 4 | 2 | 3 |
| 4 | 6 | 7 | 8 |
| 5 | 5 | 5 | 5 |

| Table 24.2 | Data for | Examp | e 24.4. |
|-------------------|----------|-------|---------|
|-------------------|----------|-------|---------|

| Table 24.3 | ANOVA table for Example 24.4. |
|-------------------|-------------------------------|
|-------------------|-------------------------------|

| Source | DF | Sum of squares | Mean squares | Component |
|-----------------|----|----------------|--------------|-----------|
| Judges | 2 | 0.13 | 0.07 | JMS |
| Between fabrics | 4 | 41.07 | 10.27 | BMS |
| Total | 14 | 49.73 | 3.55 | |
| Within fabrics | 10 | 8.67 | 0.87 | WMS |
| Error | 8 | 8.53 | 1.07 | EMS |

- Within appraisers. The consistency of each appraiser's rating across trials when each appraiser provides two or more ratings for the same unit. This coefficient expresses the degree of association among multiple ratings by an appraiser. The hypotheses tested are:
 - H_0 = There is no association among multiple ratings made by an appraiser
 - H_1 = The ratings are associated with one another
- *Kendall's correlation coefficient*. Kendall's correlation coefficient is also known as *Kendall's rank-order correlation coefficient* or *Kendall's tau correlation coefficient*. (Note: Kendall has three tau correlation coefficients denoted as a, b, and c. Tau-a is used with cross tabulations; tau-b is used with square tables and is discussed below; and tau-c is more suitable for rectangular tables.) This coefficient measures the association between all appraisers and the standard and each appraiser and the standard as follows:
 - All appraisers versus known standard. The correctness of all appraisers' ratings to a known standard. Kendall's rank-order correlation coefficient for each trial measures the degree of association between ratings from the trial and the standard. The hypotheses tested are:
 - H_0 = There is no association between ratings of all appraisers and the known standard
 - H_1 = Ratings by all appraisers are associated with the known standard
 - Each appraiser versus known standard. The correctness of each appraiser's ratings to a known standard across trials. Kendall's rank-order correlation coefficient for each trial measures the degree of association between appraisers' ratings and the standard. The hypotheses tested are:
 - H_0 = There is no association between ratings of each appraiser and the standard
 - H_1 = Ratings by each appraiser are associated with the standard

Kendall's coefficients range between –1 and 1. A positive value indicates positive association. Similarly, a negative value indicates negative association. A value of zero indicates no agreement or association.

There is often some confusion regarding when to use kappa statistics versus Kendall's coefficients. Consider the following:

- Classification: nominal—use kappa statistics
- Classification: ordinal; known standard: none—use Kendall's coefficient of concordance
- Classification: ordinal; known standard: yes—use Kendall's correlation coefficient

Furthermore, kappa treats all misclassification equally, while Kendall's correlation coefficients treat a further departure from standard more seriously.

The formulas for computing Kendall's coefficients are given by:

Coefficient of concordance:
$$W = \frac{12\sum_{i=1}^{N} R_i^2 - 3k^2 N(N+1)^2}{k^2 N(N+1) - k \sum_{j=1}^{k} T_j}$$

where:

N = The number of units

 $\sum_{i=1}^{N} R_i = \text{The sum of the squared sums of ranks for each of the ranked} N \text{ units}$

k = The number of appraisers

 T_j = Assigns the average of ratings to tied observation = $T_j = \sum_{i=1}^{s_j} (t_i^3 - t_i)$

 t_i = The number of tied ranks in the *i*th grouping of ties

 g_i = The number of groups of ties in the *j*th set of ranks

Coefficient of correlation:

$$\tau = \frac{C - D}{\sqrt{[N_{Total}(N_{Total} - 1)(0.5) - T_X]}} \sqrt{[N_{Total}(N_{Total} - 1)(0.5) - T_Y]}$$

where:

$$C =$$
 Number of concordant pairs $= \sum_{i < k} \sum_{j < i} n_{ij} n_{ki}$

D = Number of discordant pairs = $\sum_{i < k} \sum_{j > i} n_{ij} n_{ki}$

 n_{i+} = Number of observations in the *i*th row

 n_{+j} = Number of observations in the *j*th column

 n_{ij} = Number of observations in the cell in the *i*th row and *j*th column

 n_{ki} = Number of observations in the cell in the *k*th row and *i*th column

N = Total number of observations

 T_X = Number of pairs tied on $X = 0.5 \sum_i n_{i+} (n_{i+} - 1)$

 T_Y = Number of pairs tied on $Y = 0.5 \sum_j n_{+j} (n_{+j} - 1)$

k = Number of appraisers

It should be clear that computation of Kendall's coefficients can be tedious and subject to error if done manually. Consequently, an example will not be provided. However, such calculations can be accomplished using most statistical software. The above formulas are those used by Minitab.

Attribute Measurement Systems Tools and Methods

This section discusses the following two attribute measurement systems analysis methods:

- Attribute agreement analysis (that is, percent agreement)
- Attribute gage study—analytic method

Both methods will use examples courtesy of Minitab.

Attribute Agreement Analysis. Percent agreement refers to the percent of time in an attribute gage repeatability and reproducibility (GR&R) study that appraisers agree with (1) themselves (that is, repeatability), (2) other appraisers (that is, reproducibility), or (3) a known standard (that is, bias) when classifying or rating items using nominal or ordinal scales, respectively.

This concept is best illustrated by Example 24.5 using Minitab.

Attribute Gage Study—Analytic Method. An attribute gage study is used to determine the amount of bias and repeatability of a measurement system when the response is an attribute variable (that is, accept or reject). In such studies, reference values are known for each part and are selected at equal intervals across the entire range of the gage. Depending on the particular attribute gage study method chosen (for example, regression method or AIAG method), additional requirements may be placed on the characteristics of the parts, the number of parts per trial, and the number of parts used in the study. Therefore, for additional information when constructing an attribute gage study, the reader is referred to the tutorial or help files of the specific software used. Also, the reader will find the second edition of the AIAG (1995) *Measurement Systems Analysis Reference Manual* useful in understanding the underlying theory.

EXAMPLE 24.5

The data in Table 24.4 show the results of five new appraisers grading the written portion of a twelfth-grade essay test. The appraisers graded on the five-point scale: -2, -1, 0, 1, 2. In addition, each appraiser grade for a given test will be compared with a known standard grade for that test. Each appraiser graded each of the 15 tests. $\alpha = 0.05$ will be used.

Figure 24.1 illustrates the Minitab session window for this example. The output is typically divided into four parts:

• *Each appraiser versus standard*. The percent agreement table shows that both Holmes and Montgomery agreed with the standard on 15 out of 15

Continued

assessments, followed by Simpson with 14 out of 15, then Hayes with 13 out of 15, and Duncan trailing with 8 out of 15. With the exception of Duncan, all appraisers did well against the known standard. These results are confirmed with Fleiss's kappa statistics and Kendall's correlation coefficients. Notice that Duncan's overall kappa was only 0.41176 while his correlation coefficient was the lowest at 0.87506. Figure 24.2 depicts in graphical form these same results using confidence intervals. It is obvious that Duncan requires additional training on how to grade essay tests, and both Hayes and Simpson could stand a little brushing up as well.

- Between appraisers. Fleiss's kappa statistic assumes the level of disagreement among scores is the same (that is, the difference between a score of -2 and -1 is the same as the difference between a score of -2 and 2). Therefore, Fleiss's overall kappa statistic reflects a value that indicates an unacceptable measurement system (that is, 0.672965). However, since Kendall's coefficient of concordance considers the difference between scores when the order of the scale is maintained, a high value is obtained (that is, 0.966317). This value is driven primarily by the fact that four of the five appraisers agreed on a minimum of 13 out of 15 trials.
- *All appraisers versus standard*. For reasons similar to "between appraisers," Fleiss's overall kappa statistic is given as 0.831455, while Kendall's coefficient of concordance is 0.958012.
- *Within appraiser.* This table was not generated because each appraiser assessed each test only once.

EXAMPLE 24.6

Table 24.5 shows the summarized data of 20 trials for each of ten parts. For each part, a reference value has been provided and the number of acceptances has been captured. The AIAG method will be used.

Figure 24.3 depicts the results of the attribute gage study analysis. The (adjusted) repeatability was determined to be 0.0458060, while the bias in the measurement system was 0.0097955. The *t* test indicates that the bias is significantly different from zero.

It should now be clear that whenever a process improvement project is undertaken, the measurement system should be one of the first things analyzed, for the following reasons:

- All data from the process are, in effect, filtered through the measurement system
- Reducing measurement system variation often represents the most costeffective way to reduce the total observed variation

| Appraiser | Sample | Rating | Attribute | Appraiser | Sample | Rating | Attribute |
|------------|--------|--------|-----------|------------|--------|--------|-----------|
| Simpson | 1 | 2 | 2 | Duncan | 8 | 0 | 0 |
| Montgomery | 1 | 2 | 2 | Hayes | 8 | 0 | 0 |
| Holmes | 1 | 2 | 2 | Simpson | 9 | -1 | -1 |
| Duncan | 1 | 1 | 2 | Montgomery | 9 | -1 | -1 |
| Hayes | 1 | 2 | 2 | Holmes | 9 | -1 | -1 |
| Simpson | 2 | -1 | -1 | Duncan | 9 | -2 | -1 |
| Montgomery | 2 | -1 | -1 | Hayes | 9 | -1 | -1 |
| Holmes | 2 | -1 | -1 | Simpson | 10 | 1 | 1 |
| Duncan | 2 | -2 | -1 | Montgomery | 10 | 1 | 1 |
| Hayes | 2 | -1 | -1 | Holmes | 10 | 1 | 1 |
| Simpson | 3 | 1 | 0 | Duncan | 10 | 0 | 1 |
| Montgomery | 3 | 0 | 0 | Hayes | 10 | 2 | 1 |
| Holmes | 3 | 0 | 0 | Simpson | 11 | -2 | -2 |
| Duncan | 3 | 0 | 0 | Montgomery | 11 | -2 | -2 |
| Hayes | 3 | 0 | 0 | Holmes | 11 | -2 | -2 |
| Simpson | 4 | -2 | -2 | Duncan | 11 | -2 | -2 |
| Montgomery | 4 | -2 | -2 | Hayes | 11 | -1 | -2 |
| Holmes | 4 | -2 | -2 | Simpson | 12 | 0 | 0 |
| Duncan | 4 | -2 | -2 | Montgomery | 12 | 0 | 0 |
| Hayes | 4 | -2 | -2 | Holmes | 12 | 0 | 0 |
| Simpson | 5 | 0 | 0 | Duncan | 12 | -1 | 0 |
| Montgomery | 5 | 0 | 0 | Hayes | 12 | 0 | 0 |
| Holmes | 5 | 0 | 0 | Simpson | 13 | 2 | 2 |
| Duncan | 5 | -1 | 0 | Montgomery | 13 | 2 | 2 |
| Hayes | 5 | 0 | 0 | Holmes | 13 | 2 | 2 |
| Simpson | 6 | 1 | 1 | Duncan | 13 | 2 | 2 |
| Montgomery | 6 | 1 | 1 | Hayes | 13 | 2 | 2 |
| Holmes | 6 | 1 | 1 | Simpson | 14 | -1 | -1 |
| Duncan | 6 | 1 | 1 | Montgomery | 14 | -1 | -1 |
| Hayes | 6 | 1 | 1 | Holmes | 14 | -1 | -1 |
| Simpson | 7 | 2 | 2 | Duncan | 14 | -1 | -1 |
| Montgomery | 7 | 2 | 2 | Hayes | 14 | -1 | -1 |
| Holmes | 7 | 2 | 2 | Simpson | 15 | 1 | 1 |
| Duncan | 7 | 1 | 2 | Montgomery | 15 | 1 | 1 |
| Hayes | 7 | 2 | 2 | Holmes | 15 | 1 | 1 |
| Simpson | 8 | 0 | 0 | Duncan | 15 | 1 | 1 |
| Montgomery | 8 | 0 | 0 | Hayes | 15 | 1 | 1 |
| Holmes | 8 | 0 | 0 | | | | |

Table 24.4Attribute agreement analysis data for Example 24.5.

Attribute Agreement Analysis for Rating

Each Appraiser vs Standard

Assessment Agreement

| Appraiser | # | Inspected | # | Matched | Pe | ercent | | 95 | % CI |
|------------|---|-----------|---|---------|----|--------|-----|--------|---------|
| Duncan | | 15 | | 8 | | 53.33 | (2 | 26.59, | 78.73) |
| Hayes | | 15 | | 13 | | 86.67 | (! | 59.54, | 98.34) |
| Holmes | | 15 | | 15 | 1 | .00.00 | (8 | 31.90, | 100.00) |
| Montgomery | | 15 | | 15 | 1 | 00.00 | (8 | 31.90, | 100.00) |
| Simpson | | 15 | | 14 | | 93.33 | ((| 68.05, | 99.83) |

Matched: Appraiser's assessment across trials agrees with the known standard.

Fleiss' Kappa Statistics

| Appraiser Duncan | Response -2 -1 0 1 2 Overall | Kappa 0.58333 0.16667 0.44099 0.44099 0.42308 0.41176 | SE Kappa 0.258199 0.258199 0.258199 0.258199 0.258199 0.130924 | Z 2.25924 0.64550 1.70796 1.70796 1.63857 3.14508 | P(vs > 0) 0.0119 0.2593 0.0438 0.0438 0.0507 0.0008 |
|---------------------|------------------------------------------------|-------------------------------------------------------------------------|----------------------------------------------------------------------------------|---------------------------------------------------------------------------|-----------------------------------------------------------------------|
| Hayes | -2 -1 0 1 2 Overall | 0.62963 0.81366 1.00000 0.76000 0.81366 0.82955 | 0.258199 0.258199 0.258199 0.258199 0.258199 0.258199 0.134164 | 2.43855 3.15131 3.87298 2.94347 3.15131 6.18307 | 0.0074 0.0008 0.0001 0.0016 0.0008 0.0000 |
| Holmes | -2 -1 0 1 2 Overall | 1.00000 1.00000 1.00000 1.00000 1.00000 1.00000 | 0.258199 0.258199 0.258199 0.258199 0.258199 0.258199 0.131305 | 3.87298 3.87298 3.87298 3.87298 3.87298 3.87298 7.61584 | 0.0001 0.0001 0.0001 0.0001 0.0001 0.0000 |
| Montgomery | -2 -1 0 1 2 Overall | 1.00000 1.00000 1.00000 1.00000 1.00000 1.00000 | 0.258199 0.258199 0.258199 0.258199 0.258199 0.258199 0.131305 | 3.87298 3.87298 3.87298 3.87298 3.87298 3.87298 7.61584 | 0.0001 0.0001 0.0001 0.0001 0.0001 0.0000 |
| Simpson | -2 -1 0 1 2 Overall | 1.00000 1.00000 0.81366 0.81366 1.00000 0.91597 | 0.258199 0.258199 0.258199 0.258199 0.258199 0.258199 0.130924 | 3.87298 3.87298 3.15131 3.15131 3.87298 6.99619 | 0.0001 0.0001 0.0008 0.0008 0.0001 0.0000 |

Kendall's Correlation Coefficient

| Appraiser | Coef | SE Coef | Z | Р |
|------------|---------|----------|---------|--------|
| Duncan | 0.87506 | 0.192450 | 4.49744 | 0.0000 |
| Hayes | 0.94871 | 0.192450 | 4.88016 | 0.0000 |
| Holmes | 1.00000 | 0.192450 | 5.14667 | 0.0000 |
| Montgomery | 1.00000 | 0.192450 | 5.14667 | 0.0000 |
| Simpson | 0.96629 | 0.192450 | 4.97151 | 0.0000 |

Figure 24.1 Minitab session window for Example 24.5.

Between Appraisers

Assessment Agreement # Inspected # Matched Percent 95 % CI 40.00 (16.34, 67.71) 15 6 # Matched: All appraisers' assessments agree with each other. Fleiss' Kappa Statistics Kappa SE Kappa Z P(vs > 0)Response 0.680398 0.0816497 8.3331 0.0000 -2 0.602754 0.0816497 7.3822 0.0000 -1 0.707602 0.0816497 8.6663 0.0000 0 1 0.642479 0.0816497 7.8687 0.0000 0.736534 0.0816497 9.0207 0.0000 2 Overall 0.672965 0.0412331 16.3210 0.0000 Kendall's Coefficient of Concordance Coef Chi - Sq DF P 0.966317 67.6422 14 0.0000

All Appraisers vs Standard

Assessment Agreement

Inspected # Matched Percent 95 % CI 15 6 40.00 (16.34, 67.71)

Matched: All appraisers' assessments agree with the known standard.

Fleiss' Kappa Statistics

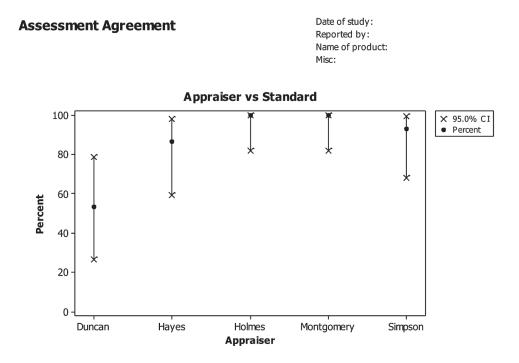
| Response | Kappa | SE Kappa | Z | P(vs > 0) |
|----------|----------|----------|---------|-----------|
| -2 | 0.842593 | 0.115470 | 7.2971 | 0.0000 |
| -1 | 0.796066 | 0.115470 | 6.8941 | 0.0000 |
| 0 | 0.850932 | 0.115470 | 7.3693 | 0.0000 |
| 1 | 0.802932 | 0.115470 | 6.9536 | 0.0000 |
| 2 | 0.847348 | 0.115470 | 7.3383 | 0.0000 |
| Overall | 0.831455 | 0.058911 | 14.1136 | 0.0000 |

Kendall's Correlation Coefficient

Coef SE Coef Z P 0.958012 0.0860663 11.1090 0.0000

* NOTE * Single trial within each appraiser. No percentage of assessment agreement within appraiser is plotted.

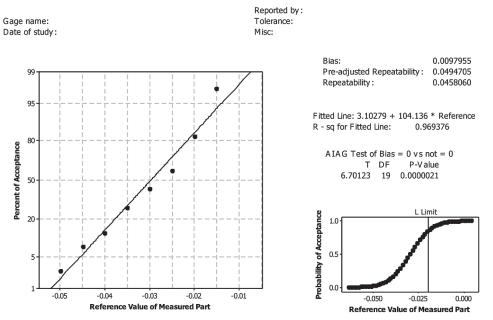
Figure 24.1 Continued.



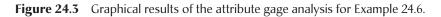


| able 24.5 Attrib | ttribute gate study data for Example 24.6. | | |
|------------------|--------------------------------------------|------------|--|
| Part number | Reference | Acceptance | |
| 1 | -0.050 | 0 | |
| 2 | -0.045 | 1 | |
| 3 | -0.040 | 2 | |
| 4 | -0.035 | 5 | |
| 5 | -0.030 | 8 | |
| 6 | -0.025 | 12 | |
| 7 | -0.020 | 17 | |
| 8 | -0.015 | 20 | |
| 9 | -0.010 | 20 | |
| 10 | -0.005 | 20 | |

| Table 24.5 | Attribute gate study data for Example 24 | 4.6. |
|------------|------------------------------------------|------|
| | rational gate study add for Example 2 | 1.0. |



Attribute Gage Study (Analytic Method) for Acceptances



VARIABLES (CONTINUOUS) MEASUREMENT SYSTEMS

Use various tools and methods (e.g., $\overline{X} - R$, $\overline{X} - s$, individual and moving range) to analyze and interpret continuous measurement systems data. (Evaluate)

Body of Knowledge VI.A.3

In order to analyze a measurement system, it is important to understand several key concepts and how they relate:

- *Accuracy* is the closeness of agreement between a measurement result and the true or accepted reference value. The components of accuracy include:
 - Bias. This is a systematic difference between the mean of the test result or measurement result and a true value. For example, if one measures the length of 10 pieces of rope that range from 1 foot to 10 feet and always concludes that the length of each piece is 2 inches shorter than the true length, then the individual exhibits a bias of 2 inches.

- *Linearity.* This is the difference in bias through the range of measurements. A measurement system that has good linearity will have a constant bias no matter the magnitude of measurement. In the previous example, the range of measurement was from 1 foot to 10 feet with a constant linear bias of 2 inches.
- Stability (of a measurement system). This represents the change in bias of a measurement system over time and usage when that system is used to measure a master part or standard. Thus, a stable measurement system is one in which the variation is in statistical control, which is typically demonstrated using control charts.
- *Precision* is the closeness of agreement between randomly selected individual measurements or test results. It is this aspect of measurement that addresses repeatability, or consistency, when an identical item is measured several times. Figure 24.4 illustrates the relationship between accuracy and precision. The components of precision include:
 - Repeatability. This is the precision under conditions where independent measurement results are obtained with the same method on identical measurement items by the same appraiser (that is, operator) using the same equipment within a short period of time. Although misleading, repeatability is often referred to as equipment variation (EV). It is also referred to as within-system variation when the conditions of measurement are fixed and defined (that is, equipment, appraiser, method, and environment).

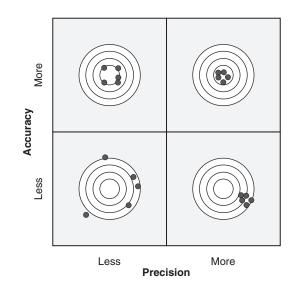


Figure 24.4 Accuracy versus precision.

- *Reproducibility.* This is the precision under conditions where independent measurement results are obtained with the same method on identical measurement items with different operators using different equipment. Although misleading, reproducibility is often referred to as *appraiser variation* (AV). The term "appraiser variation" is used because it is common practice to have different operators with identical measuring systems. Reproducibility, however, can refer to any changes in the measurement system. For example, assume the same appraiser uses the same material, equipment, and environment but uses two different measurement methods. The reproducibility calculation will show the variation due to a change in the method. It is also known as the average variation between systems, or between-conditions variation of measurement.
- *Discrimination*. This represents the measurement system's capability to detect and indicate small changes in the characteristic measured. Discrimination is also known as *resolution*. For example, a tape measure with 1-inch gradations would be unable to distinguish between object lengths that fall in between the inch marks. Hence, we would say that the measurement system can not properly discriminate between the objects. If an object to be measured were 2.5 inches, the measurement system (that is, tape measure) would produce a value of 2 or 3 inches depending on how the individual decided to round. Therefore, to measure an object that is 2.5 inches, a tape measure with finer gradations would be required.
- *Precision-to-tolerance ratio* (*PTR*). This is a measure of the capability of the measurement system. It can be calculated as

$$PTR = \frac{5.15\hat{\sigma}_{ms}}{USL - LSL}$$

where $\hat{\sigma}_{ms}$ is the estimated standard deviation of the total measurement system variability. In general, reducing the PTR will yield an improved measurement system.

- *Gage repeatability and reproducibility (GR&R) study.* This is one type of measurement system analysis done to evaluate the performance of a test method or measurement system. Such a study quantifies the capabilities and limitations of a measurement instrument, often estimating its repeatability and reproducibility. It typically involves multiple appraisers measuring a series of items multiple times.
- *Percent agreement.* This refers to the percent of time in an attribute GR&R study that appraisers agree with (1) themselves (that is, repeatability), (2) other appraisers (that is, reproducibility), or (3) a known standard (that is, bias) when classifying or rating items using nominal or ordinal scales, respectively.

Every measurement system consists of the following key components:

- Measurement instrument
- Appraiser(s) (also known as operators)
- Methods or procedures for conducting the measurement
- Environment

Three common variable measurement systems analysis methods will now be addressed:

- AIAG method
- ANOVA method
- Control chart method

Each will be addressed separately in the following sections.

AIAG Method

EXAMPLE 24.7

Figure 24.5 is an example of how to conduct a GR&R study. It uses the data collection sheet published by the Automotive Industry Action Group (AIAG) in *Measurement Systems Analysis Reference Manual* (1995). These steps provide a widely accepted procedure for analyzing measurement systems:

- 1. Label 10 parts with tags numbered 1–10 in such a way that the numbers on the tags aren't visible to the appraiser. Randomly place the parts on a work surface.
- 2. The first appraiser (we'll call him Al) chooses a part, measures the designated dimension, announces the reading, and hands it to the recorder. The recorder looks at the number on the tag and records the reading announced by Al in the appropriate column of row 1 (row labels are shown along the left margin).
- 3. Al measures the remaining nine parts and announces the reading for each. The recorder enters the values in the columns of row 1 according to tag number.
- 4. The 10 parts are again randomly placed on the work surface with the tag numbers not visible. Bill, the second appraiser, measures each one. The recorder enters Bill's readings in row 6 of the form, which is the row for trial #1 for appraiser B.
- 5. Cher, the third appraiser, repeats the procedure, and the recorder enters these values in row 11.
- 6. Now, it's Al's turn to measure the 10 parts for trial #2. These readings are entered in row 2. Bill and Cher follow with their second set of readings, which are recorded in rows 7 and 12, respectively. (The appraiser order can be randomized if desired rather than A, then B, and then C as indicated here.)

| | | G | age Re | peatabi | ility an | d Repr | oducib | ility D | ata Co | llectior | Sheet | | |
|-----|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------|-----------|---------|----------|---------|--------|---------|------------------------------------------|---------------------------|-------|-------------------|---------------------------------------|
| | | | | | | P٤ | ırt | | | | | | |
| Row | Appraiser/ Trial | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Average/ Range | Appraiser/ Trial |
| 1 | A1 | | | | | | | | | | | | A1 |
| 2 | A2 | | | | | | | | | | | | A2 |
| 3 | A3 | | | | | | | | | | | | A3 |
| 4 | Average A | | | | | | | | | | | | \overline{X}_{A} |
| 5 | Range A | | | | | | | | | | | | \overline{R}_{A} |
| 6 | B1 | | | | | | | | | | | | B1 |
| 7 | B2 | | | | | | | | | | | | B2 |
| 8 | B3 | | | | | | | | | | | | B3 |
| 9 | Average B | | | | | | | | | | | | $\overline{X}_{\scriptscriptstyle B}$ |
| 10 | Range B | | | | | | | | | | | | \overline{R}_{B} |
| 11 | C1 | | | | | | | | | | | | C1 |
| 12 | C2 | | | | | | | | | | | | C2 |
| 13 | C3 | | | | | | | | | | | | C3 |
| 14 | Average C | | | | | | | | | | | | \overline{X}_{C} |
| 15 | Range C | | | | | | | | | | | | \overline{R}_{C} |
| 16 | Part Average | | | | | | | | | | | | $\overline{\overline{X}}_{Part}$ |
| 17 | Maximum j | part ave | erage – I | Minimu | m part : | average | = | | | | | | R _{Part} |
| 18 | $(\overline{R}_A + \overline{R}_B + \overline{R}_C)$ / Number of appraisers = | | | | | | | | | $\overline{\overline{R}}$ | | | |
| 19 | | | | | | | | | $\overline{X}_{\scriptscriptstyle DIFF}$ | | | | |
| 20 | $\begin{split} \overline{X}_{DIFF} &= Max(\overline{X}_{A}, \overline{X}_{B}, \overline{X}_{C}) - Mit(\overline{X}_{A}, \overline{X}_{B}, \overline{X}_{C}) = \\ & * UCL_{R} = D_{4}\overline{\overline{R}} = (2.574)\overline{\overline{R}} = \end{split} $ | | | | | | | | | | | | |
| | $* LCL_R = D_3 \overline{R} = (0)\overline{R} = 0$ | | | | | | | | | | | | |
| 22 | $* D_3 = 0; D_4$ | $*D_3 = 0; D_4 = 2.574$ for 3 trials. | | | | | | | | | | | |

Figure 24.5Blank GR&R data collection sheet.Source: Adapted from AIAG. Used with permission of AIAG.

- 7. Each appraiser completes the third trial of measurements, and these are recorded in rows 3, 8, and 13. This completes the data entry portion of the study. Figure 24.6 shows an example of the collected data.
- 8. Calculate the average and the range for each appraiser for each part tag number. The values go in rows 4 and 5, respectively, for Al; 9 and 10, respectively, for Bill; and 14 and 15, respectively, for Cher.
- 9. Calculate the values for row 16 by averaging the values in rows 4, 9, and 14.
- 10. Calculate the values for the "Average/Range" column for rows 4, 5, 9, 10, 14, and 15 by averaging the 10 values in their respective rows.

| Gage Repeatability and Reproducibility Data Collection Sheet | | | | | | | | | | | | | |
|--------------------------------------------------------------|------------------------------------------------------------------------------------------------------------|------------------------|-----------|---------|----------|---------|-------|------|-------|--------|------------------------------------------|-------------------|---------------------------------------|
| | | Part | | | | | | | | | | | |
| Row | Appraiser/ Trial | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Average/ Range | Appraiser/ Trial |
| 1 | A1 | 0.29 | -0.56 | 1.34 | 0.47 | -0.80 | 0.02 | 0.59 | -0.31 | 2.26 | -1.36 | 0.19 | A1 |
| 2 | A2 | 0.41 | -0.68 | 1.17 | 0.50 | -0.92 | -0.11 | 0.75 | -0.20 | 1.99 | -1.25 | 0.17 | A2 |
| 3 | A3 | 0.64 | -0.58 | 1.27 | 0.64 | -0.84 | -0.21 | 0.66 | -0.17 | 2.01 | -1.31 | 0.21 | A3 |
| 4 | Average A | 0.45 | -0.61 | 1.26 | 0.54 | -0.85 | -0.10 | 0.67 | -0.23 | 2.09 | -1.31 | 0.19 | $\overline{X}_{\scriptscriptstyle A}$ |
| 5 | Range A | 0.35 | 0.12 | 0.17 | 0.17 | 0.12 | 0.23 | 0.16 | 0.14 | 0.27 | 0.11 | 0.18 | \overline{R}_{A} |
| 6 | B1 | 0.08 | -0.47 | 1.19 | 0.01 | -0.56 | -0.20 | 0.47 | -0.63 | 1.80 | -1.68 | 0.00 | B1 |
| 7 | B2 | 0.25 | -1.22 | 0.94 | 1.03 | -1.20 | 0.22 | 0.55 | 0.08 | 2.12 | -1.62 | 0.12 | B2 |
| 8 | B3 | 0.07 | -0.68 | 1.34 | 0.20 | -1.28 | 0.06 | 0.83 | -0.34 | 2.19 | -1.50 | 0.09 | B3 |
| 9 | Average B | 0.13 | -0.79 | 1.16 | 0.41 | -1.01 | 0.03 | 0.62 | -0.30 | 2.04 | -1.60 | 0.07 | $\overline{X}_{\scriptscriptstyle B}$ |
| 10 | Range B | 0.18 | 0.75 | 0.40 | 1.02 | 0.72 | 0.42 | 0.36 | 0.71 | 0.39 | 0.18 | 0.51 | $\overline{R}_{_B}$ |
| 11 | C1 | 0.04 | -1.38 | 0.88 | 0.14 | -1.46 | -0.29 | 0.02 | -0.46 | 1.77 | -1.49 | -0.22 | C1 |
| 12 | C2 | -0.11 | -1.13 | 1.09 | 0.20 | -1.07 | -0.67 | 0.01 | -0.56 | 1.45 | -1.77 | -0.26 | C2 |
| 13 | C3 | -0.15 | -0.96 | 0.67 | 0.11 | -1.45 | -0.49 | 0.21 | -0.49 | 1.87 | -2.16 | -0.28 | C3 |
| 14 | Average C | -0.07 | -1.16 | 0.88 | 0.15 | -1.33 | -0.48 | 0.08 | -0.50 | 1.70 | -1.81 | -0.25 | \overline{X}_{C} |
| 15 | Range C | 0.19 | 0.42 | 0.42 | 0.09 | 0.39 | 0.38 | 0.20 | 0.10 | 0.42 | 0.67 | 0.33 | \overline{R}_{C} |
| 16 | Part Average | 0.17 | -0.85 | 1.10 | 0.37 | -1.06 | -0.19 | 0.45 | -0.34 | 1.94 | -1.57 | 0.00 | $\overline{\overline{X}}_{Part}$ |
| 17 | Maximum j | part ave | rage – N | 1inimu | n part a | average | = | | | | | 3.51 | R _{Part} |
| 18 | $(\overline{R}_A + \overline{R}_B + \overline{R}_C)$ / Number of appraisers = $(0.18 + 0.51 + 0.33)$ / 3 = | | | | | | | | | 0.3417 | $\overline{\overline{R}}$ | | |
| 19 | | | | | | | | | | 0.4447 | $\overline{X}_{\scriptscriptstyle DIFF}$ | | |
| 20 | * $UCL_R = L$ | $\overline{P_4R} = (2$ | 2.574)(0 |).3417) | = 0.879 | 95 | | | | | | | |
| 21 | $*LCL_R = D_3 \overline{R} = (0)(0.3417) = 0$ | | | | | | | | | | | | |
| 22 | $* D_3 = 0; D_4$ | = 2.574 | 4 for 3 t | rials. | | | | | | | | | 1 |

Figure 24.6 GR&R data collection sheet with data entered and calculations completed. *Source:* Adapted from AIAG. Used with permission of AIAG.

- 11. Calculate the value for the "Average/Range" column of row 16 by averaging the 10 values in that row.
- 12. Calculate the value for the "Average/Range" column of row 17 by using the formula given in that row.
- 13. Calculate the value for the "Average/Range" column of row 18 by using the formula given in that row.

Continued

- 14. Calculate the value for the "Average/Range" column of row 19 by using the formula given in that row.
- 15. Calculate the UCL_R by using the formula shown in line 20 along with the proper control chart constant values from row 22.
- 16. Compare each of the 10 *R*-values in row 5 to the UCL_{*R*} value calculated in row 20. Any *R*-value that exceeds the UCL_{*R*} should be circled. Repeat this for the *R*-values in rows 10 and 15. The circled *R*-values are significantly different from the others, and the cause of this difference should be identified and corrected. Once this has been done, the appropriate parts can be remeasured using the same appraiser, equipment, and so on, as for the original measurements. Recompute all appropriate values on the data collection sheet as necessary.

Recall that repeatability is the variation in measurements that occurs when the same measuring system—including equipment, material, and appraiser—is used. Repeatability, then, is reflected in the *R*-values as recorded in rows 5, 10, and 15 and summarized in row 17. Repeatability is often referred to as equipment variation, but the individual *R* averages may indicate differences between appraisers. In the example in Figure 24.6, R_A is smaller than R_B or R_c . This indicates that Al has done a better job of getting the same answer upon repeated measurements of the same part than either Bill or Cher. Perhaps Al has a better technique, more skill, or sharper eyesight than the others. Investigation of this issue may provide an opportunity for variation reduction.

Reproducibility is the variation that occurs between the overall average measurements for the three appraisers. It is reflected by the X-values in rows 4, 9, and 14 and summarized in the value of \overline{X}_{DIFF} in row 19. For example, had \overline{X}_A and \overline{X}_B been closer in value and \overline{X}_C significantly different, then Cher's measurements would exhibit a bias. Again, further investigation would be necessary.

The next step in the study is to complete the GR&R report shown in Figure 24.7. A completed report based on the data from Figure 24.6 is shown in Figure 24.8. Each of the quantities in the "Measurement Unit Analysis" column will now be described:

- Equipment variation (EV). This is an estimate of the standard deviation of the variation due to repeatability and is sometimes denoted σ_{E} or σ_{rpt} .
- Appraiser variation (AV). This is an estimate of the standard deviation of the variation due to reproducibility and is sometimes denoted σ_A or σ_{tod} .
- Gage repeatability and reproducibility (GRR). This is an estimate of the standard deviation of the variation due to the measurement system and is sometimes denoted σ_{M} .
- *Part-to-part variation (PV)*. This is an estimate of the standard deviation of the variation due to the part differences and is sometimes denoted σ_{P} .
- *Total variation (TV)*. This is an estimate of the standard deviation of the total variation in the study and is sometimes denoted σ_{T} .

The "Percent Total Variation" column in Figure 24.7 shows for each type of variation the percent of total variation it consumes. Sometimes the right-hand column is based on the tolerance for the dimension. In this case, the value of *TV* is replaced by the tolerance.

Continued

| | | 0 | Gage Repeatability a | and Reproduci | bility Rep | ort |
|-----------------------------------------|--------------------------------------------|-----------------------------------|----------------------|------------------------------------------|-----------------------|-----------------------------------|
| Part No. and | l Name: | | Gage Name: | | | Date: |
| Characterist | ics: | | Gage No.: | | | Performed by: |
| Specification | ns: | - | Gage Type: | | | |
| | | | | | | |
| From data sl | heet: | $\overline{\overline{R}} =$ | | $\overline{X}_{\scriptscriptstyle DIFF}$ | | |
| | | | ement Unit Analysi | s | | Percent Total Variation |
| | | ipment Va | riation (EV) | Trials | K_1 | % EV = 100(EV/TV) |
| EV = | | | | 2 | 0.8862 | =100(/) = % |
| | =x | | | 3 | 0.5908 | |
| Reproc | lucibility - A | Appraiser V | ariation (AV) | Appraisers | <i>K</i> ₂ | % AV = 100(AV/TV) |
| AV = | $=\sqrt{(\overline{X}_{DIFF} \mathbf{x})}$ | $K_2^2 - (EV$ | $r^{2}/(nr))$ | 2 | 0.7071 | = 100(/) = % |
| = | = √(_x_) | ² - (² /(_ | _x_)) | 3 | 0.5231 | n = number of parts |
| = | · | | | | | r = number of trials |
| Gage R | Repeatability | & Reprod | ucibility (GRR) | | | %GRR = 100(GRR / TV) |
| | $=\sqrt{EV^2+AV}$ | | | Parts | K_3 | $= 100(_/_)$ = % |
| | = √²+ | 2 | | 2 | 0.7071 | =70 |
| _ | = | | | 3 | 0.5231 | |
| Part Va | ariation (PV | 7) | | 4 | 0.4467 | % PV = 100(PV/TV) |
| | $= R_{Part} \times K_3$ | | | 5 | 0.4030 | = 100(/) = % |
| | ·x | | | 6 | 0.3742 | /0 |
| | | | | 7 | 0.3534 |] |
| Total V | Variation (T) | V) | | 8 | 0.3375 | ndc = 1.41(PV/GRR) |
| $TV = \sqrt{GRR^2 + PV^2}$ | | | | 9 | 0.3249 | =1.41(/) |
| $=\sqrt{\underline{}^2+\underline{}^2}$ | | | | 10 | 0.3146 | $= \$ ndc = number of distinct |
| _ | = | | | | | categories |
| | | | | | | |

Figure 24.7 Blank GR&R report.

Source: Adapted from AIAG. Used with permission of AIAG.

Continued

In general, if the %*GRR* is:

- Less than 10%, the measurement system is considered acceptable
- Between 10% and 30%, inclusive, the measurement system is considered marginal
- Greater than 30%, the measurement system is considered inadequate

These criteria should be considered guidelines and dependent on the situation at hand.

| | (| Sage Repeat | ability and | Reproduci | bility Rep | oort | | | | |
|------------------------------------------------------------------------------|------------------------------------------------|----------------------|-------------|----------------------------|-----------------------|---------------------------------------------|---------------------------------|-----------|--|--|
| Part No. and Name: | | Gage Name: | | | | Date: | | | | |
| Characteristics: | | Gage No.: | | | | Performed by: | | | | |
| Specifications: | | Gage Type: | | | | | | | | |
| | = | 0.0417 | | | 0.4447 | | D | | | |
| From data sheet: | $\overline{R} =$ | 0.3417 ement Unit | Analysis | \overline{X}_{DIFF} | 0.4447 | Domas | R _{Part} | 3.5 | | |
| | | | | ent Total Variation | | | | | | |
| Repeatability - E | Trials | K_1 | - %EV | =100(EV/7) =100(0.2019) | | | | | | |
| $EV = R \ge K_1$ $= (0.3417)$ | | 2 | 0.8862 | | =17.62% | | | | | |
| =(0.3417) = 0.2019 | 3 | 0.5908 | _ | | | | | | | |
| Reproducibility - | Appraiser V | ariation (AV |) | Appraisers | <i>K</i> ₂ | %AV | %AV = 100(AV/TV) | | | |
| $AV = \sqrt{(\overline{X}_{DIFF})}$ | $\left(K_{2}\right)^{2} - \left(EV\right)^{2}$ | (nr)) | | 2 | 0.7071 | 1 | =100(0.2296/1.1458) =20.04% | | | |
| $=\sqrt{((0.444))}$ | 3 | 0.5231 | _ | n = number of parts | | | | | | |
| = 0.2296 | | | | | | r = nut | r = number of trials | | | |
| Gage Repeatabili | ty & Reprod | ucibility (GR | R) | | | %GRR | R = 100(GRR / | · · | | |
| $GRR = \sqrt{EV^2 + 2}$ | | _ | | Parts | <i>K</i> ₃ | 1 | = 100(0.3058/1.1458) $= 26.69%$ | | | |
| • • | $)^{2} + (0.2296)$ | 2 | | 2 | 0.7071 | 1 | - 20.0970 | | | |
| = 0.3058 | | | | 3 | 0.5231 | 7 | 1 | | | |
| Part Variation (F | <i>,</i> | | | 4 | 0.4467 | %PV | =100(PV/T) | , | | |
| $PV = R_{Part} \times K_3$ = (3.51)(0.1) | | | | 5 | 0.4030 | | =100(1.1042) = 96.37% | 2/1.1458) | | |
| =(3.51)(0.1) =1.1042 | 5140) | | | 6 | 0.3742 | 1 | _ >0.5770 | | | |
| | | | | 7 | 0.3534 | 7 | | | | |
| Total Variation (| | | | 8 | 0.3375 | ndc | =1.41(PV/c) | | | |
| $TV = \sqrt{GRR^2 + PV^2}$ = $\sqrt{(0.3058)^2 + (1.1042)^2}$ = 1.1458 | | | 9 | 0.3249 | 7 | $= 1.41(1.1042)$ $= 5.09 \Longrightarrow 5$ | 2/0.3058) | | | |
| | | | 10 | 0.3146 | ndc | | | | | |

Figure 24.8GR&R report with calculations completed.

Source: Adapted from AIAG. Used with permission of AIAG.

ANOVA Method

The data depicted in Figure 24.6 can also be analyzed using the *analysis of variance* (ANOVA) methods addressed in Chapter 25 of Kubiak and Benbow (2009).

Control Chart Method

Another method for analyzing GR&R data is with $\overline{X} - R$ control charts.

EXAMPLE 24.8

Figure 24.9 represents the results of an ANOVA using Minitab. The top source table illustrates that the interaction term (that is, parts by operators) is not significant. Thus, the term is removed, resulting in the second source table.

The third source table provides the variances for each component. Notice that the GR&R variation is relatively small with respect to the part-to-part variance. This is what we look for in a measurement systems analysis.

The last source table in Figure 24.9 depicts how much each component contributes to the total variation. The "SD" column provides the standard deviation for each component. Compare this column with the "Measurement Unit Analysis" column in Figure 24.8. The numbers are similar, though not exact. This is due to the difference in the methods used. The "% Study Var" column shows the percentage contribution of each component. Compare these numbers with the "Percent Total Variation" column in Figure 24.8. Again, the numbers are similar but not exact.

Figure 24.10 is obtained from the "% Study Var" column from the last source table in Figure 24.9 and is frequently used to depict what is called the "components of variation."

With the ready availability of statistical analysis software, the ANOVA method represents a quick and easy way to analyze data obtained from a GR&R study.

EXAMPLE 24.9

Figure 24.11 provides the source tables for this method. Notice the similarities and differences between these two tables and the last two tables in Figure 24.9, as well as the "Percent Total Variation" column from Figure 24.8. The answers are, indeed, similar.

Figure 24.12 is particularly interesting. Let's disregard the one out-of-control point on the range chart for operator B for the moment. Notice that each \overline{X} chart is out of control. This is actually desirable since the *R* chart is based on repeatability error. If the \overline{X} chart was in control, this would mean that part-to-part variation would be less than the repeatability variation. In other words, the part-to-part variation would be lost within the repeatability variation.

For a more detailed discussion on the use of the control chart method, see *The Six Sigma Handbook*, Revised and Expanded (2003), by Thomas Pyzdek.

Examples for the \overline{X} – s and individual and moving range methods will not be presented. Note: These two methods are not incorporated into Minitab's Gage R and R package. However, they can be accomplished using the traditional control chart approaches discussed in Chapter 35 of Kubiak and Benbow (2009).

Gage R&R Study - ANOVA Method

Two-Way ANOVA Table With Interaction

| Source | DF | SS | MS | F | Р |
|-------------------|----|---------|---------|---------|-------|
| Parts | 9 | 88.3619 | 9.81799 | 492.291 | 0.000 |
| Operators | 2 | 3.1673 | 1.58363 | 79.406 | 0.000 |
| Parts * Operators | 18 | 0.3590 | 0.01994 | 0.434 | 0.974 |
| Repeatability | 60 | 2.7589 | 0.04598 | | |
| Total | 89 | 94.6471 | | | |

Alpha to remove interaction term = 0.25

Two-Way ANOVA Table Without Interaction

| Source | DF | SS | MS | F | P |
|---------------|----|---------|---------|---------|-------|
| Parts | 9 | 88.3619 | 9.81799 | 245.614 | 0.000 |
| Operators | 2 | 3.1673 | 1.58363 | 39.617 | 0.000 |
| Repeatability | 78 | 3.1179 | 0.03997 | | |
| Total | 89 | 94.6471 | | | |

Gage R&R

| Source | VarComp | | | |
|-----------------|---------|--|--|--|
| Total Gage R&R | 0.09 | | | |
| Repeatability | 0.04 | | | |
| Reproducibility | 0.05 | | | |
| Operators | 0.05 | | | |
| Part-To-Part | 1.09 | | | |
| Total Variation | 1.18 | | | |

| | | Study Var | %Study Var |
|-----------------|-------------|-----------|------------|
| Source | StdDev (SD) | (6 * SD) | (%SV) |
| Total Gage R&R | 0.30237 | 1.81423 | 27.86 |
| Repeatability | 0.19993 | 1.19960 | 18.42 |
| Reproducibility | 0.22684 | 1.36103 | 20.90 |
| Operators | 0.22684 | 1.36103 | 20.90 |
| Part-To-Part | 1.04233 | 6.25396 | 96.04 |
| Total Variation | 1.08530 | 6.51180 | 100.00 |
| | | | |



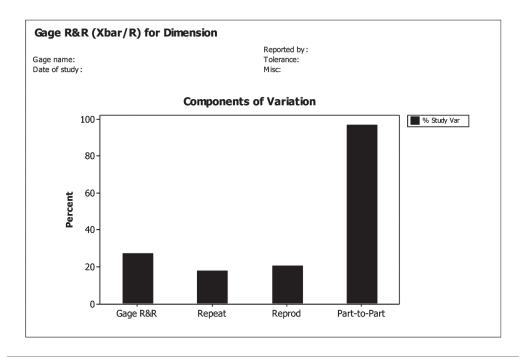
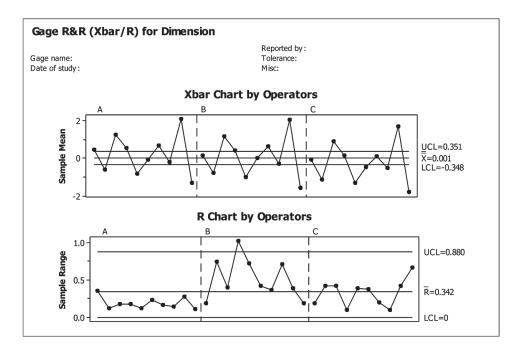


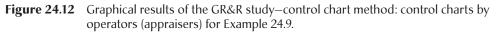
Figure 24.10 Gage R&R study–ANOVA method: components of variation..

Gage R&R Study - XBar/R Method

| Source Total Gage R&R Repeatability Reproducibility Part-To-Part Total Variation | VarComp 0.09 0.04 0.05 1.22 1.31 | | |
|-------------------------------------------------------------------------------------------------|--------------------------------------------------------------------|------------------------------------------------------------------------------|-------------------------------------------------------------------|
| Source Total Gage R&R Repeatability Reproducibility Part-To-Part Total Variation | StdDev (SD) 0.30589 0.20181 0.22988 1.10412 1.14571 | Study Var (6 * SD) 1.83536 1.21087 1.37925 6.62474 6.87428 | %Study Var (%SV) 26.70 17.61 20.06 96.37 100.00 |

Figure 24.11 Minitab session window: GR&R study–control chart method: source tables for Example 24.9.





PROCESS CAPABILITY FOR NONNORMAL DATA

Calculate capability using Weibull and other methods for non-normal data. (Apply)

Body of Knowledge VI.A.4

Transforming Nonnormal Data

If data are not normal and normality is required, the following approaches—all of which involve fairly complex calculations and are usually performed with the aid of statistical software packages—may be taken:

- Find another known distribution that fits the data—exponential, chi-square, Weibull, and *F* distributions are among the possibilities. A statistics software package is helpful in choosing the function and parameters that will provide the best fit for the data.
- Use nonlinear regression to fit a curve to the data and apply numerical integration to find the areas of the "tails" beyond the specification limits.
- Transform the data to produce a second variable that is normally distributed; various normalizing transformations are available. Among

the most used are the Box-Cox and Johnson transformations. These transformations introduce a new variable that will have a "more normal" distribution:

- The Box-Cox transforms the input data, denoted by *Y*, using $W = Y^{\lambda}$, where λ is any number typically between -5 and 5. The trick, of course, is to choose λ that produces a curve that is as close to normal as possible. Statistical software packages will use sample data to recommend a range for λ , often with a confidence interval. The user may choose a value in the range and observe the resulting curve, eventually choosing the value that best fits the situation. Also, the packages typically generate values for the capability indices. Unfortunately, the Box-Cox transformation is limited to positive data values and often does not provide a suitable transformation. It also assumes that data are in subgroups.
- The Johnson transformation is chosen from three different functions within the Johnson system. As a result, this approach usually finds an appropriate transformation. It does not assume data are in subgroups.

Assuming a Form of a Distribution

An alternative to transforming nonnormal data is to assume the data fit a specific distribution. One favored distribution is the Weibull. The Weibull is a family of distributions that can take many shapes.

The Weibull distribution has the general form

$$P(x) = \alpha \beta (x - \gamma)^{\beta - 1} e^{-\alpha (x - \gamma)^{\beta}}$$

where:

 α = Scale parameter

 β = Shape parameter

 γ = Location or threshold parameter

The elegance of the Weibull function is that it takes on many shapes depending on the value of β , as shown in Figure 24.13. For example, when:

- 0 < β < 1, the Weibull has a decreasing failure rate and can be used to model "infant mortality"
- β = 1, the Weibull is reduced to the exponential distribution with a constant failure rate
- *β*> 1, the Weibull has an increasing failure rate and can be used to model "wear-out"
- β = 3.5, the Weibull is approximately the normal distribution, has an increasing failure rate, and can be used to model "wear-out"

Although Example 24.10 assumed the use of the Weibull distribution when the data were nonnormal, any other continuous distribution deemed appropriate could have been used as well.

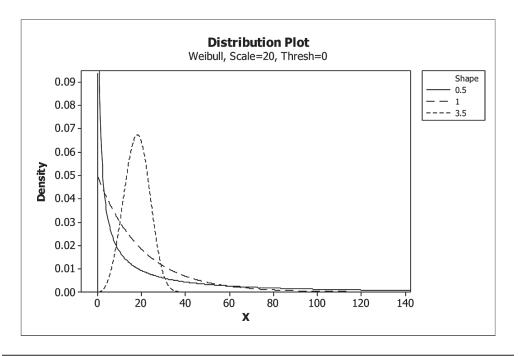


Figure 24.13 Examples of a Weibull distribution for various values of the shape parameter.

EXAMPLE 24.10

The source of this example is Bower (2001). The data for this example are temperature data and are given in Table 24.6. A quick normal probability plot of the data can confirm nonnormality. The Anderson-Darling statistic value is 1.347 with a *p*-value < 0.005. This is illustrated in Figure 24.14. This conclusion is supplemented by a graphical summary of the data, shown in Figure 24.15.

Given the data, we might want to assume they fit a specific distribution. In this case, we will assume they fit the Weibull distribution. Instead of working the numbers by hand, we will choose to let Minitab estimate the parameters of the Weibull distribution for us. This will be done using Minitab 16 and by selecting the following sequence:

- 1. Stat
- 2. Quality tools
- 3. Capability analysis
- 4. Non-normal
- 5. Fit distribution: Weibull

The above will yield Figure 24.16. This figure illustrates a plot of the data with a Weibull plot overlay. Creating a Weibull probability plot as shown in Figure 24.17 will show that we can not reject the null hypothesis that the data fit a Weibull distribution with shape parameter 2.214 and scale parameter 9.946. The Anderson-Darling statistic is 0.450 and the *p*-value > 0.250.

Part VI.A.4

| Table 24.6 | Temperatu | Femperature data for Example 24.10. | | | | | | | |
|------------|-----------|-------------------------------------|-------|-------|-------|-------|-------|--|--|
| 13.81 | 13.05 | 13.29 | 22.33 | 4.40 | 10.70 | 3.42 | 10.04 | | |
| 17.64 | 8.19 | 14.48 | 4.95 | 6.19 | 15.37 | 11.37 | 7.79 | | |
| 10.41 | 3.35 | 5.82 | 5.82 | 10.01 | 3.78 | 13.93 | 6.14 | | |
| 11.31 | 1.87 | 13.07 | 8.04 | 8.85 | 5.30 | 10.87 | 4.65 | | |
| 9.81 | 4.58 | 2.52 | 15.20 | 4.59 | 8.73 | 9.57 | 16.15 | | |
| 12.09 | 21.21 | 7.29 | 10.44 | 15.22 | 5.57 | 4.67 | 17.95 | | |
| 8.46 | 4.41 | 10.33 | 6.18 | 2.36 | 2.97 | 7.95 | 7.15 | | |
| 9.69 | 3.79 | 9.75 | 4.34 | 8.60 | 6.13 | 4.20 | 17.19 | | |
| 6.26 | 15.43 | 6.28 | 8.37 | 13.64 | 4.09 | 6.03 | 7.12 | | |
| 2.39 | 7.70 | 6.60 | 12.66 | 7.58 | 9.19 | 4.14 | 9.60 | | |
| 9.72 | 7.93 | 2.44 | 9.54 | 8.25 | 11.32 | 11.67 | 7.61 | | |
| 10.05 | 8.07 | 6.38 | 5.99 | 12.99 | 7.25 | 4.97 | 5.43 | | |
| 8.20 | 5.16 | 6.87 | 9.74 | 17.37 | 12.93 | 8.58 | 4.00 | | |
| 9.59 | 4.53 | 15.25 | 9.38 | 13.00 | 4.91 | 8.14 | 4.48 | | |
| 8.47 | 8.05 | 3.02 | 10.38 | 7.02 | 18.20 | 8.97 | 11.62 | | |

. . • -1 c. 1

Source: Bower (2001).

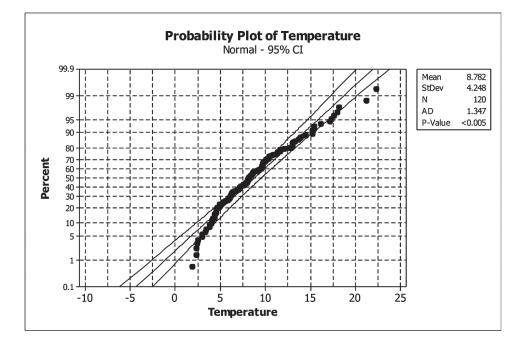


Figure 24.14 Normal probability plot of temperature data shown in Table 24.6.

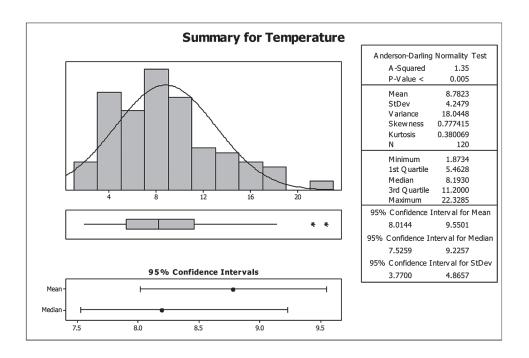


Figure 24.15 A graphical summary of the data shown in Table 24.6.

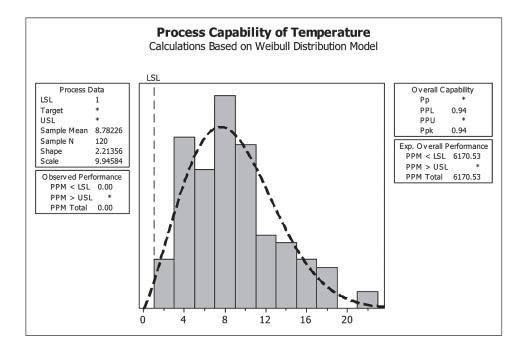


Figure 24.16 Nonnormal process capability analysis based on the Weibull distribution.

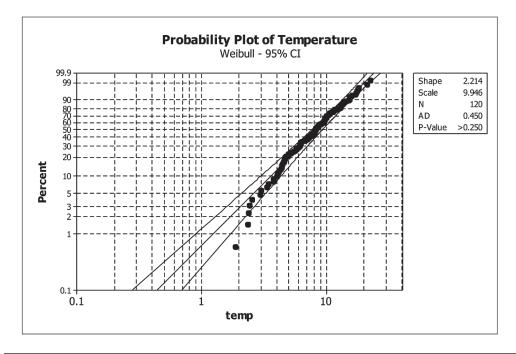


Figure 24.17 Weibull probability plot with fitted parameters.

Chapter 25

Measuring and Modeling Relationships Between Variables

This chapter covers a wide variety of tools and techniques associated with measuring and modeling the relationships between variables. Many practicing Master Black Belts will find some of this material new to them. However, every attempt has been made to present the material at the designated Bloom taxonomy level found in the body of knowledge. Remember, this handbook is designed first and foremost to prepare an individual to sit for the ASQ Certified Six Sigma Master Black Belt Examination. It is not intended to cover all topics in detail. Consequently, you may not find the material sufficiently informative in all areas. Therefore, it is recommended you pursue more in-depth resources, many of which will be found in the Bibliography.

AUTOCORRELATION AND FORECASTING

Identify autocorrelated data, including time-series modeling (e.g., ARIMA) and forecasting. (Understand)

Body of Knowledge VI.B.1

Terminology

The following terms and formulas are relevant when discussing autocorrelation, time-series modeling, ARIMA, and forecasting:

- *Time series.* A time series is a sequence of measurements of some quantity taken at different times, often at equally spaced intervals.
- Autocorrelation. The ASQ Glossary and Tables for Statistical Quality Control, Fourth Edition defines autocorrelation as "The internal correlation between members of a series of observations ordered in time."
- *Stationarity*. This refers to the statistical characteristics of the mean, variance, and autocorrelation structure over time. Time-series data

are assumed stationary when the mean, variance, and autocorrelation structure are constant over the entire range of the data.

- *Lag.* The lag represents the number of time units between differenced values.
- Differences. This represents the difference between data values for a specified lag value k. For example, for a given set of ordered data, there is a lag k between values x_t and x_{t+k}. Differencing is used to introduce stationarity to a time series. Data transformation can also be used to stabilize the variance.
- *Forecasting*. Forecasting is a prediction of future values of a time series. For example, one might want to predict the next value, the next five values, or the next ten values in the series.
- Sample autocorrelation function (ACF).

$$r_k = \frac{\sum_{t=1}^n (x_{t-k} - \overline{x})(x_t - \overline{x})}{\sum_{t=1}^n (x_t - \overline{x})^2}$$

where

n = Number of observations

 x_t = Value of x at time t

 $k = \text{Lag } j; k = 1, 2, \dots$

• Standard error of the ACF.

$$SE(r_k) = \sqrt{\frac{\left(1+2\sum_{m=1}^{k-1}r_m^2\right)}{n}}; SE(r_1) = \sqrt{\frac{1}{n}}$$

where

n = Number of observations

 r_m = Autocorrelation at lag m

 $k = \text{Lag } j; k = 1, 2, \dots$

• t_k . This is the *t* statistic at lag *k*. The formula is

$$t_k = \frac{r_k}{SE(r_k)}$$

where

 $SE(r_k)$ = Standard error of the autocorrelation at lag *k*.

- Autoregressive integrated moving average (ARIMA) models. The SAS Institute, Inc. defines ARIMA as follows, "An ARIMA model predicts a value in a response time series as a linear combination of its own past values, past errors (also called shocks or innovations), and current and past values of other time series."
- *Ljung-Box Q (LBQ) statistic.* This is a statistic that tests the null hypothesis that autocorrelations up to lag *k* are equal to zero. The statistic is computed as

$$Q_k = n(n+2)\sum_{m=1}^k \frac{r_m^2}{n-m}$$

where

n = Number of observations

 r_m = Autocorrelation at lag m; m = 1,2,...,k

k = Lag; *k* = 1,2,...

Q is distributed as $\chi^2_{\alpha,k}$ and tests the hypothesis of model adequacy. If the value of *Q* exceeds the chi-square value, the hypothesis is rejected.

The following statistics are measures of the accuracy of the fitted time-series values:

1. Mean absolute percentage error (MAPE).

$$MAPE = \frac{\left(\sum_{t=1}^{n} \frac{\left| \left(Y_{t} - \hat{Y}_{t} \right) \right|}{Y_{t}} \right) (100)}{n}; \ Y_{t} \neq 0$$

2. Mean absolute deviation (MAD).

$$MAD = \sum_{t=1}^{n} \left| \left(Y_t - \hat{Y}_t \right) \right|$$

3. Mean squared deviation (MSD).

$$MSD = \frac{\sum_{t=1}^{n} |Y_t - \hat{Y}_t|^2}{n}$$

In all three equations \hat{Y}_t is the actual value at time *t*. In the first two equations \hat{Y}_t is the fitted value at time *t* and *n* is the number of observations. In the third equation \hat{Y}_t is the forecast value at time *t* and *n* is the number of forecasted values. The smaller each of these measures is, the better the fit.

The following are common types of trend analysis models:

- 1. *Linear*. $Y_t = \beta_0 + \beta_1 t + \varepsilon_t$, where β_0 is the average change between successive time periods.
- 2. *Quadratic.* $Y_t = \beta_0 + \beta_1 t + \beta_2 t^2 + \varepsilon_t$.
- 3. *Exponential*. $Y_t = \beta_0 \beta_1^t e_t$. This model is applicable where there is either exponential growth or decay.
- 4. *S-curve*. $\frac{10^a}{\beta_0 + \beta_1 \beta_2^t}$. This model is the Pearl–Reed logistic model.

Autocorrelation

Boslaugh and Watters (2008) state that "after fitting a linear model, autocorrelation will be readily apparent in the residuals if it is present in the data. Where the residuals are rising over time, they are said to be positively correlated, or negatively correlated when they are decreasing over time."

Time-Series Analysis

A *time series* is a sequence of measurements of some quantity taken at different times, often at equally spaced intervals. Time-series models often have the following components:

 T_i = Long-term trend

 C_i = Cyclical effect (due to business or economic upturns or downturns)

 S_i = Seasonal effect

 R_i = Residual or error effect

Sometimes it is necessary to separate the signal from the noise. When this occurs, it is often useful to smooth the data. Multiple smoothing techniques are available. Here are some of the more common types:

EXAMPLE 25.1

The data given in Table 25.1 are monthly food-related data over a five-year period. Determine the autocorrelation function (ACF).

In order to determine the ACF, we must first plot the data. This is shown in Figure 25.1. An analysis of this figure indicates a lag of approximately 12 due to the high degree of seasonality. Of course, this is expected given the nature of the data. Next, we determine the differences with a lag of 12. Performing an autocorrelation on the differences follows. The results are plotted in Figure 25.2. Analyzing this figure, we find that the ACF is positive and decays slowly, which is indicative of an autoregressive process. The confidence limits shown in the figure test the hypothesis that the correlations are equal to zero. Table 25.2 provides the Minitab session window.

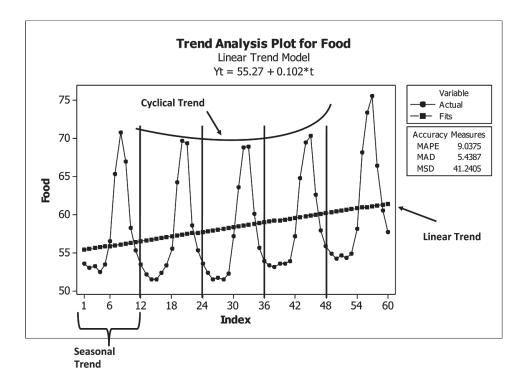
- *Moving average* of size *n*.
- *Central moving average* uses a window of size *n* and uses both past and future data to calculate the average for each point. Table 25.3 illustrates how the central moving average is computed.
- *Weighted moving average* uses a window of size *n* but assigns greater weight to data points closer to the point in question.
- *Exponential moving average* (EMA) is similar to the central moving average, but more weight is applied to measurements closer to the point in question and less weight exponentially the farther away from the point in question. The EMA is computed as

See equation on page 359

| Order | Food | Differences | Order | Food | Differences | Order | Food | Differences |
|-------|------|-------------|-------|------|-------------|-------|------|-------------|
| 1 | 53.5 | | 21 | 69.3 | 2.4 | 41 | 53.9 | 1.7 |
| 2 | 53.0 | | 22 | 58.5 | 0.3 | 42 | 57.1 | 0.0 |
| 3 | 53.2 | | 23 | 55.3 | 0.0 | 43 | 64.7 | 1.1 |
| 4 | 52.5 | | 24 | 53.6 | 0.2 | 44 | 69.4 | 0.6 |
| 5 | 53.4 | | 25 | 52.3 | 0.2 | 45 | 70.3 | 1.4 |
| 6 | 56.5 | | 26 | 51.5 | 0.0 | 46 | 62.6 | 2.5 |
| 7 | 65.3 | | 27 | 51.7 | 0.2 | 47 | 57.9 | 2.3 |
| 8 | 70.7 | | 28 | 51.5 | -0.9 | 48 | 55.8 | 1.9 |
| 9 | 66.9 | | 29 | 52.2 | -1.1 | 49 | 54.8 | 1.5 |
| 10 | 58.2 | | 30 | 57.1 | 1.6 | 50 | 54.2 | 1.1 |
| 11 | 55.3 | | 31 | 63.6 | -0.6 | 51 | 54.6 | 1.1 |
| 12 | 53.4 | | 32 | 68.8 | -0.8 | 52 | 54.3 | 0.8 |
| 13 | 52.1 | -1.4 | 33 | 68.9 | -0.4 | 53 | 54.8 | 0.9 |
| 14 | 51.5 | -1.5 | 34 | 60.1 | 1.6 | 54 | 58.1 | 1.0 |
| 15 | 51.5 | -1.7 | 35 | 55.6 | 0.3 | 55 | 68.1 | 3.4 |
| 16 | 52.4 | -0.1 | 36 | 53.9 | 0.3 | 56 | 73.3 | 3.9 |
| 17 | 53.3 | -0.1 | 37 | 53.3 | 1.0 | 57 | 75.5 | 5.2 |
| 18 | 55.5 | -1.0 | 38 | 53.1 | 1.6 | 58 | 66.4 | 3.8 |
| 19 | 64.2 | -1.1 | 39 | 53.5 | 1.8 | 59 | 60.5 | 2.6 |
| 20 | 69.6 | -1.1 | 40 | 53.5 | 2.0 | 60 | 57.7 | 1.9 |

 Table 25.1
 Data for Example 25.1 with differences calculated based on a lag of 12.

Source: Data column courtesy of Minitab.





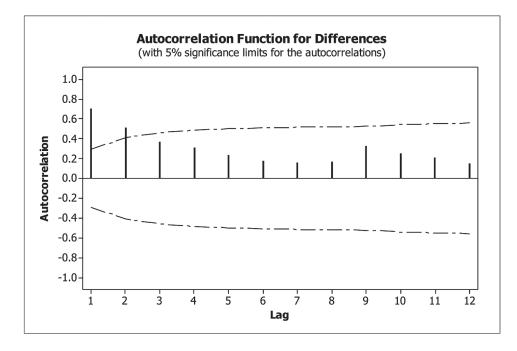


Figure 25.2 A graph of the ACF for the data in Table 25.1.

| Table 25.2 Minitab session window output for Example 25.1. | | | | | | | |
|-------------------------------------------------------------------|--------------|---------|--|--|--|--|--|
| ACF | T-statistic* | LBQ** | | | | | |
| 0.70139 | 4.85936 | 31.0177 | | | | | |
| 0.51227 | 2.51975 | 47.8488 | | | | | |
| 0.36688 | 1.60480 | 56.6333 | | | | | |
| 0.31036 | 1.29013 | 63.0319 | | | | | |
| 0.23474 | 0.94361 | 66.7589 | | | | | |
| 0.17307 | 0.68314 | 68.8224 | | | | | |
| 0.16205 | 0.63350 | 70.6655 | | | | | |
| 0.17005 | 0.65930 | 72.7342 | | | | | |
| 0.32244 | 1.23895 | 80.3177 | | | | | |
| 0.25277 | 0.94163 | 85.0713 | | | | | |
| 0.20802 | 0.76098 | 88.3565 | | | | | |
| 0.15094 | 0.54561 | 90.1222 | | | | | |

hla of o Minital 1. 25 1

*This is actually the z-statistic per Montgomery (2008).

** The LBQ values differ from Minitab, but were confirmed using Excel.

 Table 25.3
 An example of how the central moving average is computed.

| Time | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|------------------------|---|------|------|------|------|------|------|------|---|
| Data | 4 | 7 | 2 | 8 | 3 | 7 | 7 | 6 | 2 |
| CMA (<i>n</i> = 3) | | 4.33 | 5.67 | 4.33 | 6.00 | 5.67 | 6.67 | 5.00 | |

EMA =
$$\frac{p_1 + (1 - \alpha)p_2 + (1 - \alpha)^2 p_3 + (1 - \alpha)^3 p_4 + \dots}{1 + (1 - \alpha)^2 + (1 - \alpha)^2 + (1 - \alpha)^3 + \dots}$$

where

 λ = Smoothing constant such that 0 < λ < 1

 p_1 = Point in question

 p_2 = One point removed

 p_3 = Two points removed

 p_4 = Three points removed

÷

In addition to the EMA, there are double and triple exponential smoothing techniques. However, these won't be addressed here.

Decomposition. Decomposition is a method that permits the separation of a time series into trend and seasonal components. There are two general forms of decomposition:

• *Additive model*. The additive form is applicable when there is either no trend or a constant trend and there is a constant seasonal pattern. The general form of this model is

$$Y_t = T_t + S_t + \varepsilon_t$$

where

 Y_t = Actual observation at time t

 T_t = Trend component

 S_t = Seasonal component

 \mathcal{E}_{t} = Random error component

The trend component is sometimes referred to as the *trend-cycle effect*.

• *Multiplicative model*. The multiplicative form is applicable when there is either no trend or a constant trend and there is no constant seasonal pattern. The general form of this model is

$$Y_t = T_t S_t \varepsilon_t.$$

EXAMPLE 25.2

The data given in Table 25.4 are monthly trade-related data over a five-year period. Forecast twelve months using the decomposition method.

The first thing we must do is to determine the type of trend present. Figure 25.3 illustrates a linear trend, while Figure 25.4 illustrates a quadratic trend. A seasonal length of twelve appears to be appropriate. The Minitab session window for these two figures is given in Figure 25.5. A visual inspection of Figures 25.3 and 25.4 suggests that a quadratic trend might be more suitable. This is confirmed after comparing the accuracy measures shown in Figure 25.5. Also, the estimated trend component is provided in Figure 25.5.

Again, using Minitab (this time using its decomposition capability), we must determine whether to use an additive or multiplicative model. Since the trend is constant and seasonality appears to be constant, we will choose the additive model.

After making the appropriate dialog selections, a long session window is generated. This window has been placed into Figures 25.6 through 25.9.

Figure 25.6 provides the detrended trade data, predicted values, and forecasts. These are graphed and displayed in Figure 25.10. The accuracy measures are given in Figure 25.7 and are acceptable.

Figure 25.8 provides all the seasonal and deseasoned data necessary to generate the graphs shown in Figure 25.11. Note that the second chart in this figure indicates that slightly more than the first half of the first cycle is underpredicted. This is evident by the long decline trend in the first eight residuals. This is also confirmed by the "Residuals by Season" graph in Figure 25.12. Residuals for the forecasts are given in Figure 25.9. Figure 25.13 suggests that the variance may not be constant. Consequently, we might want to explore a more suitable predictive model.

The general steps to decomposition are relatively straightforward:

- 1. Prepare the data. In some instances, it may be necessary to normalize the data to account for differences in days/month, inflation, population, or other relevant factors.
- 2. Determine the type of trend (for example, linear, quadratic, exponential, S-curve). This can be used to determine whether to use an additive or multiplicative model form.
- 3. Estimate the trend component.
- 4. Detrend the data.
- 5. Determine the seasonal indices. This is commonly done using averages, but may also be performed using medians.
- 6. Forecast for a specified number of periods.

| Order | Trade | Order | Trade | Order | Trade |
|-------|-------|-------|-------|-------|-------|
| 1 | 322 | 21 | 335 | 41 | 362 |
| 2 | 317 | 22 | 338 | 42 | 367 |
| 3 | 319 | 23 | 342 | 43 | 366 |
| 4 | 323 | 24 | 348 | 44 | 370 |
| 5 | 327 | 25 | 330 | 45 | 371 |
| 6 | 328 | 26 | 326 | 46 | 375 |
| 7 | 325 | 27 | 329 | 47 | 380 |
| 8 | 326 | 28 | 337 | 48 | 385 |
| 9 | 330 | 29 | 345 | 49 | 361 |
| 10 | 334 | 30 | 350 | 50 | 354 |
| 11 | 337 | 31 | 351 | 51 | 357 |
| 12 | 341 | 32 | 354 | 52 | 367 |
| 13 | 322 | 33 | 355 | 53 | 376 |
| 14 | 318 | 34 | 357 | 54 | 381 |
| 15 | 320 | 35 | 362 | 55 | 381 |
| 16 | 326 | 36 | 368 | 56 | 383 |
| 17 | 332 | 37 | 348 | 57 | 384 |
| 18 | 334 | 38 | 345 | 58 | 387 |
| 19 | 335 | 39 | 349 | 59 | 392 |
| 20 | 336 | 40 | 355 | 60 | 396 |

Table 25.4Data for Example 25.2.

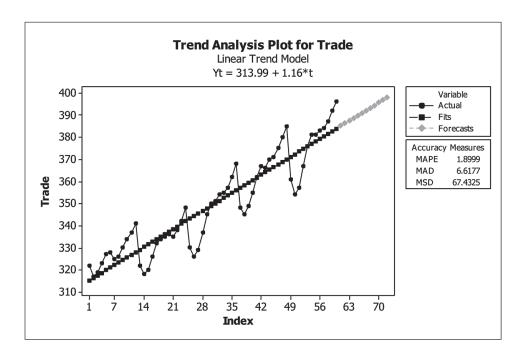


Figure 25.3 Trend analysis for Example 25.2 with a linear and additive trend model.

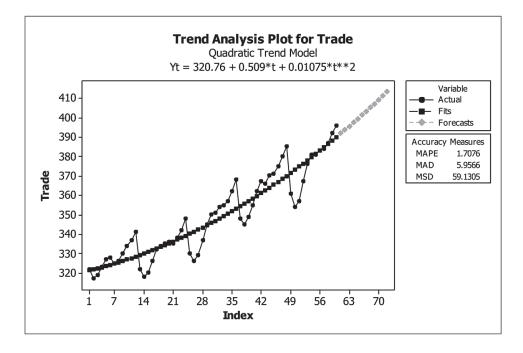


Figure 25.4 Trend analysis for Example 25.2 with a quadratic and additive trend model.

Part VI.B.1

Trend Analysis for Trade

Data Trade Length 60 NMissing 0 Fitted Trend Equation Yt = 313.99 + 1.16*t

Accuracy Measures

MAPE 1.8999 MAD 6.6177 MSD 67.4325

Trend Analysis for Trade

Data Trade Length 60 NMissing 0 Fitted Trend Equation Yt = 320.76 + 0.509*t + 0.01075*t**2 Accuracy Measures MAPE 1.7076 MAD 5.9566 MSD 59.1305

Figure 25.5 Decomposition analysis for Example 25.2. Trend analysis.

| Time | Trade | Trend | Detrend |
|--------|-------|---------|--------------------|
| 1 | 322 | 321.282 | 0.7179 |
| 2 | 317 | 321.824 | -4.8237 |
| 3 | 319 | 322.387 | -3.3868 |
| 4 | 323 | 322.971 | 0.0286 3.4225 |
| 5 | 327 | 323.577 | |
| 6 7 | 328 | 324.205 | 3.7949 |
| 8 | 325 | 324.854 | 0.1459 |
| | 326 | 325.525 | 0.4753 |
| 9 | 330 | 326.217 | 3.7833 |
| 10 | 334 | 326.930 | 7.0697 |
| 11 | 337 | 327.665 | 9.3347 |
| 12 | 341 | 328.422 | 12.5782 |
| 13 | 322 | 329.200 | -7.1998 |
| 14 | 318 | 329.999 | -11.9993 |
| 15 | 320 | 330.820 | -10.8203 |
| 16 | 326 | 331.663 | -5.6628 |
| 17 | 332 | 332.527 | -0.5268 |
| 18 | 334 | 333.412 | 0.5878 |
| 19 | 335 | 334.319 | 0.6808 |
| 20 | 336 | 335.248 | 0.7523 |
| 21 | 335 | 336.198 | -1.1976 |
| 22 | 338 | 337.169 | 0.8310 |
| 23 | 342 | 338.162 | 3.8381 |
| 24 | 348 | 339.176 | 8.8236 |
| 25 | 330 | 340.212 | -10.2123 |
| 26 | 326 | 341.270 | -15.2697 |
| 27 | 329 | 342.349 | -13.3485 |
| 28 | 337 | 343.449 | -6.4489 |
| 29 | 345 | 344.571 | 0.4292 |
| 30 | 350 | 345.714 | 4.2858 |
| 31 | 351 | 346.879 | 4.1210 |
| 32 | 354 | 348.065 | 5.9346 |
| 33 | 355 | 349.273 | 5.7268 |
| 34 | 357 | 350.503 | 6.4975 |
| 35 | 362 | 351.753 | 10.2467 |
| 36 | 368 | 353.026 | 14.9744 |
| 37 | 348 | 354.319 | -6.3194 |
| 38 | 345 | 355.635 | -10.6347 |
| 39 | 349 | 356.971 | -7.9715 -3.3298 |
| 40 | 355 | 358.330 | 2.2905 |
| 41 | 362 | 359.710 | |
| 42 | 367 | 361.111 | 5.8892 |
| 43 | 366 | 362.534 | 3.4665 |
| 44 | 370 | 363.978 | 6.0222 |
| 45 | 371 | 365.443 | 5.5565 |
| 46 | 375 | 366.931 | 8.0693 |
| 47 | 380 | 368.439 | 11.5606 |
| 48 | 385 | 369.970 | 15.0304 |
| 49 | 361 | 371.521 | -10.5213 |
| 50 | 354 | 373.094 | -19.0945 |
| 51 | 357 | 374.689 | -17.6892 |
| 52 | 367 | 376.305 | -9.3053 |
| 53 | 376 | 377.943 | -1.9430 |
| 54 | 381 | 379.602 | 1.3979 |
| 55 | 381 | 381.283 | -0.2828 |
| 56 | 383 | 382.985 | 0.0151 |
| 57 | 384 | 384.709 | -0.7085 |
| 58 | 387 | 386.454 | 0.5464 |
| 59 | 392 | 388.220 | 3.7798 |
| 60 | 396 | 390.008 | 5.9917 |

| Period | Forecast |
|--------|----------|
| 61 | 391.818 |
| 62 | 393.649 |
| 63 | 395.502 |
| 64 | 397.376 |
| 65 | 399.271 |
| 66 | 401.188 |

Forecasts

| 05 | 555.271 |
|----|---------|
| 66 | 401.188 |
| 67 | 403.127 |
| 68 | 405.087 |
| 69 | 407.068 |
| 70 | 409.071 |
| 71 | 411.096 |
| 72 | 413.142 |
| | |

| Figure 25.6 Decomposition analysis for Example 25.2. Detrended dat | ta. |
|--------------------------------------------------------------------|-----|
|--------------------------------------------------------------------|-----|

Time Series Decomposition for RESIDUALS

Additive Model Data RESIDUALS Length 60 NMissing 0 Seasonal Indices Period Index 1 -8.4826 2 -13.3368 3 -11.4410 4 -5.8160 5 0.5590 6 3.5590 7 1.7674 8 3.4757 9 3.2674 10 5.3924 8.4965 11 12.5590 12 Accuracy Measures MAPE 881.582 MAD 2.802 MSD 11.899

Figure 25.7 Decomposition analysis for Example 25.2. Decomposition of residuals.

| <u>.</u> |
|----------|
| |
| > |
| rt |
| Pa |

| Time | RESI | Trend | Seasonal | Detrend | Deseason | Predict | Error |
|------|----------|-----------|----------|----------|----------|----------|----------|
| 1 | 0.7179 | 0.0000000 | -8.4826 | 0.7179 | 9.20054 | -8.4826 | 9.20054 |
| 2 | -4.8237 | 0.0000000 | -13.3368 | -4.8237 | 8.51310 | -13.3368 | 8.51310 |
| 3 | -3.3868 | 0.0000000 | -11.4410 | -3.3868 | 8.05416 | -11.4410 | 8.05416 |
| 4 | 0.0286 | 0.0000000 | -5.8160 | 0.0286 | 5.84457 | -5.8160 | 5.84457 |
| 5 | 3.4225 | 0.0000000 | 0.5590 | 3.4225 | 2.86349 | 0.5590 | 2.86349 |
| 6 | 3.7949 | 0.0000000 | 3.5590 | 3.7949 | 0.23592 | 3.5590 | 0.23592 |
| 7 | 0.1459 | 0.0000000 | 1.7674 | 0.1459 | -1.62148 | 1.7674 | -1.62148 |
| 8 | 0.4753 | 0.0000000 | 3.4757 | 0.4753 | -3.00037 | 3.4757 | -3.00037 |
| 9 | 3.7833 | 0.0000000 | 3.2674 | 3.7833 | 0.51591 | 3.2674 | 0.51591 |
| 10 | 7.0697 | 0.0000000 | 5.3924 | 7.0697 | 1.67738 | 5.3924 | 1.67738 |
| 11 | 9.3347 | 0.0000000 | 8.4965 | 9.3347 | 0.83818 | 8.4965 | 0.83818 |
| 12 | 12.5782 | 0.0000000 | 12.5590 | 12.5782 | 0.01916 | 12.5590 | 0.01916 |
| 13 | -7.1998 | 0.0000000 | -8.4826 | -7.1998 | 1.28282 | -8.4826 | 1.28282 |
| 14 | -11.9993 | 0.0000000 | -13.3368 | -11.9993 | 1.33748 | -13.3368 | 1.33748 |
| 15 | -10.8203 | 0.0000000 | -11.4410 | -10.8203 | 0.62065 | -11.4410 | 0.62065 |
| 16 | -5.6628 | 0.0000000 | -5.8160 | -5.6628 | 0.15317 | -5.8160 | 0.15317 |
| 17 | -0.5268 | 0.0000000 | 0.5590 | -0.5268 | -1.08581 | 0.5590 | -1.08581 |
| 18 | 0.5878 | 0.0000000 | 3.5590 | 0.5878 | -2.97127 | 3.5590 | -2.97127 |
| 19 | 0.6808 | 0.0000000 | 1.7674 | 0.6808 | -1.08657 | 1.7674 | -1.08657 |
| 20 | 0.7523 | 0.0000000 | 3.4757 | 0.7523 | -2.72335 | 3.4757 | -2.72335 |
| 21 | -1.1976 | 0.0000000 | 3.2674 | -1.1976 | -4.46496 | 3.2674 | -4.46496 |
| 22 | 0.8310 | 0.0000000 | 5.3924 | 0.8310 | -4.56139 | 5.3924 | -4.56139 |
| 23 | 3.8381 | 0.0000000 | 8.4965 | 3.8381 | -4.65848 | 8.4965 | -4.65848 |
| 24 | 8.8236 | 0.0000000 | 12.5590 | 8.8236 | -3.73539 | 12.5590 | -3.73539 |
| 25 | -10.2123 | 0.0000000 | -8.4826 | -10.2123 | -1.72963 | -8.4826 | -1.72963 |
| 26 | -15.2697 | 0.0000000 | -13.3368 | -15.2697 | -1.93286 | -13.3368 | -1.93286 |
| 27 | -13.3485 | 0.0000000 | -11.4410 | -13.3485 | -1.90758 | -11.4410 | -1.90758 |
| 28 | -6.4489 | 0.0000000 | -5.8160 | -6.4489 | -0.63296 | -5.8160 | -0.63296 |
| 29 | 0.4292 | 0.0000000 | 0.5590 | 0.4292 | -0.12982 | 0.5590 | -0.12982 |
| 30 | 4.2858 | 0.0000000 | 3.5590 | 4.2858 | 0.72682 | 3.5590 | 0.72682 |
| 31 | 4.1210 | 0.0000000 | 1.7674 | 4.1210 | 2.35363 | 1.7674 | 2.35363 |
| 32 | 5.9346 | 0.0000000 | 3.4757 | 5.9346 | 2.45896 | 3.4757 | 2.45896 |
| 33 | 5.7268 | 0.0000000 | 3.2674 | 5.7268 | 2.45945 | 3.2674 | 2.45945 |
| 34 | 6.4975 | 0.0000000 | 5.3924 | 6.4975 | 1.10513 | 5.3924 | 1.10513 |
| 35 | 10.2467 | 0.0000000 | 8.4965 | 10.2467 | 1.75015 | 8.4965 | 1.75015 |
| 36 | 14.9744 | 0.0000000 | 12.5590 | 14.9744 | 2.41534 | 12.5590 | 2.41534 |
| 37 | -6.3194 | 0.0000000 | -8.4826 | -6.3194 | 2.16321 | -8.4826 | 2.16321 |
| 38 | -10.6347 | 0.0000000 | -13.3368 | -10.6347 | 2.70209 | -13.3368 | 2.70209 |
| 39 | -7.9715 | 0.0000000 | -11.4410 | -7.9715 | 3.46947 | -11.4410 | 3.46947 |
| 40 | -3.3298 | 0.0000000 | -5.8160 | -3.3298 | 2.48620 | -5.8160 | 2.48620 |
| 41 | 2.2905 | 0.0000000 | 0.5590 | 2.2905 | 1.73144 | 0.5590 | 1.73144 |
| 42 | 5.8892 | 0.0000000 | 3.5590 | 5.8892 | 2.33019 | 3.5590 | 2.33019 |
| 43 | 3.4665 | 0.0000000 | 1.7674 | 3.4665 | 1.69911 | 1.7674 | 1.69911 |
| 44 | 6.0222 | 0.0000000 | 3.4757 | 6.0222 | 2.54654 | 3.4757 | 2.54654 |
| 45 | 5.5565 | 0.0000000 | 3.2674 | 5.5565 | 2.28915 | 3.2674 | 2.28915 |
| 46 | 8.0693 | 0.0000000 | 5.3924 | 8.0693 | 2.67693 | 5.3924 | 2.67693 |
| 47 | 11.5606 | 0.0000000 | 8.4965 | 11.5606 | 3.06405 | 8.4965 | 3.06405 |
| 48 | 15.0304 | 0.0000000 | 12.5590 | 15.0304 | 2.47135 | 12.5590 | 2.47135 |
| 49 | -10.5213 | 0.0000000 | -8.4826 | -10.5213 | -2.03867 | -8.4826 | -2.03867 |
| 50 | -19.0945 | 0.0000000 | -13.3368 | -19.0945 | -5.75769 | -13.3368 | -5.75769 |
| 51 | -17.6892 | 0.0000000 | -11.4410 | -17.6892 | -6.24820 | -11.4410 | -6.24820 |
| 52 | -9.3053 | 0.0000000 | -5.8160 | -9.3053 | -3.48936 | -5.8160 | -3.48936 |
| 53 | -1.9430 | 0.0000000 | 0.5590 | -1.9430 | -2.50202 | 0.5590 | -2.50202 |
| 54 | 1.3979 | 0.0000000 | 3.5590 | 1.3979 | -2.16116 | 3.5590 | -2.16116 |
| 55 | -0.2828 | 0.0000000 | 1.7674 | -0.2828 | -2.05013 | 1.7674 | -2.05013 |
| 56 | 0.0151 | 0.0000000 | 3.4757 | 0.0151 | -3.46060 | 3.4757 | -3.46060 |
| 57 | -0.7085 | 0.0000000 | 3.2674 | -0.7085 | -3.97588 | 3.2674 | -3.97588 |
| 58 | 0.5464 | 0.0000000 | 5.3924 | 0.5464 | -4.84599 | 5.3924 | -4.84599 |
| 59 | 3.7798 | 0.0000000 | 8.4965 | 3.7798 | -4.71676 | 8.4965 | -4.71676 |
| 60 | 5.9917 | 0.0000000 | 12.5590 | 5.9917 | -6.56736 | 12.5590 | -6.56736 |
| | | | | | | | |
| | | | | | | | |

| Figure 25.8 | Decomposition analysis for Example 25.2. Adjusted data. |
|-------------|---------------------------------------------------------|
| | |

Residuals

Forecasts

| Period | Forecast |
|--------|----------|
| 61 | -8.4826 |
| 62 | -13.3368 |
| 63 | -11.4410 |
| 64 | -5.8160 |
| 65 | 0.5590 |
| 66 | 3.5590 |
| 67 | 1.7674 |
| 68 | 3.4757 |
| 69 | 3.2674 |
| 70 | 5.3924 |
| 71 | 8.4965 |
| 72 | 12.5590 |
| | |

Figure 25.9 Decomposition analysis for Example 25.2. Forecasts for residuals.

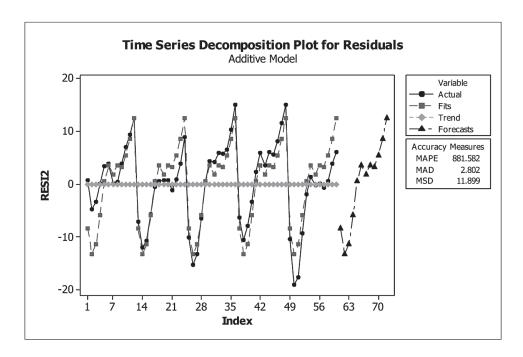


Figure 25.10 Decomposition analysis for Example 25.2. Time series decomposition.

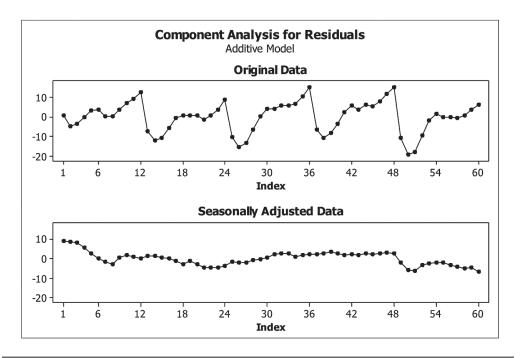


Figure 25.11 Decomposition analysis for Example 25.2. Component analysis for residuals.

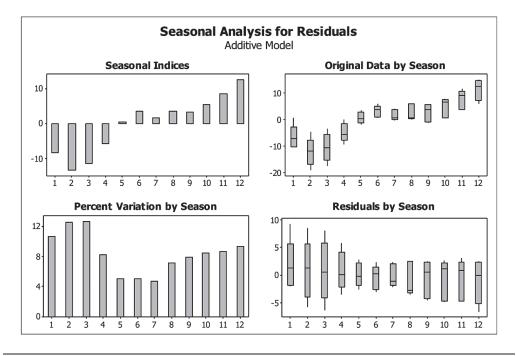


Figure 25.12 Decomposition analysis for Example 25.2. Seasonal analysis for residuals.

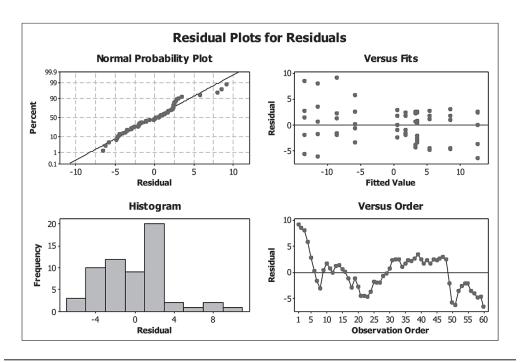


Figure 25.13 Decomposition analysis for Example 25.2. Residual plots.

Autoregressive Integrated Moving Average (ARIMA). ARIMA models are the most general class of models that can be used for forecasting a time series. The taxonomy for ARIMA models is ARIMA(p, d, q) where

p = Number of autoregressive terms

- *d* = Number of nonseasonal differences
- q = Number of lagged forecast errors in the prediction equation

Consider the following form of an ARIMA(*p*, *d*, *q*):

$$\hat{y}_{T+\tau} = \delta + \sum_{i=1}^{p+d} \phi_i y_{T+\tau-i} + \varepsilon_{t+\tau} - \sum_{i=1}^{q} \theta_i \varepsilon_{T+\tau-i}$$

where

 τ = Number of predicted time periods

 δ = Constant in the prediction equation

- ϕ = Coefficients of the prediction equation autoregressive terms
- θ = Coefficients of the lagged forecast error terms

The above time series needed to be differenced to be stationary. As such, it is said to be an "integrated" version of a stationary series.

EXAMPLE 25.3

Recall the monthly food data given in Table 25.1. Develop an ARIMA(1).

Using Minitab, we first generate the ACF and partial ACF (PACF) of residuals. These are depicted in Figures 25.14 and 25.15, and the session window is shown in Figure 25.16. A review of these figures indicates that the AR(1) coefficient is significantly different from zero, and all the Ljung-Box statistics show the *p*-values to not be significant, indicating that the residuals appear to be uncorrelated. The residuals for lag 9 in both Figures 25.14 and 25.15 are likely due to random error. The time-series plot for the food data is illustrated in Figure 25.17. Confidence limits are shown for predicted values. Strong seasonality continues to be present in the forecast, and the overall forecast values are higher than the previous month. Figure 25.18 provides the residual plots. Taken together, the "Versus Fits" and "Versus Order" plots suggest that this model could benefit from some additional tuning.

MULTIPLE REGRESSION ANALYSIS

Apply and interpret multiple regression analysis, including using variance inflation factors (VIFs) to identify collinearity issues. (Apply)

Body of Knowledge VI.B.2

Regression analysis is a technique that typically uses continuous predictor variable(s) (that is, regressor variables) to predict the variation in a continuous response variable. Regression analysis uses the method of least squares to determine the values of the linear regression coefficients and the corresponding model.

The *method of least squares* is a technique for estimating a parameter that minimizes the sum of the squared differences between the observed and the predicted values derived from the model. These differences are known as *residuals* or *experimental errors*. Experimental errors associated with individual observations are assumed independent and normally distributed. It should be noted that analysis of variance, regression analysis, and analysis of covariance are all based on the method of least squares.

The *linear regression equation* is a mathematical function or model that indicates the linear relationship between a set of predictor variables and a response variable. *Linear regression coefficients* are numbers associated with each predictor variable in a linear regression equation that tell how the response variable changes with each unit increase in the predictor variable. The model also gives some sense of the degree of linearity present in the data.

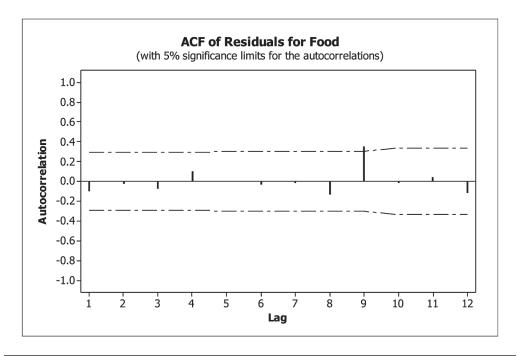


Figure 25.14 ACF of residuals for Example 25.3.

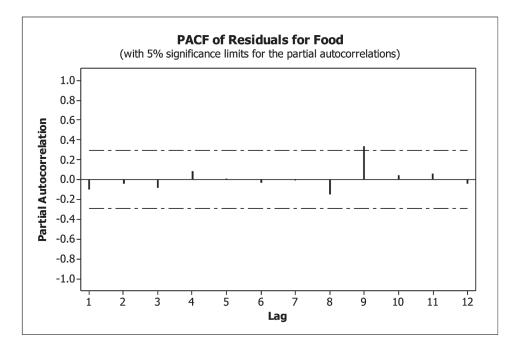


Figure 25.15 PACF of residuals for Example 25.3.

ARIMA Model: Food

Estimates at each iteration Iteration SSE Parameters 0 95.2343 0.100 0.847 1 77.5568 0.250 0.702 2 64.5317 0.400 0.556 3 56.1578 0.550 0.410 4 52.4345 0.700 0.261 52.2226 0.733 0.216 5 6 52.2100 0.741 0.203 7 52.2092 0.743 0.201 8 52.2092 0.743 0.200 9 52.2092 0.743 0.200 Relative change in each estimate less than 0.0010 Final Estimates of Parameters Туре Coef SE Coef Т Ρ 0.1001 7.42 0.000 AR 1 0.7434 0.1520 1.31 0.196 Constant 0.1996 Differencing: 0 regular, 1 seasonal of order 12 Number of observations: Original series 60, after differencing 48 SS = 51.0364 (backforecasts excluded) MS = 1.1095 DF = 46 Residuals: Modified Box-Pierce (Ljung-Box) Chi-Square statistic 12 24 Laq 36 48 * Chi-Square 11.3 19.1 27.7 10 * DF 22 34 * P-Value 0.338 0.641 0.768 Forecasts from period 60 95% Limits Period Forecast Lower Upper Actual 56.4121 54.3472 58.4770 61 55.5981 53.0251 58.1711 62 63 55.8390 53.0243 58.6537 64 55.4207 52.4809 58.3605 65 55.8328 52.8261 58.8394 66 59.0674 56.0244 62.1104 67 69.0188 65.9559 72.0817 74.1827 71.1089 77.2565 68 69 76.3558 73.2760 79.4357 70 67.2359 64.1527 70.3191 71 61.3210 58.2360 64.4060 72 58.5100 55.4240 61.5960



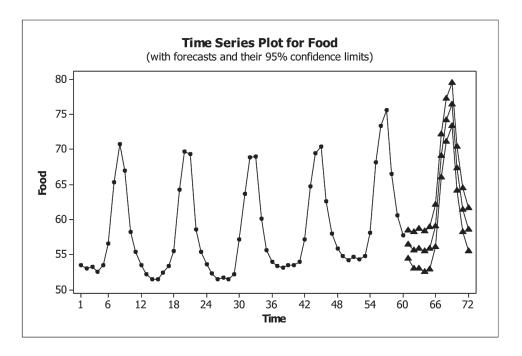


Figure 25.17 Time-series plot for Example 25.3 with forecasts.

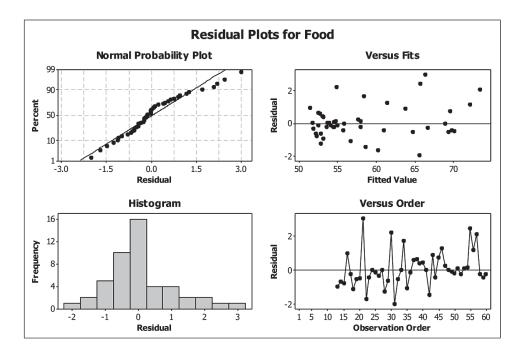


Figure 25.18 Residual plots for Example 25.3.

Multiple Linear Regression

When multiple predictor variables are used, regression is referred to as *multiple linear regression*. For example, a multiple linear regression model with *k* predictor variables uses a linear regression equation expressed as

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_k x_k + \varepsilon$$

where

Y is the response

 x_1, x_2, \dots, x_k are the values of the predictor variables

 $\beta_0, \beta_1, \beta_2, \dots, \beta_k$ are the linear regression coefficients

 ε is the error term

The random error terms in regression analysis are assumed to be independent and normally distributed with a zero mean and constant variance. These assumptions can be readily checked through residual analysis or residual plots. Note: Other regression techniques are available when the input and/or output variables are not continuous.

Unlike simple linear regression, multiple linear regression is extraordinarily tedious. Fortunately, statistical software packages make the drudgework simple. Consequently, we will skip the lengthy formulas. The interested reader should refer to a text such as Montgomery and Runger (2006) for the fascinating details. Instead, we will focus our attention on key measures that help us develop and refine models. Although we will use Minitab as a basis, most of these measures are common to most software packages.

Multicollinearity

Multicollinearity occurs when two or more predictor variables in a multiple regression model are correlated. Multicollinearity may cause the coefficient estimates and significance tests for each predictor variable to be underestimated and difficult to interpret. Note: The presence of multicollinearity does not reduce the predictive capability of the model—only calculations regarding individual predictor variables. However, including correlated variables does add to the overall model complexity and will likely increase the cost of data collection, perhaps needlessly.

The most effective way to reduce multicollinearity is to remove highly correlated predictor variables from the model. The "best subsets" approach is very helpful in this regard.

Adjusted R²

Recall that the *coefficient of determination*, R^2 , indicates how much variation in the response variable is accounted for by the regression model. It is computed as follows:

$$R^{2} = 1 - \frac{\sum_{i=1}^{n} (y_{i} - \hat{y}_{i})^{2}}{\sum_{i=1}^{n} (y_{i} - \overline{y}_{i})^{2}}$$

where

n = Number of observations

 $y_i = i$ th observed response value

 \overline{y}_i = Mean observed response value

 $\hat{y}_i = i$ th fitted value

By contrast, the adjusted R^2 coefficient of determination takes into consideration the number of predictor variables in the model and is helpful when comparing models with a different number of predictor variables. It is computed as follows:

$$R_{adj}^{2} = 1 - \left(\frac{\sum_{i=1}^{n} (y_{i} - \hat{y}_{i})^{2}}{\left(\sum_{i=1}^{n} y_{i} - \overline{y}\right)^{2}}\right) \left(\frac{n-1}{n-p-1}\right)$$

where

n = Number of observations

 $y_i = i$ th observed response value

 \overline{y}_i = Mean observed response value

 $\hat{y}_i = i$ th fitted value

p = Number of terms in the model including the constant term.

Notice when p = 2 (that is, simple linear regression), R_{adj}^2 equals the adjusted R^2 .

Variance Inflation Factor (VIF)

The *variance inflation factor* measures the increase in the variance of a regression coefficient when predictor variables are correlated. The formula for the VIF is given by

$$\operatorname{VIF}(B_{j}) = \frac{1}{1 - R_{j}^{2}}; j = 1, 2, \dots, k$$

where R_j^2 is the coefficient of multiple determination. It is determined by regressing the predictor variable, x_j on the other remaining k - 1 predictor variables. Consequently, when R_j^2 is large, VIF(β_j) is also large. This causes the variances of the coefficients to become inflated. The following rules of thumb are useful for interpreting and acting on the VIF:

- VIF = 1. No correlation exists between the predictor variables.
- 1 < VIF < 5. Moderate correlation exists between the predictor variables.
- $5 \le \text{VIF} \le 10$. High correlation exists between the predictor variables.
- VIF > 10. Very high correlation exists between the predictor variables.

Minitab recommends that for VIF > 10, reduce the multicollinearity by removing unnecessary predictor variables from the model. However, from a practical standpoint, VIF ≥ 5 might be more appropriate.

Prediction Error Sum of Squares (PRESS)

The PRESS statistic is used to assess the fit of a regression model for a given set of observations that were not used to estimate the model's parameters. The smaller the PRESS value, the better. The PRESS statistic is computed as

$$PRESS = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

where

 $y_i = i$ th observed value

 $\hat{y}_i = i$ th predicted value based on a model fitted to the remaining n-1 points

n = Number of observations

Mallows's C_p

This statistic is used to measure the goodness of the prediction. It is computed by using

Mallows's
$$C_p = \frac{SSE_p}{MSE_m} - (n-2p)$$

where

 SSE_{p} = Sum of the squared error for the model under consideration

 MSE_m = Mean squared error for the model with all predictors included

n = Number of observations

p = Number of terms in the model including the constant term

One should look for a Mallows's C_p value that is close to the value of p. A small C_p value indicates the model is more precise in estimating both regression coefficients and predicting future values.

Best Subsets

Best subsets is a very useful tool to help select a regression model among many other viable models. It is based on the concept that for a model comprising *m* predictor variables, there are $2^m - 1$ possible subsets of predictor variables possible. For example, consider a model with *m* = 4; there would be 15 possible subsets. The permutations would be:

- $P_1^4 = 4$
- $P_2^4 = 6$
- $P_3^4 = 4$
- $P_4^4 = 1$

The last six columns of Table 25.5 illustrate the best subset. For example, model 5 is composed of variables 1, 4, and 6, while model 7 is composed of variables 1, 2, 4, and 6. Minitab presents the top two models with one variable in R_{adj}^2 sequence, then the top two models with two variables in R_{adj}^2 sequence, and so on. In the case of model 11, there was only one model with all six variables in it.

EXAMPLE 25.4

Given the data in Tables 25.5 and 25.6, select the best model and confirm Mallows's C_p . Assume n = 54.

A review of Table 25.5 indicates there are two viable models to consider based on both the R_{adj}^2 and C_p values. These would include models 5 and 7. Model 5 has a slightly lower R_{adj}^2 , but a higher C_p than model 7. However, it contains only three predictor variables. By contrast, model 7 contains four predictor variables, three of which are the same as model 5. To err in the favor of simplicity, one would likely select model 5.

To compute the C_p value for model 5, we apply the formula from above:

$$C_{p} = \frac{45698}{917} - (54 - 2(4))$$
$$= 49.83 - 46$$
$$= 3.83$$
$$= 3.8$$

This matches the C_p value for model 5 found in Table 25.5.

| Model # | Vars | R-Sq | R-Sq (adj) | C _p | s | Var 1 | Var 2 | Var 3 | Var 4 | Var 5 | Var 6 |
|------------|------|------|---------------|----------------|--------|----------|----------|----------|----------|----------|----------|
| 1 | 1 | 92.8 | 92.6 | 12.1 | 33.078 | | | | | | X |
| 2 | 1 | 87.2 | 87.0 | 59.4 | 43.913 | | | | X | | |
| 3 | 2 | 93.7 | 93.4 | 6.3 | 31.252 | | | | X | | X |
| 4 | 2 | 93.6 | 93.3 | 7.1 | 31.475 | Х | | | | | X |
| 5 | 3 | 94.2 | 93.8 | 3.8 | 30.232 | Х | | | Х | | X |
| 6 | 3 | 93.8 | 93.4 | 7.1 | 31.194 | | Х | | Х | | Х |
| 7 | 4 | 94.5 | 94.0 | 3.3 | 29.741 | Х | Х | | Х | | Х |
| 8 | 4 | 94.2 | 93.8 | 5.5 | 30.419 | Х | | | Х | Х | Х |
| 9 | 5 | 94.5 | 93.9 | 5.0 | 29.974 | Х | Х | | Х | Х | Х |
| 10 | 5 | 94.5 | 93.9 | 5.2 | 30.036 | Х | Х | Х | Х | | Х |
| 11 | 6 | 94.5 | 93.8 | 7 | 30.277 | Х | Х | Х | Х | Х | Х |

Table 25.5 Data for Example 25.4.

Table 25.6Data for Example 25.4.

| | | Regression wit | h all predictors | | |
|----------------|----|-------------------|------------------|--------|-------|
| Source | DF | SS | MS | F | Р |
| Regression | 6 | 743197 | 123866 | 135.12 | 0.000 |
| Residual error | 47 | 43086 | 917 | | |
| Total | 53 | 786283 | | | |
| | I | Regression for th | e specific mode | el | |
| Source | DF | SS | MS | F | Р |
| Regression | 3 | 740586 | 246862 | 270.10 | 0.000 |
| Residual error | 50 | 45698 | 914 | | |
| Total | 53 | 786283 | | | |

Worked Examples

Due to the tediousness of the calculations involved in computing the coefficients in multiple regression, software programs are often employed. Two examples will be worked using Minitab. The first example will construct a multiple regression, and the second will demonstrate the use of best subsets.

EXAMPLE 25.5

The data in Table 25.7 were collected to determine whether heat flux can be predicted from wind direction. The response variable is in the first column and wind direction is given in columns 3–5. Construct a multiple linear regression model. Use $\alpha = 0.05$.

Using Minitab, we generate Figure 25.19. A review of this figure indicates the regression is significant given the low *p*-value. Only the South and North predictor variables are significant, and their corresponding VIF values are very low. This indicates almost no correlation between the predictor variables. The R_{adj}^2 is reasonably high, suggesting that the model fits quite well. The predicted R_{adj}^2 is also high and close to the R_{adj}^2 , indicating that the model does not overfit and has a reasonable predictive capability.

Two residual observations stand out. These are 4 and 22. These are evident at the bottom of Figure 25.19, in the top right of Figure 25.20, and in Figure 25.21. Standard-ized residuals greater than 2 or less than –2 are considered large. They may be outliers. However, it is also appropriate to go back and recheck the data for correctness. Figure 25.22 displays the standardized residuals versus the fitted responses. As the fitted values increase, the standardized residuals begin collecting toward the zero line. This may be an indication that the residuals lack a constant variance.

EXAMPLE 25.6

The data in Table 25.7 were collected to determine whether heat flux can be predicted from insulation, wind direction, and time. The response variable is in the first column, insulation is in the second column, wind direction is given in columns 3–5, and time is provided in column 6. Identify a model from best subsets using all five predictor variables. Using Minitab, we generate Figure 25.23. A brief review of this figure indicates that selecting a model isn't particularly clear. The model with five variables has the highest R_{adj}^2 , the second lowest C_{pr} and the lowest MSE. The best (that is, first) model with four variables has a slightly lower R_{adj}^2 and C_p , but also a slightly higher MSE. However, the model contains the East predictor variable, which was shown in Example 25.5 to not be significant. The next best model to pursue would be the second model with four variables since it has the highest R_{adj}^2 , the lowest C_p , and the lowest MSE of the remaining models.

However, before selecting a final model, we would want to verify that none of the model assumptions regarding residuals are violated. This can be accomplished using Minitab's various residual plots.

The importance of this example is that it demonstrates that real-life solutions are not always readily evident.

| Table 25.7 Date | ita for Examples 2 | 5.5 and 25.6. | | | |
|-------------------------|--------------------|---------------|-------|-------|-------|
| Heat flux | Insulation | East | South | North | Time |
| 271.8 | 783.35 | 33.53 | 40.55 | 16.66 | 13.20 |
| 264.0 | 748.45 | 36.50 | 36.19 | 16.46 | 14.11 |
| 238.8 | 684.45 | 34.66 | 37.31 | 17.66 | 15.68 |
| 230.7 | 827.80 | 33.13 | 32.52 | 17.50 | 10.53 |
| 251.6 | 860.45 | 35.75 | 33.71 | 16.40 | 11.00 |
| 257.9 | 875.15 | 34.46 | 34.14 | 16.28 | 11.31 |
| 263.9 | 909.45 | 34.60 | 34.85 | 16.06 | 11.96 |
| 266.5 | 905.55 | 35.38 | 35.89 | 15.93 | 12.58 |
| 229.1 | 756.00 | 35.85 | 33.53 | 16.60 | 10.66 |
| 239.3 | 769.35 | 35.68 | 33.79 | 16.41 | 10.85 |
| 258.0 | 793.50 | 35.35 | 34.72 | 16.17 | 11.41 |
| 257.6 | 801.65 | 35.04 | 35.22 | 15.92 | 11.91 |
| 267.3 | 819.65 | 34.07 | 36.50 | 16.04 | 12.85 |
| 267.0 | 808.55 | 32.20 | 37.60 | 16.19 | 13.58 |
| 259.6 | 774.95 | 34.32 | 37.89 | 16.62 | 14.21 |
| 240.4 | 711.85 | 31.08 | 37.71 | 17.37 | 15.56 |
| 227.2 | 694.85 | 35.73 | 37.00 | 18.12 | 15.83 |
| 196.0 | 638.10 | 34.11 | 36.76 | 18.53 | 16.41 |
| 278.7 | 774.55 | 34.79 | 34.62 | 15.54 | 13.10 |
| 272.3 | 757.90 | 35.77 | 35.40 | 15.70 | 13.63 |
| 267.4 | 753.35 | 36.44 | 35.96 | 16.45 | 14.51 |
| 254.5 | 704.70 | 37.82 | 36.26 | 17.62 | 15.38 |
| 224.7 | 666.80 | 35.07 | 36.34 | 18.12 | 16.10 |
| 181.5 | 568.55 | 35.26 | 35.90 | 19.05 | 16.73 |
| 227.5 | 653.10 | 35.56 | 31.84 | 16.51 | 10.58 |
| 253.6 | 704.05 | 35.73 | 33.16 | 16.02 | 11.28 |
| 263.0 | 709.60 | 36.46 | 33.83 | 15.89 | 11.91 |
| 265.8 | 726.90 | 36.26 | 34.89 | 15.83 | 12.65 |
| 263.8 | 697.15 | 37.20 | 36.27 | 16.71 | 14.06 |

Table 25.7Data for Examples 25.5 and 25.6.

Regression Analysis: HeatFlux versus East, South, North

The regression equation is HeatFlux = 389 + 2.12 East + 5.32 South - 24.1 North Predictor Coef SE Coef Т Р VIF Constant 389.17 66.09 5.89 0.000 2.125 1.214 1.75 0.092 1.122 East
 5.3185
 0.9629
 5.52
 0.000
 1.226

 -24.132
 1.869
 -12.92
 0.000
 1.091
 South North S = 8.59782 R-Sq = 87.4% R-Sq(adj) = 85.9% PRESS = 3089.67 R-Sq(pred) = 78.96% Analysis of Variance DF Source SS MS F Ρ Regression 3 12833.9 4278.0 57.87 0.000 Residual Error 25 1848.1 73.9 Total 28 14681.9 Source DF Seq SS East 1 153.8 South 1 349.5 North 1 12330.6 Unusual Observations Obs East HeatFlux Fit SE Fit Residual St Resid 433.1230.70210.205.0320.502.94R2237.8254.50237.164.2417.342.32R 22 37.8 R denotes an observation with a large standardized residual.

Figure 25.19 Minitab session window for Example 25.5.

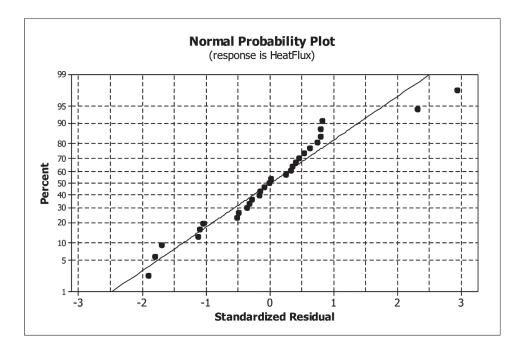


Figure 25.20 Normal probability plot for Example 25.5.

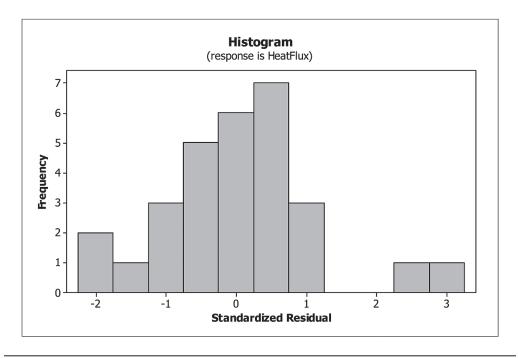


Figure 25.21 Histogram plot of standardized residuals for Example 25.5.

Part VI.B.2

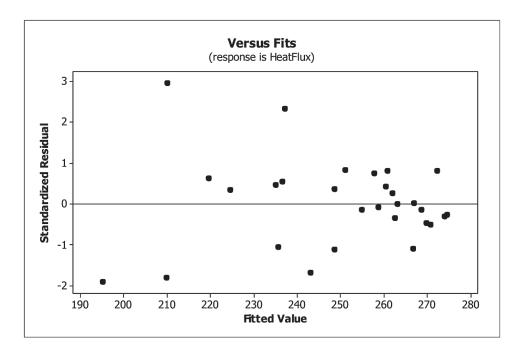


Figure 25.22 Plot of standardized residuals versus fits for Example 25.5.

Response is HeatFlux

| | | | | | I | | | | |
|------|------|-----------|---------|--------|---|-----|---|---|---|
| | | | | | n | | | | |
| | | | | | s | | | | |
| | | | | | u | | | | |
| | | | | | 1 | | | | |
| | | | | | а | | S | Ν | |
| | | | | | t | Е | 0 | 0 | т |
| | | | | | i | a | u | r | i |
| | | | Mallows | | 0 | s | t | t | m |
| Vars | R-Sq | R-Sq(adj) | Ср | S | n | t | h | h | е |
| 1 | 72.1 | 71.0 | 38.5 | 12.328 | | | | Х | |
| 1 | 39.4 | 37.1 | 112.7 | 18.154 | Х | | | | |
| 2 | 85.9 | 84.8 | 9.1 | 8.9321 | | | х | х | |
| 2 | 82.0 | 80.6 | 17.8 | 10.076 | | | | | Х |
| 3 | 87.4 | 85.9 | 7.6 | 8.5978 | | x | Х | | |
| 3 | 86.5 | 84.9 | 9.7 | 8.9110 | Х | ••• | X | | |
| 4 | 89.1 | 87.3 | 5.8 | 8.1698 | | x | X | | |
| 4 | 88.0 | 86.0 | 8.2 | | X | Δ | | X | x |
| 5 | 89.9 | 87.7 | 6.0 | 8.0390 | | v | | X | |
| 5 | 09.9 | 01.1 | 0.0 | 0.0390 | Λ | Λ | Λ | Λ | Λ |

Figure 25.23 Minitab session window for best subsets for Example 25.6.

LOGISTIC REGRESSION ANALYSIS

Apply and interpret logistic regression analysis, including binary, ordinal, and nominal data considerations. (Apply)

Body of Knowledge VI.B.3

In Kubiak and Benbow (2009) and in the previous subsection, we looked at simple and multiple linear regression analysis, respectively. Though commonly used, these techniques are only applicable when dealing with continuous variables. When the response variables are discrete, we must resort to the following types of *logistic regression*:

- *Binary logistic regression*. The response variable for this type of regression has two distinct categories usually of the following type: yes/no, on/off, and so forth.
- *Nominal logistic regression.* The response variable for this type of regression has three or more distinct categories, but with no natural ordering. Such categories might include math/science/arts, apparel/ tools/automotive, and so forth.
- *Ordinal logistic regression.* The response variable for this type of regression has three or more distinct categories, but with a natural ordering. Such categories might include survival level 1/level 2/ level 3, first grade/second grade/third grade, and so forth.

Recall that with linear regression an equation is derived that permits the prediction of an exact response variable value. However, logistic regression provides probabilities for each level of the response variable by finding a linear relationship between the predictor and response variables using a link function.

A *link function* transforms probabilities of a response variable from the closed interval, (0,1), to a continuous scale that is unbounded. Once the transformation is complete, the relationship between the predictor and response variable can be modeled with linear regression. Three types of link functions are commonly used:

• Logit. The logit link function is of the form

$$g(p_i) = \log\left(\frac{p_i}{1-p_i}\right)$$

• Normit or probit. The normit, or probit, link function is of the form

$$g(p_i) = \Phi^{-1}(p_i)$$

• *Gompit*. The gompit link function is of the form

$$g(p_i) = \log\left[-\log(1-p_i)\right]$$

In these link functions, we have

 p_i = Probabilities associated with each response variable

 Φ^{-1} = Inverse cumulative standard normal distribution

Link functions should be chosen to provide the best goodness-of-fit test results. Each of these tools will now be explored through the following examples.

Although more complicated to perform and interpret, logistic regressions permit us to develop models that fit categorical regressor and response variables, thus overcoming the limitation inherent in simple and multiple regression analyses.

EXAMPLE 25.7

Using the resting pulse data collected in Table 25.8, perform a binary logistic regression analysis using Minitab. Use $\alpha = 0.05$.

Choose *Stat* > *Regression* > *Binary Logistic Regression*. Select "Resting Pulse" for *Response*. Select "Smokes" and "Weight" for *Model*. Select "Delta chi-square vs. probability" and "Delta chi-square vs. leverage" in the *Graphs* dialog box. Select "In addition, list of factor level values, tests for terms with more than 1 degree of freedom, and 2 additional goodness-of-fit tests" in the *Results* dialog box. The resulting analysis is depicted in Figures 25.24–25.26.

From Figure 25.24, we see that 70 individuals had a low resting pulse, while 22 were high. The logistic regression table indicates that the regression coefficients are not 0, due to the low p-values. However, the odds ratio for weight is close to 1, indicating that a 1-pound increase in weight has little practical effect on the resting pulse. The coefficient of -1.19297, along with an odds ratio of 0.3, indicates that individuals who smoke tend to have a higher resting pulse than individuals who don't smoke. Alternately, the odds ratio can be interpreted as the odds of smokers having 30% of the odds of nonsmokers having a low pulse for the same weight. The log-likelihood test indicates that there is sufficient evidence to conclude that at least one coefficient is not zero. Note that the *p*-value is 0.023. All of the goodness-of-fit tests have *p*-values much greater than $\alpha = 0.05$, indicating that we should accept the null hypothesis that the model fits the data adequately. This is confirmed by observing the similarities of the observed and expected frequencies. The values for the summary measures range between 0 and 1. The higher the value, the better the predictability of the model. The highest summary measure value is 0.39, indicating that despite the adequacy of the model fit, it offers a low level of predictability.

Taken together, Figures 25.25 and 25.26 indicate that the model does not fit two observations very well. These two observations are evident by their high delta chi-square values.

| Resting pulseResting smokes1LowNo2LowNo3LowYes4LowYes5LowNo6LowNo7HighNo8LowNo9LowNo10LowNo11HighYes | | | | | | | | | | | | | |
|------------------------------------------------------------------------------------------------------------|--------|--------|------------------|--------|--------|--------|------------------|--------|--------|--------|------------------|--------|--------|
| | Weight | Number | Resting pulse | Smokes | Weight | Number | Resting pulse | Smokes | Weight | Number | Resting pulse | Smokes | Weight |
| Low Low Low Low Low High Low Low Low How High | 140 | 24 | Low | Yes | 185 | 47 | Low | No | 150 | 70 | Low | No | 120 |
| Low Low Low High Low Low Low Low High | 145 | 25 | High | No | 140 | 48 | High | Yes | 180 | 71 | Low | No | 130 |
| Low Low High Low Low Low High | 160 | 26 | Low | No | 120 | 49 | Low | No | 160 | 72 | High | Yes | 131 |
| Low Low High Low Low Low High | 190 | 27 | Low | Yes | 130 | 50 | Low | No | 135 | 73 | Low | No | 120 |
| Low High Low Low High | 155 | 28 | High | No | 138 | 51 | Low | No | 160 | 74 | Low | No | 118 |
| High Low Low High | 165 | 29 | High | Yes | 121 | 52 | Low | Yes | 130 | 75 | Low | No | 125 |
| Low Low Low High | 150 | 30 | Low | No | 125 | 53 | Low | Yes | 155 | 76 | High | Yes | 135 |
| Low Low High | 190 | 31 | High | No | 116 | 54 | Low | Yes | 150 | 77 | Low | No | 125 |
| Low High | 195 | 32 | Low | No | 145 | 55 | Low | No | 148 | 78 | High | No | 118 |
| High | 138 | 33 | High | Yes | 150 | 56 | High | No | 155 | 79 | Low | No | 122 |
| | 160 | 34 | Low | Yes | 112 | 57 | Low | No | 150 | 80 | Low | No | 115 |
| 12 Low No | 155 | 35 | Low | No | 125 | 58 | High | Yes | 140 | 81 | Low | No | 102 |
| 13 High Yes | 153 | 36 | Low | No | 190 | 59 | Low | No | 180 | 82 | Low | No | 115 |
| 14 Low No | 145 | 37 | Low | No | 155 | 60 | Low | Yes | 190 | 83 | Low | No | 150 |
| 15 Low No | 170 | 38 | Low | Yes | 170 | 61 | High | No | 145 | 84 | Low | No | 110 |
| 16 Low No | 175 | 39 | Low | No | 155 | 62 | High | Yes | 150 | 85 | High | No | 116 |
| 17 Low Yes | 175 | 40 | Low | No | 215 | 63 | Low | Yes | 164 | 86 | Low | Yes | 108 |
| 18 Low Yes | 170 | 41 | Low | Yes | 150 | 64 | Low | No | 140 | 87 | High | No | 95 |
| 19 Low Yes | 180 | 42 | Low | Yes | 145 | 65 | Low | No | 142 | 88 | High | Yes | 125 |
| 20 Low No | 135 | 43 | Low | No | 155 | 66 | High | No | 136 | 89 | Low | No | 133 |
| Low No | 170 | 44 | Low | No | 155 | 67 | Low | No | 123 | 06 | Low | No | 110 |
| 22 Low No | 157 | 45 | Low | No | 150 | 68 | Low | No | 155 | 91 | High | No | 150 |
| 23 Low No | 130 | 46 | Low | Yes | 155 | 69 | High | No | 130 | 92 | Low | No | 108 |

Results for: EXH_REGR.MTW

Binary Logistic Regression: RestingPulse versus Smokes, Weight

Link Function: Logit Response Information Variable Value Count RestingPulse Low 70 (Event) High 22 Total 92 Factor Information Factor Levels Values Smokes 2 No, Yes Logistic Regression Table Odds 95% CI Coef SE Coef Z P Ratio Lower Upper Predictor Constant -1.98717 1.67930 -1.18 0.237 Smokes -1.19297 0.552980 -2.16 0.031 0.30 0.10 0.90 Yes Weight 0.0250226 0.0122551 2.04 0.041 1.03 1.00 1.05 Log-Likelihood = -46.820Test that all slopes are zero: G = 7.574, DF = 2, P-Value = 0.023 Goodness-of-Fit Tests Chi-Square DF P 40.8477 47 0.724 51.2008 47 0.312 Method Pearson Deviance 4.7451 8 0.784 Hosmer-Lemeshow Brown:
 General Alternative
 0.9051
 2
 0.636

 Symmetric Alternative
 0.4627
 1
 0.496
 Table of Observed and Expected Frequencies: (See Hosmer-Lemeshow Test for the Pearson Chi-Square Statistic) Group 1 2 7 8 9 10 Total Value 3 4 5 6 Low Obs 4 6 6 8 8 6 8 12 10 2 70 Exp 4.4 6.4 6.3 6.6 6.9 7.2 8.3 12.9 9.1 1.9 High 5 3 1 3 2 3 0 0 22 Obs 4 1 2.4 2.1 1.8 1.7 Exp 4.6 3.6 2.7 2.1 0.9 0.1 Total 9 10 9 9 9 9 10 15 10 2 92 Measures of Association: (Between the Response Variable and Predicted Probabilities)

Figure 25.24 Minitab session window for Example 25.7.

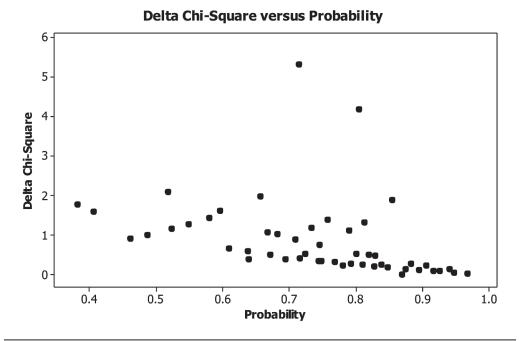


Figure 25.25 Delta chi-square versus probability plot for Example 25.7.

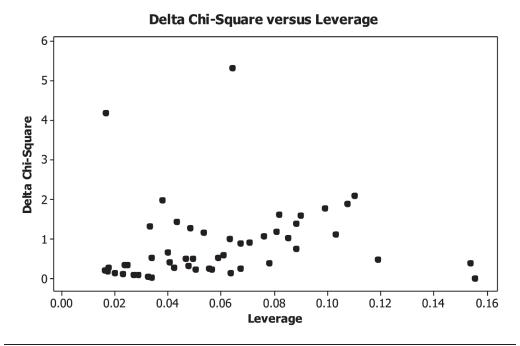


Figure 25.26 Delta chi-square versus leverage plot for Example 25.7.

EXAMPLE 25.8

Using the subject and teaching method data collected in Table 25.9, perform a nominal logistic regression analysis using Minitab. Use $\alpha = 0.05$.

Choose Stat > Regression > Nominal Logistic Regression. Select "Subject" for Response. Select "Teaching Method" and "Age" for Model. Select "Teaching Method" for Factors (optional). Select "In addition, list of factor level values, tests for terms with more than 1 degree of freedom" in the Results dialog box. The resulting analysis is depicted in Figure 25.27.

Since the reference event is "science," Minitab produced logit functions for math and science and for language arts and science, as seen in Figure 25.27. The high *p*-values for the first logit function indicate that there is sufficient evidence to conclude that a change in the teaching method or age affected the choice of math as a favorite subject compared to science. However, due to the low *p*-values, the second logit function suggests that the teaching method or age affected the choice of language arts as a favorite subject compared to science. The large odds ratio of 15.96 indicates that the odds of choosing language arts over science are almost 16 times greater when the teaching method changes to a lecture. The log-likelihood test indicates that there is sufficient evidence to conclude that at least one coefficient is not zero. Both goodness-of-fit tests have *p*-values much greater than $\alpha = 0.05$, indicating that we should accept the null hypothesis that the model fits the data adequately.

| Subject | Teaching method | Age | Subject | Teaching method | Age |
|---------|--------------------|-----|---------|--------------------|-----|
| Math | Discuss | 10 | Science | Lecture | 12 |
| Science | Discuss | 10 | Science | Lecture | 12 |
| Science | Discuss | 10 | Science | Discuss | 12 |
| Math | Lecture | 10 | Arts | Lecture | 12 |
| Math | Discuss | 10 | Math | Discuss | 12 |
| Science | Lecture | 10 | Math | Discuss | 12 |
| Math | Discuss | 10 | Arts | Lecture | 12 |
| Math | Lecture | 11 | Arts | Discuss | 13 |
| Arts | Lecture | 11 | Math | Discuss | 13 |
| Science | Discuss | 11 | Arts | Lecture | 13 |
| Arts | Lecture | 11 | Arts | Lecture | 13 |
| Math | Discuss | 11 | Math | Discuss | 13 |
| Science | Lecture | 11 | Science | Discuss | 13 |
| Science | Discuss | 11 | Math | Lecture | 13 |
| Arts | Lecture | 11 | Arts | Lecture | 13 |

Table 25.9Data for Example 25.8.

Nominal Logistic Regression: Subject versus TeachingMethod, Age

Response Information

| Variable Subject | | Count 10 | (Reference | Event) |
|---------------------|-------|-------------|--------------|---------|
| | math | 11 | (11020201000 | 2.0110, |
| | arts | 9 | | |
| | Total | 30 | | |

Factor Information

Factor Levels Values TeachingMethod 2 discuss, lecture

Logistic Regression Table

| | | | | | Odds | 95% CI |
|-------------------------|-----------|----------|-------|-------|-------|-----------|
| Predictor | Coef | SE Coef | Z | P | Ratio | Lower |
| Logit 1: (math/science) | | | | | | |
| Constant | -1.12266 | 4.56425 | -0.25 | 0.806 | | |
| TeachingMethod | | | | | | |
| lecture | -0.563115 | 0.937591 | -0.60 | 0.548 | 0.57 | 0.09 |
| Age | 0.124674 | 0.401079 | 0.31 | 0.756 | 1.13 | 0.52 |
| Logit 2: (arts/science) | | | | | | |
| Constant | -13.8485 | 7.24256 | -1.91 | 0.056 | | |
| TeachingMethod | | | | | | |
| lecture | 2.76992 | 1.37209 | 2.02 | 0.044 | 15.96 | 1.08 |
| Age | 1.01354 | 0.584494 | 1.73 | 0.083 | 2.76 | 0.88 |

| Predictor | Upper | |
|-------------------------|--------|--|
| Logit 1: (math/science) | | |
| Constant | | |
| TeachingMethod | | |
| lecture | 3.58 | |
| Age | 2.49 | |
| Logit 2: (arts/science) | | |
| Constant | | |
| TeachingMethod | | |
| lecture | 234.91 | |
| Age | 8.66 | |

Log-Likelihood = -26.446Test that all slopes are zero: G = 12.825, DF = 4, P-Value = 0.012

Goodness-of-Fit Tests

| Method | Chi-Square | DF | P |
|----------|------------|----|-------|
| Pearson | 6.95295 | 10 | 0.730 |
| Deviance | 7.88622 | 10 | 0.640 |

Figure 25.27 Minitab session window for Example 25.8.

EXAMPLE 25.9

Using the toxicity data collected in Table 25.10, perform an ordinal logistic regression analysis using Minitab. Use $\alpha = 0.05$.

Choose *Stat* > *Regression* > *Nominal Logistic Regression*. Select "Survival" for *Response*. Select "Region" and "Toxic Level" for *Model*. Select "Region" for *Factors* (*optional*). Select "In addition, list of factor level values, tests for terms with more than 1 degree of freedom" in the *Results* dialog box. The resulting analysis is depicted in Figure 25.28.

From Figure 25.28, we see that due to the high *p*-value of 0.685, we can not conclude that "region" has a significant effect on survival time. In contrast, the low *p*-value for "toxic level" indicates that toxic level affects survival time. The log-likelihood test indicates that there is sufficient evidence to conclude that at least one coefficient is not zero. Both goodness-of-fit tests have *p*-values much greater than $\alpha = 0.05$, indicating that we should accept the null hypothesis that the model fits the data adequately. The values for the summary measures range between 0 and 1. The higher the value, the better the predictability of the model. In this example, the summary measure values range from 0.032 to 0.59, indicating that the model fit offers a low-to-moderate level of predictability.

MODEL FITTING FOR NONLINEAR PARAMETERS

Apply and interpret fits of models that are non-linear. (Apply)

Body of Knowledge VI.B.4

When we refer to linear and nonlinear regression, we are referring to whether or not the parameters of the model are linear or nonlinear. With linear regression, the parameters have a closed-form solution. However, with nonlinear regression they do not. Instead, the parameters must be solved for using iterative optimization techniques that may or may not converge. Consequently, one does not solve nonlinear manually, and the mathematical details far exceed Bloom's taxonomy level even in basic terms. As such, this section will address the basics of nonlinear regression and supplement those basics with a comprehensive example using Minitab.

The use of nonlinear regression requires substantial knowledge to determine the functional form of the model and often requires heavy reliance on subject matter experts to estimate the initial parameter values. Therefore, it can be fraught with trial and error.

Commonly used model forms include power series, sinusoidal, exponential growth and decay, and polynomial, among others. Virtually all software packages permit the user to enter their own functional model form as well.

| Number | Survival | Region | Toxic level | Number | Survival | Region | Toxic level |
|--------|----------|--------|----------------|--------|----------|--------|----------------|
| 1 | 1 | 1 | 62.00 | 38 | 2 | 1 | 40.50 |
| 2 | 1 | 2 | 46.00 | 39 | 2 | 2 | 60.00 |
| 3 | 2 | 1 | 48.50 | 40 | 3 | 1 | 57.50 |
| 4 | 3 | 2 | 32.00 | 41 | 2 | 1 | 48.75 |
| 5 | 2 | 1 | 63.50 | 42 | 2 | 1 | 44.50 |
| 6 | 1 | 1 | 41.25 | 43 | 1 | 1 | 49.50 |
| 7 | 2 | 2 | 40.00 | 44 | 2 | 2 | 33.75 |
| 8 | 3 | 1 | 34.25 | 45 | 2 | 1 | 43.50 |
| 9 | 2 | 1 | 34.75 | 46 | 2 | 2 | 48.00 |
| 10 | 1 | 2 | 46.25 | 47 | 3 | 1 | 34.00 |
| 11 | 2 | 1 | 43.50 | 48 | 1 | 1 | 50.00 |
| 12 | 2 | 2 | 46.00 | 49 | 3 | 2 | 35.00 |
| 13 | 2 | 1 | 42.50 | 50 | 1 | 1 | 49.00 |
| 14 | 1 | 2 | 53.00 | 51 | 2 | 2 | 43.50 |
| 15 | 1 | 2 | 43.50 | 52 | 3 | 2 | 37.25 |
| 16 | 1 | 1 | 56.00 | 53 | 3 | 2 | 39.00 |
| 17 | 2 | 1 | 40.00 | 54 | 3 | 1 | 34.50 |
| 18 | 1 | 2 | 48.00 | 55 | 2 | 1 | 47.50 |
| 19 | 2 | 1 | 46.50 | 56 | 1 | 2 | 42.00 |
| 20 | 2 | 2 | 72.00 | 57 | 2 | 2 | 45.50 |
| 21 | 2 | 2 | 31.00 | 58 | 2 | 2 | 38.50 |
| 22 | 1 | 1 | 48.00 | 59 | 2 | 1 | 36.50 |
| 23 | 2 | 2 | 36.50 | 60 | 2 | 2 | 37.50 |
| 24 | 2 | 2 | 43.75 | 61 | 3 | 1 | 38.50 |
| 25 | 2 | 1 | 34.25 | 62 | 2 | 2 | 47.00 |
| 26 | 2 | 1 | 41.25 | 63 | 2 | 2 | 39.75 |
| 27 | 2 | 2 | 41.75 | 64 | 1 | 1 | 60.00 |
| 28 | 2 | 2 | 45.25 | 65 | 2 | 2 | 41.00 |
| 29 | 2 | 1 | 43.50 | 66 | 2 | 1 | 41.00 |
| 30 | 2 | 2 | 53.00 | 67 | 3 | 1 | 30.00 |
| 31 | 3 | 1 | 38.00 | 68 | 2 | 2 | 45.00 |
| 32 | 2 | 2 | 59.00 | 69 | 2 | 2 | 51.00 |
| 33 | 2 | 1 | 52.50 | 70 | 2 | 2 | 35.25 |
| 34 | 2 | 2 | 42.75 | 71 | 1 | 2 | 40.50 |
| 35 | 2 | 2 | 31.50 | 72 | 2 | 2 | 39.50 |
| 36 | 2 | 2 | 43.50 | 73 | 3 | 2 | 36.00 |
| 37 | 2 | 2 | 40.00 | | | | |

| Table 25.10 | Data for | Examp | le 25.9. |
|-------------|----------|-------|----------|
|-------------|----------|-------|----------|

Ordinal Logistic Regression: Survival versus Region, ToxicLevel

Link Function: Logit Response Information Variable Value Count Survival 1 15 2 46 3 12 Total 73 Factor Information Factor Levels Values Region 2 1, 2 Logistic Regression Table Odds 95% CI
 Predictor
 Coef
 SE Coef
 Z
 P
 Ratio
 Lower
 Upper

 Const(1)
 -7.04343
 1.68017
 -4.19
 0.000

 Const(2)
 -3.52273
 1.47108
 -2.39
 0.017
 Region 0.201456 0.496153 0.41 0.685 1.22 0.46 3.23 2 ToxicLevel 0.121289 0.0340510 3.56 0.000 1.13 1.06 1.21 Log-Likelihood = -59.290Test that all slopes are zero: G = 14.713, DF = 2, P-Value = 0.001 Goodness-of-Fit Tests Method Chi-Square DF Р Pearson 122.799 122 0.463 100.898 122 0.918 Deviance Measures of Association: (Between the Response Variable and Predicted Probabilities) Pairs Number Percent Summary Measures Concordant 1127 79.3 Somers' D 0.59 288 20.3 Goodman-Kruskal Gamma 0.59 Discordant 7 Ties 0.5 Kendall's Tau-a 0.32
 Ties
 7
 0.5

 Total
 1422
 100.0

Figure 25.28 Minitab session window for Example 25.9.

Nonlinear regression requires the response variable to be continuous, and each predictor variable must be either continuous or an indicator variable (that is, 0 or 1). The underlying assumptions for nonlinear regression are the same as with linear regression (that is, the residuals are independent and identically distributed normal with a constant variance).

Probably the biggest issue with the use of nonlinear regression lies in the ability to estimate the model's parameters. Nonlinear regression parameters must be estimated using iterative optimization techniques (that is, $\beta_j^{k+1} = \beta_j^k + \Delta \beta_j$). These methods focus on minimizing the *sum of the squared errors* (SSE). This means that a number of iterations, a specified tolerance on the SSE, and set of initial starting conditions are required. The number of iterations and tolerance set the stopping conditions for the optimization algorithm used. The algorithm stops after it has performed the specified number of iterations, has reached its minimum SSE within the tolerance, or has encountered a condition that prevents it from continuing further.

As with most optimization techniques, we are likely to encounter difficulties. These include:

- The algorithm may converge to a local, not a global, minimum SSE for a given set of starting conditions. Note: In some cases there may even be multiple global minimums.
- The algorithm may not converge for some starting conditions.
- Different starting conditions can yield different results (that is, minimum SSE) for the same model. Therefore, careful thought and consideration must be given to their choice.
- Convergence of the algorithm does not imply that the model fit is optimal or the SSE is minimum. Always check the residual plots to ensure that the model fit is accurate and values for parameters appear reasonable.

The above notwithstanding, it may be possible to transform or linearize a nonlinear model. Consider the following model:

 $y = \beta_1 e^{\beta_2 x}$

By taking the natural logarithm of both sides, the above model can be linearized:

$$\ln(y) = \ln(\beta_1) + \beta_2 x$$

Table 25.11 summaries the differences between linear and nonlinear regression. Although, on the surface, linear regression may seem easier, one should not select it on that basis alone. Remember, the functional form of the model should determine what type of regression should be used. Once the functional form is determined, an adequate software package will do the rest.

| | 0 |
|---------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------|
| Linear regression | Nonlinear regression |
| The model form: the parameters appear as linear combinations. | The model form: the parameters appear as functions |
| There is a closed-form solution for the parameters. | Parameters must be solved iteratively using optimization algorithms. |
| Not applicable. | Optimization algorithms depend on starting conditions and may not converge. |
| Not applicable. | Optimization algorithms may converge to a local, rather than a global sum of the squared errors (SSE) minimum. |
| The solution is unique. | The SSE minimum may not be unique. |
| When errors are uncorrelated, the parameter estimates are unbiased. | When errors are uncorrelated, the parameter estimates are generally biased. |

| Table 25.11 | Differences | hetween | linear and | l nonlinear | regression |
|-------------|-------------|---------|------------|-------------|-------------|
| 10010 23.11 | Differences | Detween | micui unc | inormitua | regression. |

EXAMPLE 25.10

Researchers are interested in understanding the relationship between electron mobility and the natural logarithm of the density of a semiconductor. Previous research has suggested the following model:

$$y = \frac{\beta_1 + \beta_2 x + \beta_3 x^2 + \beta_4 x^3}{1 + \beta_5 x + \beta_6 x^2 + \beta_7 x^3}$$

The data for this example are provided in Table 25.12. Column 1 contains the response variable data and column 2 provides the data for the predictor variable. Use Minitab to apply the data and interpret the results. Assume $\alpha = 0.05$.

The session window for this example is shown in Figure 25.29. The Gauss–Newton method imposes a maximum number of iterations of 200 and a tolerance of 0.00001 as its stopping criteria. However, the algorithm converged after 27 iterations. The starting values for each of the parameters are: 1300, 1500, 500, 75, 1, 0.4, and 0.05, respectively. Ideally, these were established with process knowledge and subject matter expert input. The mobility equation with parameters is:

$$y = \frac{1288.14 + 1491.08x + 583.24x^2 + 75.42x^3}{1 + 0.97x + 0.40x^2 + 0.05x^3}$$

Confidence intervals, instead of *p*-values, are provided for each of the parameters. Knowledge of the process is required to understand the reasonableness of these intervals. In some cases, the upper bounds of the confidence intervals could not be

Continued

computed for the given alpha value. However, Minitab might be able to compute the upper confidence interval limit for lower alpha values. This alternative might be worth trying.

The correlation matrix indicates that several of the predictor variables are highly correlated. This may suggest the need to rethink the model.

The *S* value is expressed in the same units as the response variable and, consequently, it is easier to interpret than the final *SSE*. The *S* value indicates that the observed values are approximately 13.7146 units from the fitted values.

The fitted plot line is shown in Figure 25.30 and includes both confidence and prediction intervals. The prediction intervals are bigger than the confidence intervals due to the additional level of uncertainty present. Overall, the fitted line follows the data closely.

The residual plots for this regression are depicted in Figure 25.31. The normal probability plot and histogram indicate that some degree of departure from normality may be present. The "residuals versus fits" plot suggests a constant variance although some of the earlier fits were more tightly grouped. The first 13 points of the "residuals versus order" plot are not random. They should be investigated to determine their correctness and subsequent impact on the model. However, beyond the thirteenth point, the graph appears random, indicating no systematic effects in the data.

| Mobility | Density ln | Mobility | Density ln | Mobility | Density ln |
|----------|---------------|----------|---------------|----------|---------------|
| 80.574 | -3.067 | 390.724 | -1.460 | 1273.514 | -0.103 |
| 84.248 | -2.981 | 567.534 | -1.274 | 1288.339 | 0.010 |
| 87.264 | -2.921 | 635.316 | -1.212 | 1327.543 | 0.119 |
| 87.195 | -2.912 | 733.054 | -1.100 | 1353.863 | 0.377 |
| 89.076 | -2.840 | 759.087 | -1.046 | 1414.509 | 0.790 |
| 89.608 | -2.797 | 894.206 | -0.915 | 1425.208 | 0.963 |
| 89.868 | -2.702 | 990.785 | -0.714 | 1421.384 | 1.006 |
| 90.101 | -2.699 | 1090.109 | -0.566 | 1442.962 | 1.115 |
| 92.405 | -2.633 | 1080.914 | -0.545 | 1464.350 | 1.572 |
| 95.854 | -2.481 | 1122.643 | -0.400 | 1468.705 | 1.841 |
| 100.696 | -2.363 | 1178.351 | -0.309 | 1447.894 | 2.047 |
| 101.060 | -2.322 | 1260.531 | -0.109 | 1457.628 | 2.200 |
| 401.672 | -1.501 | | | | |

Table 25.12Data for Example 25.10.

Nonlinear Regression: Mobility = (b1 + b2 * 'Density Ln' + ...

Method

| Algorithm | Gauss-Newton |
|----------------|--------------|
| Max iterations | 200 |
| Tolerance | 0.00001 |

Starting Values for Parameters

| Value |
|-------|
| 1300 |
| 1500 |
| 500 |
| 75 |
| 1 |
| 0.4 |
| 0.05 |
| |

Equation

```
Mobility = (1288.14 + 1491.08 * 'Density Ln' + 583.238 * 'Density Ln' ** 2 +
75.4167 * 'Density Ln' ** 3) / (1 + 0.966295 * 'Density Ln' + 0.397973 *
'Density Ln' ** 2 + 0.0497273 * 'Density Ln' ** 3)
```

Parameter Estimates

| Parameter | Estimate | SE Estimate | 95% | CI |
|-----------|----------|-------------|-----------|----------|
| b1 | 1288.14 | 4.6648 | (1278.59, | 1297.71) |
| b2 | 1491.08 | 39.5711 | (1381.50, | 1548.27) |
| b3 | 583.24 | 28.6986 | (502.36, | 625.87) |
| b4 | 75.42 | 5.5675 | (59.58, | 83.57) |
| b5 | 0.97 | 0.0313 | (0.88, | *) |
| b6 | 0.40 | 0.0150 | (0.36, | *) |
| b7 | 0.05 | 0.0066 | (0.03, | 0.06) |

Mobility = (b1 + b2 * 'Density Ln' + b3 * 'Density Ln' ** 2 + b4 * 'Density Ln'
 ** 3) / (1 + b5 * 'Density Ln' + b6 * 'Density Ln' ** 2 + b7 * 'Density
 Ln' ** 3)

Correlation Matrix for Parameter Estimates

| | b | 1 b2 | b3 | b4 | b5 | b6 |
|------|-----------|--------------|------------|------------|-----------|----------|
| b2 | -0.03705 | 0 | | | | |
| b3 | -0.08970 | 2 0.994680 | | | | |
| b4 | -0.09564 | 2 0.987680 | 0.997585 | | | |
| b5 | -0.17821 | 8 0.971011 | 0.977058 | 0.965442 | | |
| b6 | 0.03035 | 5 0.952063 | 0.962728 | 0.961647 | 0.948161 | |
| b7 | 0.10437 | 9 0.848710 | 0.827557 | 0.839019 | 0.725042 | 0.772118 |
| | | | | | | |
| Lac | k of Fit | | | | | |
| | | | | | | |
| The | re are no | replicates | | | | |
| Min | itab cann | ot do the la | ack of fit | test based | on pure e | rror. |
| | | | | | | |
| Sum | nary | | | | | |
| | - | | | | | |
| Ite | rations | 27 | | | | |
| Fina | al SSE | 5642.71 | | | | |
| DFE | | 30 | | | | |
| MSE | | 188.090 | | | | |
| S | | 13.7146 | | | | |
| | | | | | | |

Figure 25.29 Minitab session window for Example 25.10.

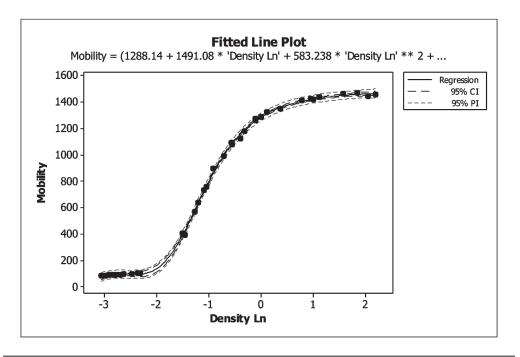


Figure 25.30 Nonlinear fitted line plot for Example 25.10.

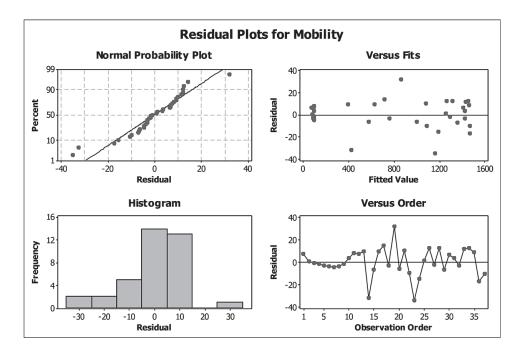


Figure 25.31 Residual plots for Example 25.10.

GENERAL LINEAR MODELS (GLMs)

Apply and interpret GLMs using assumptions and assumptions testing. Compare and contrast GLMs with various other models, including ANOVA results, (crossed, nested, and mixed models) simple linear regression, multiple regression, ANCOVA, and continuous MSA. (Apply)

Body of Knowledge VI.B.5

The *general linear model* (GLM) is a statistical tool that synthesizes many of the other common statistical techniques for analyzing both continuous and discrete data. Several of these tools include:

- Analysis of variance (ANOVA)
- Analysis of covariance (ANCOVA)
- Simple linear regression
- Multiple linear regression
- Continuous measurement systems analysis

As with all models, the GLM is based on certain statistical assumptions that must be adhered to for the results of the models to be valid. The underlying assumptions for the GLM include:

- The predictor variables can be either continuous or discrete, but the response variable must be continuous.
- The relationship between the predictor and response variables is linear.
- The residuals or errors are independent and identically distributed normal with zero mean and constant variance σ^2 .

Like nonlinear regression, one does not manually solve general linear model problems in practice. Application of the GLM is typically performed using common statistical software packages.

Table 25.13 compares the capabilities of several statistical techniques against the general linear model when using Minitab. This table indicates the flexibility of the GLM. Notice that it is the only tool that can handle the unbalanced design as well as address covariates without introducing the need for indicator or dummy variables. Further, each of the tools in Table 25.13 save GLM can be conducted within the GLM framework of Minitab. Consequently, the GLM should be included in the repertoire of every Lean Six Sigma professional.

| Table 25.13 | A comparison o | of various sta | atistical techni | ques to the GL | .M. | |
|--------------------------------|----------------|----------------|------------------|--------------------------------|----------------------------------|-------------------|
| Capability | GLM | ANOVA | ANCOVA | Simple linear regression | Multiple linear regression | Continuous MSA |
| Balanced design | Yes | Yes | Yes | Yes | Yes | Yes |
| Unbalanced design | Yes | No | No | No | No | Yes |
| Covariates | Yes | No | Yes | No | Indicator variables | No |
| Fixed model | Yes | Yes | No | Yes | Yes | Yes |
| Random model | Yes | Yes | No | No | No | Yes |
| Mixed model | Unrestricted | Yes | No | No | No | Yes |
| Nested model | Yes | Fully | No | No | No | Yes |
| Crossed model | Yes | Yes | No | No | No | Yes |
| Crossed and nested model | Yes | No | No | No | No | No |

| Table 25.13 A | A comparison | of various | statistical | techniques | to the GLM. |
|----------------------|--------------|------------|-------------|------------|-------------|
|----------------------|--------------|------------|-------------|------------|-------------|

Testing the Assumptions

Before adopting any model, it is important to test the underlying assumptions. Minitab recognizes this and provides the user with a variety of graphs that help the user understand whether the major key assumption (that is, residuals or errors are independent and identically distributed normal with zero mean and constant variance σ^2) has been violated. These graphs are:

- *Normal probability plot of residuals.* This graph is used to check for nonnormality of the residuals. Ideally, the residual points should plot along the straight line or reasonably close. Departure from the line at the ends is indicative of skewness while points far away from the line suggest the presence of outliers. Whenever the slope of the plot changes, this is an indication of an unidentified variable.
- *Histogram of residuals.* This graph is used to check for nonnormality of the residuals. Generally, look for long tails to indicate skewness, or an isolated bar to indicate the presence of an outlier.
- *Residuals versus fits.* This graph is used to check heteroscedasticity (that is, nonconstant variance) in the residuals. The graph should

appear random. Patterns in the graph indicate the possible presence of heteroscedasticity while points far away from the line suggest the presence of outliners. It is often useful to plot the fits against the "standardized residuals" since Minitab considers residuals greater than 2.0 units or less than –2.0 units away from the zero line to be possible outliers and automatically flags them.

• *Residuals versus order.* This graph is particularly useful in time-series analysis where it might be possible that the order of the observations could influence the results. The graph should appear random. Correlation may exist between the residuals when increasing or decreasing trends are present or when rapid changes in the signs of adjacent residuals occurs.

EXAMPLE 25.11

This example was obtained from Minitab. An experiment was conducted to determine the effect of glass type and temperature on the light output of an oscilloscope. The data for the experiment are provided in Table 25.14. Analyze the data using the GLM method and assign "temperature" as the covariate. Assume $\alpha = 0.05$.

The Minitab session window is provided in Figure 25.32. The table of factors indicates three levels for glass type, thus providing some minimal assurance of correct data entry. The analysis of variance source table lists both the sequential and adjusted sums of squares. Recall that the sequential sums of squares are the cumulative sums of squares given that prior terms are in the model and depend on the model order while the adjusted sums of squares are the sums of squares given that all other terms are in the model and do not depend on the model order. This table also indicates that all effects in the model are significant, including second-order effects. (Remember, second-order effects could be calculated because the quantitative variable was at three levels.)

The R^2 value is very high, indicating a good model fit. The covariate coefficients table is provided next, confirming the significance of the effects. Minitab identified standardized residuals that are worth noting. (Minitab flags standardized residuals that are greater than 2.0 or less than –2.0.) These data should be investigated for correctness or to determine whether they are outliers.

Figure 25.33 provides four key residual plots. Notice the central values on the normal probability plot wavering about the line, thus throwing some degree of doubt on the assumption of normality. This pattern is reflected in the histogram below it with the high degree of leptokurtosis. The "residuals versus fits" plot seems to confirm the assumption of constant variance while the "residuals versus order" plot identifies several unusual patterns, suggesting the presence of correlation between the residuals.

Figure 25.34 depicts the plot of "residuals versus temperature." The plot pattern appears to be sufficiently random around the zero line. However, Figure 25.35, the plot of "residuals versus glass type," illustrates what appears to be a nonrandom dispersion across the three glass types. Notice that type 1 is tight around the zero line, which contrasts greatly with type 2. Type 3, on the other hand, appears to be middle of the road. Given that glass type is a discrete variable, it may be exerting a systematic influence on the response variable.

| Table 2. | Data for Example 23.11. | | | | | | | |
|-----------------|-------------------------|---------------|-----------------|-------------|---------------|-----------------|-------------|---------------|
| Light output | Temperature | Glass type | Light output | Temperature | Glass type | Light output | Temperature | Glass type |
| 580 | 100 | 1 | 550 | 100 | 2 | 546 | 100 | 3 |
| 1090 | 125 | 1 | 1070 | 125 | 2 | 1045 | 125 | 3 |
| 1392 | 150 | 1 | 1328 | 150 | 2 | 867 | 150 | 3 |
| 568 | 100 | 1 | 530 | 100 | 2 | 575 | 100 | 3 |
| 1087 | 125 | 1 | 1035 | 125 | 2 | 1053 | 125 | 3 |
| 1380 | 150 | 1 | 1312 | 150 | 2 | 904 | 150 | 3 |
| 570 | 100 | 1 | 579 | 100 | 2 | 599 | 100 | 3 |
| 1085 | 125 | 1 | 1000 | 125 | 2 | 1066 | 125 | 3 |
| 1386 | 150 | 1 | 1299 | 150 | 2 | 889 | 150 | 3 |
| | | | | | | | | |

| Table 25.14 | Data for Example 25.11. |
|-------------|--------------------------|
| | Edita for Example 25.11. |

- *Residuals versus variables.* This graph is used to detect whether a variable is systematically influencing the response variable or a higher-order term of the variable should be included in the model. Ideally, the residuals should be random about the zero line. A pattern indicates that the variable is systematically influencing the response variable. Curvature in the plot suggests the inclusion of a higher-order term of the variable.
- *Unusual observation table.* Minitab provides a table of unusual observations. It determines these by standardizing the residuals and identifying those that are greater than 2.0 or less than –2.0. This provides the user with opportunity to recheck the data for correctness or identify outliers.

EXAMPLE 25.12

This example was obtained from Minitab. An experiment was conducted to test the efficacy of insecticides made by four companies in killing mosquitoes. The chemical composition of each insecticide differs by company. The experiment was conducted by placing 400 mosquitoes in a jar, applying a specific insecticide, and counting the live mosquitoes four hours later. Three replications for each product were performed. Analyze the data using the GLM method and assume $\alpha = 0.05$. The data for the experiment are given in Table 25.15.

The Minitab session window is provided in Figure 25.36. The table of factors indicates that both factors are fixed; there are four levels for "company," and eleven for the nested factor of "product" within "company," thus providing some minimal assurance of correct data entry. Table 25.15 indicates the design is unbalanced.

The analysis of variance source table signifies that both factors are significant for the given alpha value, and the R^2 value is quite high, indicating that the model fits the data fairly well.

General Linear Model: LightOutput versus GlassType

Factor Type Levels Values GlassType fixed 3 1, 2, 3

Analysis of Variance for LightOutput, using Adjusted SS for Tests

| Source | DF | Seq SS | Adj SS | Adj MS | F | Р |
|-----------------------------------|----|---------|--------|--------|--------|-------|
| Temperature | 1 | 1779756 | 262884 | 262884 | 719.21 | 0.000 |
| Temperature*Temperature | 1 | 190579 | 190579 | 190579 | 521.39 | 0.000 |
| GlassType | 2 | 150865 | 41416 | 20708 | 56.65 | 0.000 |
| GlassType*Temperature | 2 | 226178 | 51126 | 25563 | 69.94 | 0.000 |
| GlassType*Temperature*Temperature | 2 | 64374 | 64374 | 32187 | 88.06 | 0.000 |
| Error | 18 | 6579 | 6579 | 366 | | |
| Total | 26 | 2418330 | | | | |

S = 19.1185 R-Sq = 99.73% R-Sq(adj) = 99.61%

| Term Constant Temperature Temperature*Temperature GlassType | Coef -4968.8 83.867 -0.28516 | SE Coef 191.3 3.127 0.01249 | T -25.97 26.82 -22.83 | P 0.000 0.000 0.000 | | | | | |
|-------------------------------------------------------------------------|---------------------------------------|--------------------------------------|--------------------------------|------------------------------|--|--|--|--|--|
| 1 2 | 1322.8 1553.8 | 270.5 270.5 | 4.89 5.74 | 0.000 | | | | | |
| Temperature*GlassType | | | | | | | | | |
| 1 | -24.400 | 4.423 | -5.52 | 0.000 | | | | | |
| 2 | -27.867 | 4.423 | -6.30 | 0.000 | | | | | |
| Temperature*Temperature*GlassType | | | | | | | | | |
| 1 2 | 0.11236 0.12196 | 0.01766 0.01766 | 6.36 6.91 | 0.000 0.000 | | | | | |

Unusual Observations for LightOutput

| Obs | LightOutput | Fit | SE Fit | Residual | St Resid |
|-----|-------------|---------|--------|----------|----------|
| 11 | 1070.00 | 1035.00 | 11.04 | 35.00 | 2.24 R |
| 17 | 1000.00 | 1035.00 | 11.04 | -35.00 | -2.24 R |

R denotes an observation with a large standardized residual.



The session window output notes that, "means that do not share a letter are significantly different." Therefore, the "Grouping Information Using Tukey Method" table is interpreted as:

- Companies A and B are not significantly different from each other.
- Companies A and B are significantly different from C and D.
- Companies C and D are significantly different from each other.
- Company D's insecticide killed significantly more mosquitoes than the other three companies'. Remember, the mean is the average number of surviving mosquitoes. Therefore, the smaller the mean, the better.

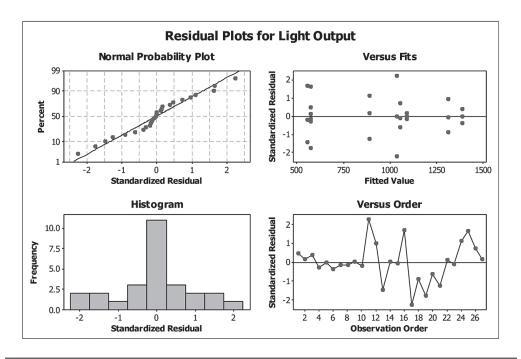


Figure 25.33 Minitab residual plots for light output for Example 25.11.

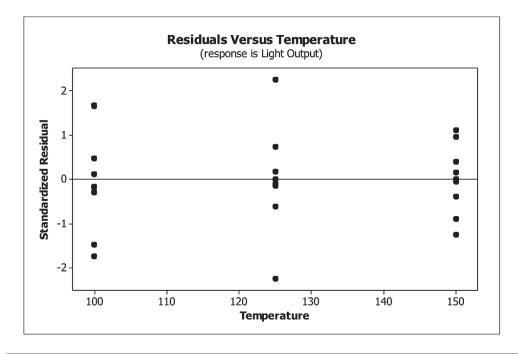


Figure 25.34 Minitab plot of residuals versus variable temperature for Example 25.11.

Part VI.B.5

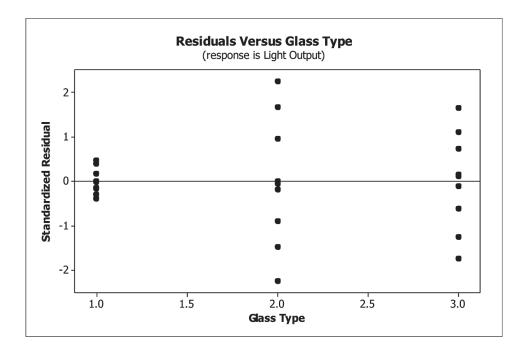


Figure 25.35 Minitab plot of residuals versus variable glass type for Example 25.11.

Continued

Tukey's "Simultaneous Confidence Intervals" are interpreted as follows:

- Company A subtracted from companies B, C, and D. The confidence interval for company B contains zero, indicating that the means are not significantly different at the stated alpha value.
- Company B subtracted from companies C and D. None of the confidence intervals contain zero, indicating a significant difference in the means at the stated alpha value.
- Company C subtracted from company D. None of the confidence intervals contain zero, indicating a significant difference in the means at the stated alpha value.

Tukey's "Simultaneous Tests" parallel the "Simultaneous Confidence Intervals" and cause us to draw the same conclusions.

Figure 25.37 provides the residual plots. The normal probability plot casts doubt on the normality assumption. Notice the wavering of the plot across the line and curvature at the tails. This conclusion is supported by a review of the histogram, which is essentially flat. The "residuals versus fits" plot demonstrates random dispersion around the zero line. Similarly, the "residuals versus order" plot appears at first glance to be relatively random. However, upon closer inspection, notice that observations 10–14 and 26–32 alternate signs. This might suggest that some degree of correlation exists between the residuals.

| NMosquito | Company | Product | NMosquito | Company | Product |
|-----------|---------|---------|-----------|---------|---------|
| 151 | А | A1 | 94 | С | C1 |
| 135 | А | A1 | 84 | С | C2 |
| 137 | А | A1 | 87 | С | C2 |
| 118 | А | A2 | 82 | С | C2 |
| 132 | А | A2 | 79 | D | D1 |
| 135 | А | A2 | 74 | D | D1 |
| 131 | А | A3 | 73 | D | D1 |
| 137 | А | A3 | 67 | D | D2 |
| 121 | А | A3 | 78 | D | D2 |
| 140 | В | B1 | 63 | D | D2 |
| 152 | В | B1 | 90 | D | D3 |
| 133 | В | B1 | 81 | D | D3 |
| 151 | В | B2 | 96 | D | D3 |
| 132 | В | B2 | 83 | D | D4 |
| 139 | В | B2 | 89 | D | D4 |
| 96 | С | C1 | 94 | D | D4 |
| 108 | С | C1 | | | |

Table 25.15Data for Example 25.12.

General Linear Model: NMosquito versus Company, Product

| Factor Company Product(Company) | Type fixed fixed | | А, В, | C, D | , B2, C1 | , C2, D1 | , D2, | D3, | D4 |
|---------------------------------------|------------------------|-----------|----------|----------|----------|-----------|-------|-----|----|
| Analysis of Varia | nce for | r NMosqui | to, usi | ng Adjus | ted SS f | for Tests | | | |
| Source | | - | Adj SS | Adj MS | | P | | | |
| Company | 3 22 | 2813.3 2 | 2813.3 | 7604.4 | 132.78 | 0.000 | | | |
| Product (Company) | 7 1 | 1500.6 | 1500.6 | 214.4 | 3.74 | 0.008 | | | |
| Error | | L260.0 | 1260.0 | 57.3 | | | | | |
| Total | 32 25 | 5573.9 | | | | | | | |
| | | | | | | | | | |
| S = 7.56787 R-S | Sq = 95. | .07% R- | -Sq(adj) | = 92.83 | 00 | | | | |

Figure 25.36 Minitab session window for Example 25.12.

Grouping Information Using Tukey Method and 95.0% Confidence
 Company
 N
 Mean
 Grouping

 B
 6
 141.2
 A

 A
 9
 133.0
 A

 C
 6
 91.8
 B

 D
 12
 80.6
 C
 Means that do not share a letter are significantly different. Tukey 95.0% Simultaneous Confidence Intervals Response Variable NMosquito All Pairwise Comparisons among Levels of Company Company = A subtracted from: -50 -25 0 Company = B subtracted from: -50 -25 0 Company = C subtracted from: -50 -25 0 Tukey Simultaneous Tests Response Variable NMosquito All Pairwise Comparisons among Levels of Company Company = A subtracted from: SE of Difference Adjusted
 Company
 of Means
 Difference
 T-Value
 P-Value

 B
 8.17
 3.989
 2.05
 0.2016

 C
 -41.17
 3.989
 -10.32
 0.0000
 3.337 -15.71 0.0000 D -52.42 Company = B subtracted from: Difference SE of Adjusted of Means Difference T-Value P-Value Company -49.33 4.369 -11.29 0.0000 С D 3.784 -16.01 0.0000 -60.58 Company = C subtracted from: Difference SE of Adjusted Company of Means Difference T-Value P-Value D -11.25 3.784 -2.973 0.0329

Figure 25.36 Continued.

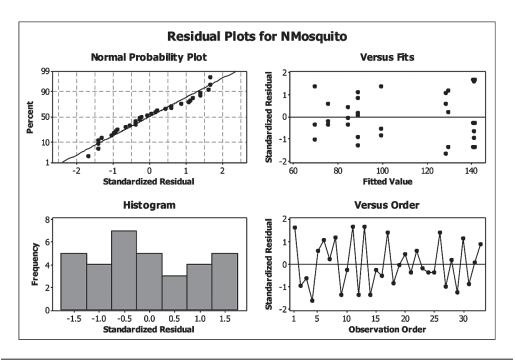


Figure 25.37 Minitab residual plots for NMosquito for Example 25.12.

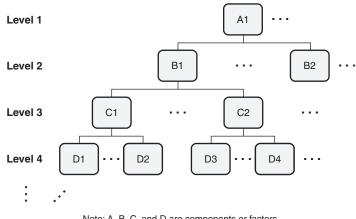
COMPONENTS OF VARIATION

Select, calculate, and interpret components of variation and nested design studies. (Evaluate)

Body of Knowledge VI.B.6

Components of variation (COV) is a statistical technique that allows us to separate process variation by the inputs. Consequently, it helps us identify major sources of variation and thus prioritize our process improvement activities. Underlying COV is the use of control charts that quantify the magnitude of the variation of each of the components. Therefore, we will not be using control charts in the traditional sense to identify out-of-control points.

Components of variation is useful in the situation where our problem can be formulated as a nested hierarchal structure. Such a structure is shown in Figure 25.38. Notice that Figure 25.38 resembles a family tree. It is for this reason that we speak of the relationship between each level as a parent–child relationship. For example, A1 in level 1 has two children in level 2, which are B1 and B2. Similarly,



Note: A, B, C, and D are components or factors.

Figure 25.38 Example of a nested hierarchy.

 Table 25.16
 The relationship between parent and child at each level of the nested hierarchy.

| Level | | Relati | onship | |
|-------|--------|--------|--------|--------|
| 1 | Parent | | | |
| 2 | Child | Parent | | |
| 3 | | Child | Parent | |
| 4 | | | Child | Parent |
| 5 | | | | Child |

the same is true for levels 2 and 3, and levels 3 and 4, and so on. This concept is generalized in Table 25.16 for a five-level nested hierarchal structure.

The idea behind components of variation is to decompose and quantify the variation at each level (or component) to understand how it contributes to the total variation. In equation form, this is simply:

Total variation = Level 1 variation + Level 2 variation + Level 3 variation + ...

Total variation =
$$\sum_{i=1}^{n} \hat{\sigma}_{i}^{2}$$

where *n* is the number of components or factors. The question becomes one of how to compute $\hat{\sigma}_i^2$. To do this, we must start at the bottom of the nested hierarchy and work toward the top. Also, we must recognize that the total variation at the *i*th level (where *i* is not the bottom level) is the variation contributed by the *i*th level itself plus the variation from all subordinate levels. Therefore, we must remove all variation from all subordinate levels to obtain the true variation contributed at the *i*th level.

EXAMPLE 25.13

The process for filling requests for medical prescriptions is taking longer to complete than promised to the customers. Customers are upset, and the CEO has been receiving numerous complaints. Of course, when complaints reach the CEO's desk the filling process receives all the attention it is due. The Lean Six Sigma department has been asked to analyze the process and to make recommendations regarding where to begin improvement activities. The process was analyzed and the following was determined:

- Five processing centers are used to fill prescriptions.
- There are two filling departments at each processing center.
- Three prescriptions from each filling department at each processing center were timed. All times are in days.

Data were collected, and the results are shown in Table 25.17. Determine the variation for each of the components.

We must start by defining the equations for the total variation at each level of the nested hierarchal structure. These have been computed and identified in Figure 25.39.

Components of variance calculations are easy to do in Excel. The completed table for this example is shown in Table 25.18. Let's examine this table briefly. Columns 1–3 represent the nested hierarchal structure in column form. Column 4 represents the measured values. Columns 5 and 6 are the computed ranges and averages that would be plotted on a control chart for the prescriptions. At the bottom of the column is the $\bar{R}_{\text{Prescription}}$, which will be needed to compute $\hat{\sigma}_{\text{Prescription}}^2$. Columns 7 and 8 are similar to columns 5 and 6, but apply to the filling departments. Column 9 represents the moving range for the processing centers.

Using Table 25.18 and the formulas on the bottom half of Figure 25.39, we can now compute the values of $\hat{\sigma}^2$ for each of the components of variation:

$$\hat{\sigma}_{\text{Prescription}}^2 = \left(\frac{\bar{R}}{d_2}\right)_{\text{Prescription}}^2 = \left(\frac{0.36}{1.693}\right)^2 = 0.045$$

$$\hat{\sigma}_{\text{Filling department}}^2 = \left(\frac{\bar{R}_{\text{Filling department}}}{d_2}\right)_{\text{Filling department}}^2 - \frac{\hat{\sigma}_{\text{Prescription}}^2}{3} = \left(\frac{1.42}{1.128}\right)^2 - \left(\frac{0.045}{3}\right) = 1.564$$

$$\hat{\sigma}_{\text{Processing center}}^2 = \left(\frac{\bar{R}_{\text{Processing center}}}{d_2}\right)^2 - \frac{\hat{\sigma}_{\text{Filling department}}^2}{2} - \frac{\hat{\sigma}_{\text{Prescription}}^2}{6} = \left(\frac{0.33}{1.128}\right)^2 - \left(\frac{1.56}{2}\right) - \left(\frac{0.045}{6}\right) = -0.703$$

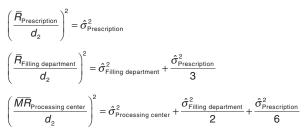
Since the processing center variance is negative, and variances can not be negative, it will be set to zero. Thus, the total variation for the prescriptions and filling departments is 1.609. Therefore, the percent contribution to total process variance from the filling departments is 97.2%, and the percent contribution from the prescriptions is 2.8%. These percentages are depicted graphically in Figure 25.40.

The Lean Six Sigma department recommended further investigation into the filling departments' processes.

| Table 25.17 | Data for Exa | mpie 25.15. | | | | | |
|----------------------|-----------------------|-------------------------|-----------------|----------------------|-----------------------|-------------------------|-----------------|
| Processing center | Filling department | Prescription request | Time to fill | Processing center | Filling department | Prescription request | Time to fill |
| 1 | 1 | 1 | 8.45 | 3 | 6 | 16 | 9.84 |
| 1 | 1 | 2 | 9.33 | 3 | 6 | 17 | 10.10 |
| 1 | 1 | 3 | 9.26 | 3 | 6 | 18 | 9.77 |
| 1 | 2 | 4 | 10.56 | 4 | 7 | 19 | 9.94 |
| 1 | 2 | 5 | 10.68 | 4 | 7 | 20 | 9.96 |
| 1 | 2 | 6 | 10.63 | 4 | 7 | 21 | 10.29 |
| 2 | 3 | 7 | 10.85 | 4 | 8 | 22 | 10.70 |
| 2 | 3 | 8 | 11.02 | 4 | 8 | 23 | 10.97 |
| 2 | 3 | 9 | 10.67 | 4 | 8 | 24 | 10.59 |
| 2 | 4 | 10 | 9.46 | 5 | 9 | 25 | 11.27 |
| 2 | 4 | 11 | 9.16 | 5 | 9 | 26 | 10.72 |
| 2 | 4 | 12 | 9.39 | 5 | 9 | 27 | 10.81 |
| 3 | 5 | 13 | 11.20 | 5 | 10 | 28 | 8.81 |
| 3 | 5 | 14 | 11.14 | 5 | 10 | 29 | 8.87 |
| 3 | 5 | 15 | 11.16 | 5 | 10 | 30 | 9.08 |

Table 25.17 Data for Example 25.13.

Total variation at *i*th level



Variation at ith level

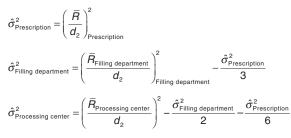


Figure 25.39 Components of variation equations for Example 25.13.

| | I | I | | | | | | I | | | | | | | | | | | I | | |
|--------------------------------------------|-------------------------------------------|------|------|------|-------|-------|-------|--------|-------|-------|-------|------|------|------|-------|-------|-------|------|------|------|--|
| | Moving range for processing centers | | | | | | | | 0.27 | | | | | 647 | 0.44 | | | | | | |
| | Average for filling departments | | | 000 | 70% | | | | | 10.00 | 60.01 | | | | | 10 54 | +C.UI | | | | |
| | Range for filling departments | | | 1.61 | | | | - 1.51 | | | 1.26 | | | | | | | | | | |
| | Average for prescriptions | | 9.01 | | | 10.62 | | | 10.85 | | | 9.34 | | | 11.17 | | | 9.90 | | | |
| | Range for prescriptions | | 0.88 | | | 0.12 | | 0.12 | | | 0.35 | | | 0.3 | | | 0.06 | | | 0.33 | |
| mple 25.13. | Time (in days) | 8.45 | 9.33 | 9.26 | 10.56 | 10.68 | 10.63 | 10.85 | 11.02 | 10.67 | 9.46 | 9.16 | 9.39 | 11.2 | 11.14 | 11.16 | 9.84 | 10.1 | 9.77 | | |
| Excel computation table for Example 25.13. | Prescription | 1 | 2 | 3 | 4 | 5 | 9 | 7 | 8 | 6 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | | |
| Excel computat | Filling department | 1 | 1 | 1 | 2 | 2 | 2 | 3 | З | 3 | 4 | 4 | 4 | 5 | 5 | 5 | 9 | 9 | 9 | | |
| Table 25.18 | Processing center | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | | |

Continued

Part VI.B.6

| Table 25.18 Continued. | Continued. | | | | | | | |
|--------------------------------|-----------------------|--------------|-------------------|----------------------------|------------------------------|-------------------------------------|---------------------------------------|-------------------------------------------|
| Processing center | Filling department | Prescription | Time (in days) | Range for prescriptions | Average for prescriptions | Range for filling departments | Average for filling departments | Moving range for processing centers |
| 4 | 7 | 19 | 9.94 | | | | | |
| 4 | 7 | 20 | 96.6 | 0.35 | 10.06 | | | |
| 4 | 7 | 21 | 10.29 | | | 070 | 10.41 | CF 0 |
| 4 | 8 | 22 | 10.7 | | | 60.0 | 10.41 | CT.U |
| 4 | 8 | 23 | 10.97 | 0.38 | 10.75 | | | |
| 4 | 8 | 24 | 10.59 | | | | | |
| IJ | 6 | 25 | 11.27 | | | | | |
| IJ | 6 | 26 | 10.72 | 0.55 | 10.93 | | | |
| 5 | 6 | 27 | 10.81 | | | 5 | 000 | 010 |
| 5 | 10 | 28 | 8.81 | | | 7.01 | 06.6 | 0.40 |
| 5 | 10 | 29 | 8.87 | 0.27 | 8.92 | | | |
| 5 | 10 | 30 | 9.08 | | | | | |
| | | ł | Averages | 0.36 | | 1.42 | | 0.33 |

Part VI.B.6

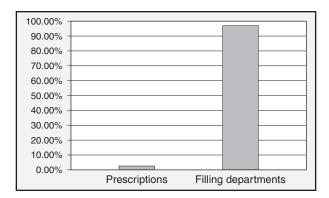


Figure 25.40 Graphical comparison of components of variation for Example 25.13.

To begin, we must recall that from application of control charts we can estimate $\hat{\sigma}_k^2$ at the bottom of a nested hierarchal structure *k* levels deep using

$$\hat{\sigma}_k^2 = \left(\frac{\overline{R}}{d_2}\right)^2$$

where

 d_2 = number of children reporting to a parent

It is difficult to generalize the COV approach mathematically, so Example 25.13 has been provided to illustrate the concept.

SIMULATION

Apply simulation tools such as Monte Carlo, dynamic process simulation, queuing theory, etc. (Apply)

Body of Knowledge VI.B.7

Simulation is the act of modeling the characteristics or behaviors of a physical or abstract system. The simulation may be conducted in various manners ranging from a simple manual representation such as an exercise taught in a Black Belt class to a highly complex mathematical model involving the use of a high-speed computer system.

Simulation becomes a good alternative when process problems become too difficult to solve analytically or when we want to focus on a problem as a system.

This does not mean that mathematics have to be excluded from simulations. On the contrary, many simulations require a host of mathematical formulas to describe system complexities. However, simulations help us gain a better understanding of the system or component interactions taking place and rapidly explore a variety of system configurations.

From a Lean Six Sigma perspective, when we think of simulation we are really thinking of process modeling. Process modeling allows us to build accurate and detailed graphical computer representations or models of chemical, physical, biological, or technical processes.

There are two types of simulations. These are *discrete* and *continuous*. Discrete simulations are also known as *event-based simulations*. Event-based simulations move through time by advancing from event to event. Examples of such events include the arrival of an order, assembly of a component, delivery of an order, rejection of a product at final inspection, and so on. Each event can be color-coded to increase the level of visibility, and statistics by event can be captured and analyzed. Further, the properties of events (that is, arrivals and departures) can be defined statistically through probability distributions or empirical distributions defined by data collected.

Continuous simulations are time-based. That is, they advance through time incrementally and determine whether an event has occurred.

The advantage of simulation models is that we can start at high levels of a process and build or expand as time, money, and effort permit. An additional advantage is that simulations are run "off-line." Unlike experimental designs, we can run simulation after simulation without ever affecting a process. Consequently, there is no adverse impact to the production, manufacturing, or service environment. Also, process-modeling software has decreased sharply in price and can now be run on a desktop or laptop computer, so significant cost advantages exist.

However, the technical sophistication of graphical simulations and the capabilities they provide has progressed to the point that they have become the proverbial double-edged sword. The advantage is that processes can be modeled easily and swiftly. The disadvantage is that processes can be modeled easily and swiftly. This means that anyone with little skill and knowledge can build a sophisticated process model, run it, and implement changes based on the results. This is unfortunate. Decades ago, two types of skills were required to develop and use simulations. The first was the skill to understand and write the simulation in an archaic non-graphical, non-user-friendly interface language. The second was the skill to generate and interpret the statistical results. Also, one had to validate and verify the simulation. Today, these steps can be easily omitted, often out of ignorance, simply because the ability to build the model has been so greatly simplified.

One of the earliest and most often-used applications of simulation is with *queueing models*. Some of the simpler queueing models have closed-form analytical solutions. However, change a few parameters and these models can quickly become mathematically intractable. Consequently, simulation becomes that last resort particularly when:

- Arrivals are not following the Poisson distribution.
- Departures are not following the exponential distribution.

- Limitations exist on queue lengths, or other special conditions exist (for example, preemption, balking, reneging, or jockeying) that are difficult to incorporate in analytical models.
- There is the need to study other than steady-state conditions such as startup. In fact, even the simplest queueing model, M/M/1 (that is, single-queue, single-server with an infinite buffer) has an extremely complete mathematical solution when steady-state conditions are not assumed.

To compare and evaluate queueing simulations, it is important to compute several meaningful queueing performance measurements. Several of these include:

- The probability that there are no customers in the system. This is usually designated as *P*₀.
- The probability that there are *n* customers in the system. This is usually designated as *P*_n.
- The probability that an arriving customer will have to wait for service. This is usually designated as *P*_w.
- The average number of customers in the system. This is usually designated as *L*. Also, it is important to compute the variance of *L*, *V*(*L*).
- The average number of customers in the queue. This is usually designated as L_q . Also, it is important to compute the variance of $L_{qr} V(L_q)$. Note: In queueing theory, the queue length includes the customer being served.
- The average time a customer spends in the system. This is usually designated as *W*. Also, it is important to compute the variance of *W*, *V*(*W*).
- The average time a customer spends in the queue. This is usually designated as W_q . Also, it is important to compute the variance of W_q , $V(W_q)$.
- The utilization factor for each server. This is usually designated as *ρ*. This is the fraction of time that each server is busy.

The above statistics not only help compare models, but also can be useful in determining steady-state conditions. For basic analytical models, these statistics can be easily computed, but when using simulation, the statistics must be computed dynamically during the simulation. Fortunately, most software packages available today compute these measures automatically.

Simulation has added significant capability to the Lean Six Sigma professional's arsenal of tools and techniques. Simulation is highly versatile and its application is limited only by the user's imagination. It has been used across all business sectors including:

Manufacturing

- Retail
- Pharmaceutical
- Banking
- Government
- Military
- Education

Furthermore, it is well suited for use and has been highly successful in transactional environments.

Some Master Black Belts and other Lean Six Sigma professionals have been heard to say that simulation lacks glamour. Perhaps this is due to its contrast to straight analytical solutions. However, one should keep in mind that the selection of tools in project applications should not be a matter of preference, but a matter of need. Don't avoid the use of a very viable tool because of personal preference. This will only shortchange the project and underachieve the potential results.

LINEAR PROGRAMMING

Understand how linear programming principles, such as critical path analysis, can be used in modeling diverse types of problems (e.g., planning, routing, scheduling, assignment, design) to optimize system performance. (Understand)

Body of Knowledge VI.B.8

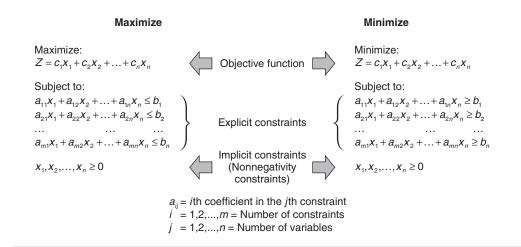
Linear programming is a constrained optimization technique and comprises both a linear objective function and linear constraints. It has found uses in a wide variety of problems, including assignment, scheduling, transportation, and mixture problems. The general form of a linear programming model is shown in Figure 25.41 for both maximization and minimization problems. Notice that Figure 25.41 doesn't illustrate equality constraints. However, such constraints as

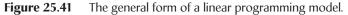
$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1$$

are written as two separate constraints as follows:

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \le b_1$$
$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \ge b_1$$

Linear programming is based on the idea that the linear set of constraints forms a convex polyhedron whose corner points are basic feasible solutions to the set of





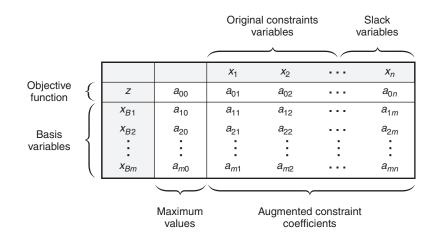


Figure 25.42 The general form of the simplex tableau.

constraints. The *simplex method* is used to systematically explore the basic feasible solutions in search of a single point that either maximizes or minimizes the objective function. In essence, the simplex method solves simultaneous linear equations using linear algebra. The real crux of the method lies with the selection of variables to enter the basis and those to depart.

Figure 25.42 illustrates the general form of the simplex tableau. Slack variables have been introduced, which have created the augmented matrix. The augmented matrix is essentially an identity matrix, and the corresponding slack variables represent the initial basic feasible solution. Then, using the simplex algorithm, variables are systematically moved into and out of the basis based on their contribution to the objective function. Figure 25.43 provides the simplex algorithm in flowchart form.

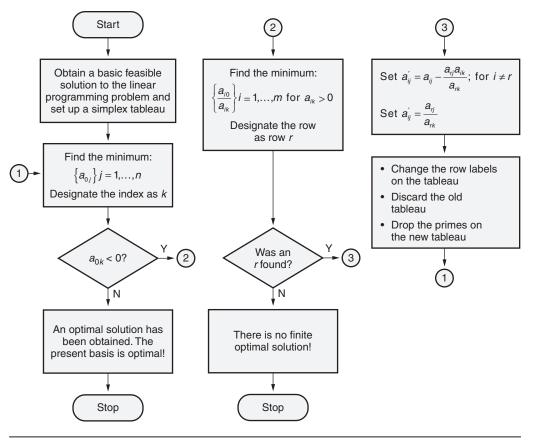


Figure 25.43 The simplex algorithm flowchart. *Source:* Zionts (1974).

EXAMPLE 25.14

This example has been obtained from Zionts (1974).

Suppose that a dog food manufacturer, Canine Products, Inc., produces two blends of dog food, Frisky Pup and Husky Hound. Two raw materials, cereal and meat, are available. The manufacturer wants to find a production mix that maximizes profits.

Frisky Pup dog food is a blend of 1 pound cereal and 1.5 pounds meat, and sells for \$0.70 per 2.5-pound package. Husky Hound dog food is a blend of 2 pounds cereal and 1 pound meat and sells for \$0.60 per 3-pound package. Cereal costs \$0.10 per pound; meat costs \$0.20 per pound.

Husky Hound dog food costs \$0.18 cents per package for packaging, and Frisky Pup dog food costs \$0.14 per package for packaging. Blending of the meat and cereal is accomplished automatically during the packaging. Frisky Pup requires the use of a special packaging machine. In a one-month period the company has available 240,000 pounds of cereal and 180,000 pounds of meat for which it has contracted at the above prices. If the meat or cereal is not completely used, there is no alternate use for it. The special packaging machine for Frisky Pup dog food can package as many as 110,000 packages per month. The Husky Hound packaging facility is of sufficient capacity to handle any mixture of products, given the raw material available.

The marketing manager of Canine Products estimates that all of Canine production—whatever the mixture of the products, given the raw materials available—will be sold at the indicated prices. Assuming that all of the variable costs of production are indicated above, what is the production plan that maximizes contribution to overhead and profits?

To begin, we must first develop the constraint set. We'll start by defining the variables. Let

 x_1 = Number of packages of Frisky Pup produced per month

 x_2 = Number of packages of Husky Hound produced per month

Now we can compute the number of pounds of each variable:

$$x_1$$
 pounds = $\left(\frac{1 \text{ pound}}{\text{package}}\right)(x_1 \text{ packages})$

$$2x_2 \text{ pounds} = \left(\frac{2 \text{ pound}}{\text{package}}\right)(x_2 \text{ packages})$$

The constraint on the total cereal required is given by

Total cereal required =
$$x_1 + 2x_2 \le 240,000$$

and the constraint on the total meat required is given by

Total meat required =
$$1.5x_1 + x_2 \le 180,000$$

Also, we know there is an upper limit on the amount of cereal. Therefore,

 $x_1 \le 110,000$

Finally, we must not forget to include the nonnegativity constraints:

 $x_1 \ge 0, x_2 \ge 0$

We know from the problem statement that Frisky Pup sells at \$0.70 per package, and the variable costs per package are \$0.14. Similarly, for Husky Hound we have \$0.60 and \$0.18, respectively. Our objective function is now of the form

Maximize: $z = (\$0.70 - \$0.14)x_1 + (\$0.60 - \$0.18)x_2$ $z = \$0.56x_1 + \$0.42x_2$

Continued

At last, our linear programming model has become

Maximize:
$$Z = 0.56x_1 + 0.42x_2$$

Subject to: $x_1 + 2x_2 \le 240,000$
 $1.5x_1 + x_2 \le 180,000$

 $x_1 \le 110,000$

 $x_1, x_2 \ge 0$

With our model finally assembled, we can construct the first tableau by building the augmented matrix and adding the slacking variables. This is shown in Figure 25.44.

By applying the algorithm in the flowchart illustrated in Figure 25.43, we can generate a successive series of tableaux until we reach an optimal solution or determine that no finite optimal solution exists. From tableau 1 in Figure 25.44, x_1 enters the basis because it is the largest negative coefficient, a_{31} becomes the pivot entry, and x_5 is chosen to depart the basis.

Some specific a_{ij} entries for the first tableau are

$$a'_{00} = a_{00} - \frac{a_{30}a_{01}}{a_{31}} = 0 - \frac{(110,000)(-0.56)}{1} = 61,600$$

$$a'_{02} = a_{02} - \frac{a_{32}a_{01}}{a_{31}} = -0.42 - \frac{(0)(-0.56)}{1} = -0.42$$

$$a'_{10} = a_{10} - \frac{a_{30}a_{11}}{a_{31}} = 240,000 - \frac{(110,000)(1)}{1} = 130,000$$

$$a'_{01} = a_{01} - \frac{a_{31}a_{01}}{a_{31}} = 0$$

$$a'_{30} = \frac{a_{30}}{a_{31}} = \frac{110,000}{1} = 110,000$$

$$a'_{25} = a_{25} - \frac{a_{35}a_{21}}{a_{31}} = 0 - \frac{(1)(1.5)}{1} = -1.5$$

This process continues until there are no more negative coefficients left in the objective function or it is determined that a finite optimal solution does not exist.

Tableau 4 provides the optimal solution as

 $x_1 = 60,000$ packages of Frisky Pup

 $x_2 = 90,000$ packages of Husky Hound

 $x_5 = 50,000$ packages of Frisky Pup

z = \$71,400

Notice that the slack variable, x_5 , has a value associated with it. This means that there should be idle capacity equivalent to 50,000 packages of Frisky Pup dog food.

| | | | ¥ | | | | | |
|-----------|-----------------------|---------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|---|
| Tableau 1 | | | <i>x</i> ₁ | <i>x</i> ₂ | <i>x</i> ₃ | <i>x</i> ₄ | <i>x</i> ₅ | |
| | Ζ | 0 | -0.56 | -0.42 | 0 | 0 | 0 | |
| | <i>x</i> 3 | 240,000 | 1 | 2 | 1 | 0 | 0 | |
| | <i>x</i> ₄ | 180,000 | 1.5 | 1 | 0 | 1 | 0 | |
| | <i>x</i> ₅ | 110,000 | 1 | 0 | 0 | 0 | 1 | |
| | | 1 | | ¥ | | | | |
| Tableau 2 | | | <i>x</i> ₁ | <i>x</i> ₂ | <i>x</i> ₃ | <i>x</i> ₄ | <i>x</i> ₅ | |
| | Ζ | 61,600 | 0 | -0.42 | -0 | 0 | 0.56 | |
| | <i>x</i> ₃ | 130,000 | 0 | 2 | 1 | 0 | -1 | |
| | <i>x</i> ₄ | 15,000 | 0 | 1 | 0 | 1 | -1.5 | |
| | <i>x</i> ₁ | 110,000 | 1 | 0 | 0 | 0 | 1 | |
| | | | | | | | ¥ | _ |
| Tableau 3 | | | <i>x</i> ₁ | <i>x</i> ₂ | <i>x</i> ₃ | <i>x</i> ₄ | <i>x</i> ₅ | |
| | Ζ | 67,900 | 0 | 0 | 0 | 0.42 | -0.07 | |
| | <i>x</i> 3 | 100,000 | 0 | 0 | 1 | -2 | 2 | |
| | <i>x</i> ₂ | 15,000 | 0 | 1 | 0 | 1 | -1.5 | |
| | <i>x</i> ₁ | 110,000 | 1 | 0 | 0 | 0 | 1 | |
| | | | | | | | | • |
| Tableau 4 | | | <i>x</i> ₁ | <i>x</i> ₂ | <i>x</i> ₃ | <i>x</i> ₄ | <i>x</i> ₅ | |
| | Ζ | 71,400 | 0 | 0 | 0.035 | 0.35 | 0 | |
| | <i>x</i> ₅ | 50,000 | 0 | 0 | 0.5 | -1 | 1 | |
| | <i>x</i> ₂ | 90,000 | 0 | 1 | 0.75 | -0.5 | 0 | |
| | <i>x</i> ₁ | 60,000 | 1 | 0 | -0.5 | 1 | 0 | |

Figure 25.44 The simplex tableaux for Example 25.14.

Generally, solving complex linear programming problems takes sophisticated mathematical software, much of which is readily available off-the-shelf. However, for relatively small problems, Microsoft's Excel Solver add-in functionality is convenient and works well. Furthermore, it is might be sufficient for brief introductory training sessions to the concept.

RELIABILITY MODELING

Use reliability modeling and tools to enhance reliability of a product or process and reliability growth modeling. (Apply)

Body of Knowledge VI.B.9

Borror (2009) defines *reliability* as "the probability that a product or service will perform properly under specified conditions for a specified period of time." The main factors that affect a system's reliability include:

- *Design and configuration of the system*. We will see a bit later that certain system configurations increase reliability. This includes the standby and *k* out of *n* systems.
- *Reliability of the individual components.* Individual component reliability greatly affects the overall system reliability. We will see this clearly when we look at a series configuration.
- *Operating environment of the system*. Systems are designed to operate within specific environmental conditions. Exceeding those conditions can dramatically affect a system's reliability. One way to combat the problem is through component derating. *Derating* is the practice of using components at lower stress levels than those stated by the component specifications.
- *Manufacturing or latent defects.* Many times, component defects will not be evident upon inspection or stress testing by the manufacturer. Only when operating in field conditions do latent defects become evident. *Latent defects* are hidden defects in either material and/or workmanship that may cause a component to malfunction or fail, but are not discoverable through inspection.
- *Preventive maintenance*. Preventive maintenance is a necessary practice to maintain reliability levels, and applies to systems that are repairable.

Estimating Reliability

Suppose *N* identical components are tested. During a specified time interval *t*, we observe *x* failures and (N - x) survivors. Since reliability is defined as the cumulative probability function of success, then at time *t* the reliability, *R*(*t*) is

$$R(t) = \frac{N - x}{N}$$

Therefore, the reliability function at time t is the fraction of all components that have survived for a time greater than t. Also, R(t) is used as the estimate of the probability that a randomly selected component will survive for a time greater than t.

The cumulative distribution function of failure can be defined as

$$F(t) = \frac{x}{N}$$

From the two equations above, we now know that

$$1 = R(t) + F(t)$$
$$R(t) = 1 - F(t)$$
$$F(t) = 1 - R(t)$$

System Configurations

Numerous system configurations exist. However, when examined closely, these configurations are really combinations of several basic ones. These include the:

- Series
- Parallel
- k-out-of-n
- Standby

Series Systems. A typical series system is composed of *n* components (or subsystems) connected end-to-end as exhibited in Figure 25.45a. A failure of any component results in the failure of the entire system. Generally, it is assumed that the

EXAMPLE 25.15

A system is in a series configuration with the following individual component reliabilities:

 $R_1 = 0.96$ $R_2 = 0.92$

$$R_3 = 0.94$$

$$R_4 = 0.90$$

Compute the system reliability.

Applying the above formula, we obtain

 $R(t) = 0.96 \times 0.92 \times 0.94 \times 0.90 = 0.7472$

Because of the multiplicative nature of the probabilities, the overall system reliability is lower than the lowest individual component reliability.

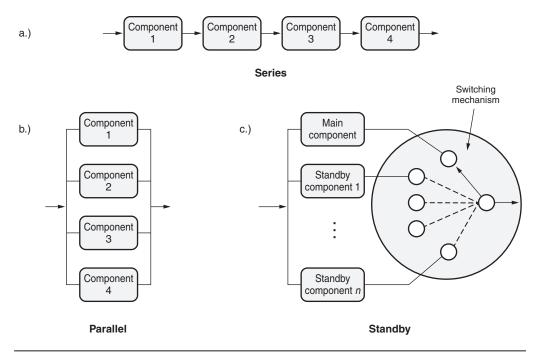


Figure 25.45 Common reliability system configurations. *Source:* Borror (2009).

components in a series configuration are independent. Thus, the reliability of the system is the product of the reliability of the individual components and is given by

$$R(t) = R_1(t) \times R_2(t) \times \dots \times R_n(t) = \prod_{i=1}^n R_i(t)$$

Parallel Systems. In a parallel system, components are connected in parallel so that the failure of one or more paths still allows the remaining paths to perform properly. A parallel system is exhibited in Figure 25.45b. The system fails when all components fail. Generally, it is assumed that the components in a parallel configuration are independent. Thus, the reliability of the system is given by

$$R(t) = 1 - [F_1(t) \times F_2(t) \times \dots \times F_n(t)] = 1 - \prod_{i=1}^n F_i(t)$$

k-out-of-*n* Systems. In a *k*-out-of-*n* systems configuration, *k*-out-of-*n* components must operate for the system to function properly. This is a direct application of the binomial distribution. Generally, it is assumed that the components in a *k*-out-of-*n* system are both independent and identical. Thus, the reliability of the system is given by

$$R(t) = \sum_{i=k}^{n} C_{i} p^{i} (1-p)^{n-i}$$

where

$$_{n}C_{i} = \frac{n!}{i!(n-i)!}; i = 1, 2, ..., n$$

 p^i = Reliability of the *i*th component

EXAMPLE 25.16

Assume a parallel system with three components. The individual component reliabilities are:

$$R_1 = 0.95$$

 $R_2 = 0.93$

 $R_3 = 0.91$

Compute the system reliability.

Applying the above formula, we obtain

 $R(t) = 1 - \left[(1 - 0.95)(1 - 0.93)(1 - 0.91) \right] = 0.99969$

Notice how the parallel configuration has increased the system reliability. In fact, the overall system reliability is now greater than the highest individual component reliability.

EXAMPLE 25.17

A system has four independent and identical components. For the system to function properly, a minimum of two components is required. Each component operates with a reliability of 0.90. Compute the system reliability.

Applying the given formula, we obtain

$$R(t) = \sum_{i=2}^{3} {}_{4}C_{i}(0.90)^{i}(1-0.90)^{4-i}$$

= ${}_{4}C_{2}(0.90)^{2}(0.10)^{2} + {}_{4}C_{3}(0.90)^{3}(0.10)^{1} + {}_{4}C_{4}(0.90)^{4}(0.10)^{0}$
= $(6)(0.90)^{2}(0.10)^{2} + (4)(0.90)^{3}(0.10)^{1} + (1)(0.90)^{4}(0.10)^{0}$
= $0.0486 + 0.2916 + 0.6561$
= 0.9963

Since the *k*-out-of-*n* configuration is essentially a parallel system conditioned on *k* components working, the overall system reliability is once again higher than the highest individual component reliability.

Standby Systems. In standby systems, the standby components function only upon the failure of the main component. A standby system is exhibited in Figure 25.45c. Generally, it is assumed that:

- Standby components are identical.
- The switch never fails.
- Standby components never fail in the standby mode.

Thus, the reliability of the system with *n* standby components is given by

$$R(t) = e^{-\lambda t} \sum_{i=0}^{n} \frac{(\lambda t)^{i}}{i!}$$

 λ = Component failure rate

Reliability Growth Models

Reliability growth is the improvement in product reliability over a period of time. This improvement in reliability is due to changes in the product design. In the

EXAMPLE 25.18

A standby system has three components in the standby mode. All components have a constant failure rate of $\lambda = 0.02$. Compute the system reliability at 75 hours of continuous operation.

Applying the above formula, we obtain

$$R(75) = e^{-(0.02)(75)} \sum_{i=0}^{3} \frac{\left[(0.02)(75) \right]^{i}}{i!}$$
$$= e^{-1.5} \sum_{i=0}^{3} \frac{(1.5)^{i}}{i!}$$
$$= e^{-1.5} \left[\frac{1.5^{0}}{0!} + \frac{1.5^{1}}{1!} + \frac{1.5^{2}}{2!} + \frac{1.5^{3}}{3!} \right]$$
$$= e^{-1.5} [1.0 + 1.5 + 1.125 + 0.5625]$$
$$= e^{-1.5} [4.1875]$$
$$= (0.22313)(4.1875)$$
$$= 0.9344$$

Notice that the index, *i*, runs from 0 to 3. This accounts for four terms in the above equations, but there were only three standby components. Remember, the fourth component is the main component. In a standby configuration, all components must fail for the entire system to fail.

early stage of new product development, problems exist in the design that negatively affect reliability. Early development activities such as FMEA, reliability prediction, and early testing on prototypes and engineering models will begin to surface these problems. Testing will resolve some problems while surfacing additional ones. More changes to the design are made, and the cycle repeats. This is known at the *test, analyze, and fix* (TAAF) process, which will result in reliability growth.

Models that can be used to track reliability growth include the Duane model and the AMSAA (that is, Crow) model. Both require the total unit test time, *T*, and the total number of failures, *r*, data from all early testing to be recorded. These models are compared in Table 25.19.

Notice that both the objectives and key assumptions are the same for both models. In particular, the second assumption states, "Design changes will be incorporated immediately after a failure and before testing resumes." This is a key assumption in both models, but in practice can be easily violated.

| Comparison categories | Duane growth model | AMSAA growth model |
|--------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Objective | Find failures during test and learn from those failures by redesigning to eliminate them | Same |
| Key assumptions | The relationship between the MTBF and test time will be a straight line on log-log paper | Same |
| | Design changes will be incorporated immediately after a failure and before testing resumes | |
| Parameters | Growth rate, <i>a</i> = change in MTBF/time interval over which change occurred <i>K</i> , a constant that is a function of the initial MTBF <i>T</i> , the test time | λ = Initial failure rate = $\left(\frac{1}{\text{MTBF}}\right)$ β = Growth rate T = Test time |
| Equations | Cumulative MTBF: MTBF _c = $\left(\frac{1}{K}\right)T^{a}$ Instantaneous MTBF: MTBF _i = $\frac{\text{MTBF}}{1-a}$ Test time: $T = \left[(\text{MTBF}_{i})(K)(1-a)\right]^{\frac{1}{a}}$ | Cumulative failure rate: $\lambda_c = \lambda T^{\beta-1}$ Instantaneous failure rate: $\lambda_i = \lambda \beta T^{\beta-1}$ Test time: $T = \left(\frac{\lambda_i}{\lambda\beta}\right)^{\frac{1}{\beta-1}}$ |

Table 25.19 A comparison of the Duane and AMSAA growth models

QUALITATIVE ANALYSIS

Use appropriate qualitative analysis tools (affinity diagrams, force field analysis, etc.) and analyze the results. (Analyze)

Body of Knowledge VI.B.10

Many of our qualitative tools and techniques help us build relationships between input and output variables, though most often in a highly subjective manner. For example, inputs may be gathered or even synthesized from facts or research, and the output may be simply a decision resulting from a studious analysis and review of the input. Such is the case with the PEST and SWOT analyses discussed in Chapter 1.

The affinity diagram provides another example where relationships are subjective. Tague (2005) cites an example wherein a team is brainstorming a list of potential performance measures. Although the brainstorming resulted in numerous measures, these measures were unorganized. Over several days of review by individual team members, relationships between the unorganized list of performance measures began to emerge. Consequently, the list of performance measures could be subcategorized in natural groups such as safety, product quality, maintenance, product volume, and cost. This simple categorization established relationships between variables where previously no relationship was evident.

Tague (2005) and Andersen (2007) cite numerous qualitative tools, several of which build relationships between input and output variables. A few notable tools are identified below. However, many of them have been addressed in their books or in *The Certified Six Sigma Black Belt Handbook*, Second Edition. The tools discussed in this book are:

- Effective–achievable (this is shown in Table 6.6 and is also called an impact–effort chart).
- Force field analysis (this is shown in Figure 7.3).
- PEST analysis (this is shown in Table 1.3).
- SWOT analysis (this is shown in Table 1.2).
- Systems thinking relationship diagram (this is shown in Figure 7.1).

The following tools were all addressed in detail in *The Certified Six Sigma Black Belt Handbook*, Second Edition. As such, they will not be discussed again here. These tools include:

- Affinity diagram.
- Cause-and-effect diagram.

- Circle diagram (this is similar to the systems thinking relationship diagram shown in Figure 7.1 in this book).
- Flowchart.
- FMEA.
- House of quality.
- Interrelationship digraphs.
- Matrix diagram.
- Prioritization matrix.
- Process decision program chart.
- Pugh matrix (also called *Pugh analysis* or *decision matrix*).
- SIPOC diagram.
- SWOT analysis.
- Tree diagram.

Notice that six of the tools in the above list are included in the *seven management and planning tools*.

Chapter 26 Design of Experiments (DOE)

This chapter covers a wide variety of tools and techniques associated with experimental designs. Many practicing Master Black Belts will find some of this material new to them. However, every attempt has been made to present the material at the designated Bloom taxonomy level found in the body of knowledge. Remember, this handbook is designed first and foremost to prepare an individual to sit for the ASQ Certified Six Sigma Master Black Belt Examination. It is not intended to cover all topics in detail. Consequently, you may not find the material sufficiently informative in all areas. Therefore, it is recommended you pursue more in-depth resources, many of which will be found in the Bibliography.

Also, several topics covered in this chapter were originally found in the Certified Six Sigma Black Belt Body of Knowledge issued in 2001. Some of these topics were removed in the 2007 release and have now been reincorporated into the Certified Six Sigma Master Black Belt Body of Knowledge.

FACTOR ANALYSIS

Apply and interpret factor relationship diagrams. (Apply)

Body of Knowledge VI.C.1

Note: Although this subsection is titled "Factor Analysis," it has nothing to do with factor analysis in the traditional sense. This is an error in the development of the body of knowledge. The title "Factor Relationship Diagram" would be more appropriate.

The *factor relationship diagram* is a visual tool that assists in the planning of experiment designs. Figure 26.1 illustrates the basic form of a factor relationship diagram. Notice that it is the familiar tree diagram. Typically, a factor relationship diagram is composed of two structures: the design structure and the unit structure. The design structure contains those factors that are deliberately manipulated during the experimental design. The unit structure is determined

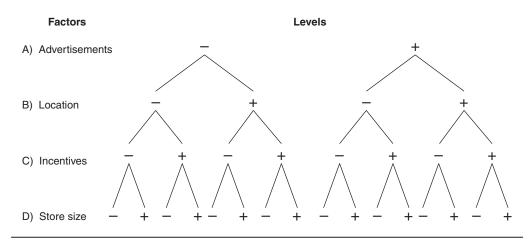


Figure 26.1 A basic factor relationship diagram (tree diagram).

by how we manage noise and other unmanipulated sources of variation during the experimental design. In addition to these two structures, we have the *inference space*. Hild (2009) states, "The inference space represents the range of conditions over which the experiment's results can expect to repeat." Horizontal lines on the factor relationship diagram represent the inference space. Overall, the factor relationship diagrams help us:

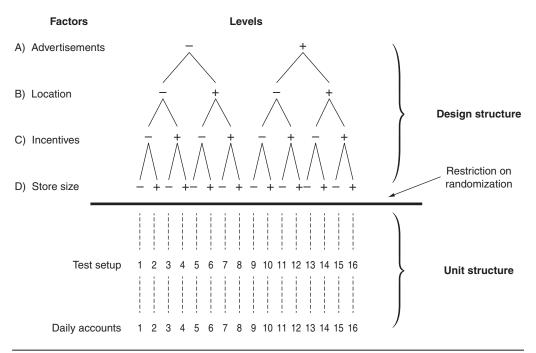
- Graphically understand the experimental design setup
- Allocate degrees of freedom
- Place appropriate restriction on randomization
- Decide on repeats or replicates

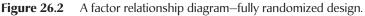
EXAMPLE 26.1

A retail clothing store is experiencing a large amount of variation in the number of new credit card accounts opened at its stores. The CEO is disappointed since the sales department spent a significant portion of its budget sending advertisements and incentives to some customers. The retail company generally operates two sizes of stores based on its ability to locate and secure leased space or purchase land to build. Construct some feasible designs using the factor relationship diagram approach.

Figure 26.2 illustrates a fully randomized design based on the problem description. Notice the horizontal line indicating the inference space. Figure 26.3 provides a completely randomized design diagram with two replicates, and Figure 26.4 provides the completely randomized design diagram with repeats.

Once these designs have been completed, the analyst can decide how to proceed, data can be collected, and the designs can be analyzed using ANOVA methods as usual.





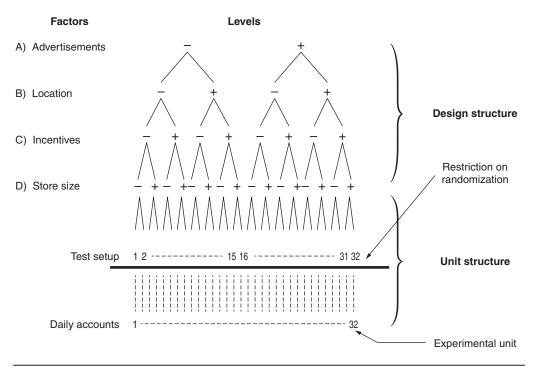


Figure 26.3 A factor relationship diagram–fully randomized design with two replicates.

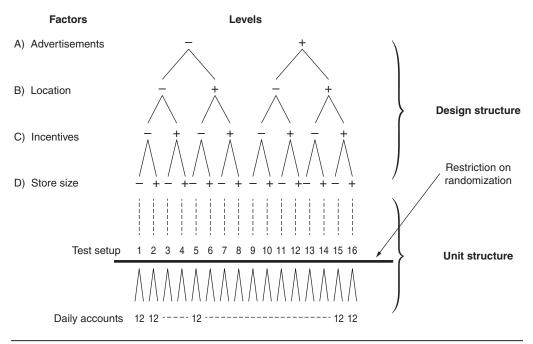


Figure 26.4 A factor relationship diagram–fully randomized design with repeats.

COMPLEX BLOCKING STRUCTURES

Recognize other designs for handling more complex blocking structures, including balanced incomplete block design (BIBD). (Understand)

Body of Knowledge VI.C.2

Montgomery (2009) succinctly defines blocking "as a technique used to improve the precision with which comparisons among the factors of interest are made. Often blocking is used to reduce or eliminate the variability transmitted from nuisance factors—that is, factors that may influence the experimental response but in which we are not directly interested."

Numerous blocking designs exist. However, in this section we will address some of the more common models. These include:

- Randomized complete block design (RCBD)
- Balanced incomplete block design (BIBD)

- Latin squares
- Greco-Latin squares

Hyper-Greco-Latin squares and Yourdon squares will not be discussed since they are an extension of the Latin squares concept.

Randomized Complete Block Design (RCBD)

The ASQ *Glossary and Tables for Statistical Quality Control*, Fourth Edition (2005) defines a randomized block design as "an experimental design consisting of *b* blocks with *t* treatments assigned via randomization to the experiment units within each block. This is a method for controlling the variability of experimental units. For the completely randomized design, no stratification of the experimental units is made. In the randomized block design, the treatments are randomly allocated within each block; that is, the randomization is restricted."

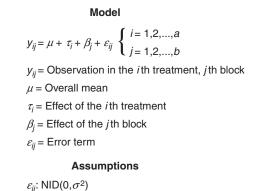
The randomized complete block design is probably the most used and the most useful of the experimental designs. Some of the advantages and disadvantages of using the RCBD include:

- Advantages:
 - No limit on the number of treatments and blocks.
 - Provides more accurate results than the completely randomized design due to the blocking.
 - The statistical analysis is easy even with missing data.
 - Allows the unbiased error to be calculated for specific treatments.
 - Some treatments may be replicated more times than others.
- Disadvantages:
 - It is not particularly suitable for a large number of treatments because the blocks become too large.
 - It is unsuitable when the complete block contains considerable variability.
 - The interaction between the blocks and the treatments affects the error.
 - The experimental design may be less efficient than the completely randomized design when there is missing data.

Figure 26.5 provides the model and an example of the design table for the randomized complete block design.

Balanced Incomplete Block Design (BIBD)

The ASQ *Glossary and Tables for Statistical Quality Control,* Fourth Edition (2005) defines an incomplete block design as "a design in which the design space is



| Block 1 | Block 2 | | Block n |
|------------------------|------------------------|---|------------------------|
| <i>Y</i> ₁₁ | <i>Y</i> ₁₂ | | У _{1<i>b</i>} |
| <i>y</i> ₂₁ | y ₂₂ | | y _{2b} |
| | | - | : |
| <i>Y</i> _{a1} | y _{a2} | | У _{аb} |

y

Figure 26.5Randomized complete block design.Source: Montgomery (2009).

Model

$$y_{ij} = \mu + \tau_i + \beta_j + \varepsilon_{ij} \begin{cases} i = 1, 2, ..., a \\ j = 1, 2, ..., b \end{cases}$$

$$y_{ij} = \text{Observation in the ith treatment, jth block}$$

$$\mu = \text{Overall mean}$$

$$\tau_i = \text{Effect of the ith treatment}$$

$$\beta_j = \text{Effect of the jth block}$$

$$\varepsilon_{ij} = \text{Error term}$$
Assumptions
$$\varepsilon_{ij} : \text{NID}(0, \sigma^2)$$

Figure 26.6 Balanced incomplete block design. *Source:* Montgomery (2009).

subdivided into blocks in which there are insufficient experimental units available to run a complete replicate of the experiment." Montgomery (2005a) builds on this and goes on to say that a balanced incomplete block design (BIBD) "is an incomplete block design in which any two treatments appear together an equal number of times."

Incomplete designs occur for several reasons such as physical limitations on running experiments, material shortages, or material defects discovered just before the experiment is about to run and time does not permit replacement.

Figure 26.6 provides the model and an example of the design table for the balanced incomplete block design. Notice that the mathematical model is constructed the same as the randomized complete block design; however, there are gaps in the data.

Latin Squares

The ASQ *Glossary and Tables for Statistical Quality Control,* Fourth Edition (2005) defines a Latin square design as "a design involving three factors in which the combination of the levels of any one of them with the levels of the other two appears once and only once. It is often used to reduce the impact of two blocking factors by balancing out their contributions. A basic assumption is that these block factors do not interact with the factor of interest or with each other. This design is particularly useful when the assumptions are valid for minimizing the amount of experimentation." Some of the advantages and disadvantages of using Latin squares include:

- Advantages:
 - They handle the situation when we have several nuisance factors and we either can not combine them into a single factor or we wish to keep them separate.
 - They allow experiments with a relatively small number of runs.
- Disadvantages:
 - The number of levels of each blocking variable must equal the number of levels of the treatment factor.
 - The Latin square model assumes there are no interactions between the blocking variables or between the treatment variable and the blocking variable.

Figure 26.7 provides the model and an example of the design table for a 4×4 Latin square.

Model
$$y_{ijk} = \mu + \alpha_i + \tau_j + \beta_k + \varepsilon_{ijk} \begin{cases} i = 1, 2, ..., p \\ j = 1, 2, ..., p; p \ge 3 \\ k = 1, 2, ..., p \end{cases}$$

 y_{ijk} = Observation in the *i*th row, *k*th column for the *j*th treatment

 $\mu = Overall mean$

 α_i = Effect of the *i*th row

 $\tau_i = \text{Effect of the } j \text{ th treatment}$

 β_k = Effect of the *k* th column

 ε_{iik} = Error term

Assumptions

 $\varepsilon_{ijk} \sim \text{NID}(0, \sigma^2)$

No interactions between the blocking variables or between the blocking variables and the treatment variable

| | | Colu | umn | |
|-----|---|------|-----|---|
| | С | D | А | В |
| Row | В | С | D | А |
| | А | В | С | D |
| | D | А | В | С |

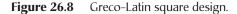
Figure 26.7 Latin square design.

| Model | i = 1,2,,p | | | | |
|-------------------------------------------------------------------------------------------------|--------------------------------------------------------------------|--|--|--|--|
| Model $y_{ijk} = \mu + \theta_i + \tau_j + \omega_k + \psi_l + \varepsilon_{ijk}$ | $j = 1, 2,, p; p \ge 3$ $k = 1, 2,, p; p \ne 6$ l = 1, 2,, p | | | | |
| <i>y_{ijkl}</i> = Observation in the <i>i</i> th row for Latin letter <i>j</i> and Gree | / and /th column | | | | |
| μ = Overall mean | | | | | |
| θ_i = Effect of the <i>i</i> th row | | | | | |
| τ_j = Effect of the Latin letter treation | atment <i>j</i> | | | | |
| ω_k = Effect of the Greek letter tr | reatment k | | | | |
| ψ_l = Effect of the <i>j</i> th column | | | | | |
| $\varepsilon_{iikl} = \text{Error term}$ | | | | | |

| Assump | otions |
|--------|--------|
|--------|--------|

 $\varepsilon_{ijkl} \sim \mathsf{NID}(0,\sigma^2)$

No interactions between the blocking variables or between the blocking variables and the treatment variable



Greco-Latin Squares

The ASQ *Glossary and Tables for Statistical Quality Control,* Fourth Edition (2005) defines a Greco-Latin square design as "an extension of a Latin square design that involves four factors in which the combination of the level of any one of them with the levels of the other three appears once and only once."

The Greco-Latin square is basically one Latin square superimposed on another. One square contains Greek letters that appear once and only once while the other square contains Latin letters that appear once and only once. The squares are orthogonal and the analysis of variance is similar to that of the Latin square.

Figure 26.8 provides the model and an example of the design table for a 4×4 Greco-Latin square.

OTHER DOE APPROACHES

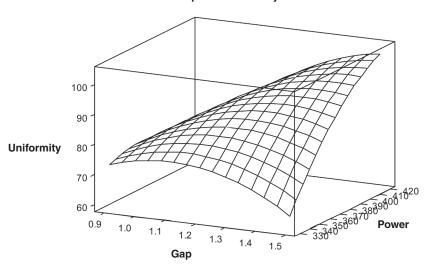
Recognize when to apply approaches such as response surface methodology (RSM), mixture experiments, evolutionary operations (EVOP), split-plot designs, Taguchi, D-optimal designs, etc. (Understand)

Body of Knowledge VI.C.3

| | | Colu | umn | |
|-----|----|------|-----|----|
| | Aα | Bβ | Cγ | Dδ |
| Row | Вδ | Aγ | Dβ | Cα |
| HOW | Cβ | Dα | Aδ | Βγ |
| | Dγ | Cδ | Вα | Aβ |

Response Surface Methodology (RSM)

Response surface methodology, or RSM, is a sequential procedure that couples the concepts of experimental designs and optimization theory. The purpose of the methodology is to determine the optimum operating conditions for a given set of process variables. This is accomplished by modeling the functional relationship between a set of independent variables and the response variable using experimental design techniques. This relationship establishes the response surface, say, $E(y) = f(x_1, x_2)$, where the actual analysis is performed. Figure 26.9 represents an example of a response surface, while a corresponding plot of the contour lines is shown in Figure 26.10. The importance of contour lines will become evident shortly. The surface is then searched systematically using an optimization technique such as the *method of steepest ascent*. Once an optimum point is found on the surface, the operating conditions are reset; a new response surface is modeled, and the optimum for that new response surface is sought. This procedure continues until no further cost-effective gain in the response variable is evident. Typically, response surfaces are modeled initially using first-order approximation models before moving to higher-order approximation models. This concept is depicted in Figure 26.11. In fact, depending on the nature of the response surface (that is, hilly and bumpy), it would not be improbable for a given RSM problem to involve models of varying orders throughout the sequential procedure in order to refine the estimate of the optimum. For this reason, it is important that the experimental designs test for evidence of quadratic, cubic, and/or higher levels of interaction. Formulas for first- and second-order models are given in Table 26.1.



Surface plot of uniformity

Figure 26.9 Example of a response surface. *Source:* Montgomery (2009).

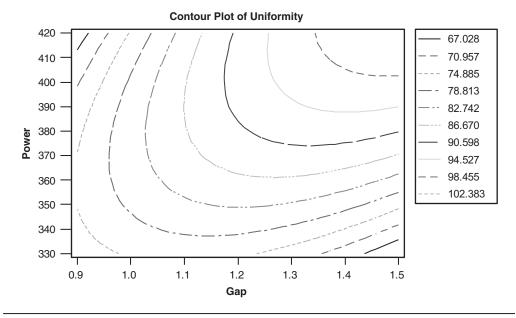


Figure 26.10Example of a contour plot.Source: Montgomery (2009).

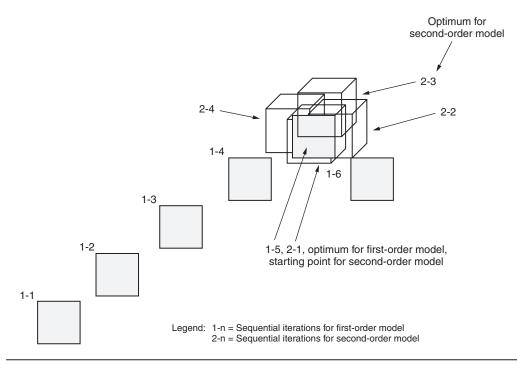


Figure 26.11 Shifting to a higher-order fitted response surface model near the region of the optimum.

| | Model | Fitted model |
|-----------------|--------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------|
| First order | $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_k x_k + \varepsilon$ | $\hat{y} = \hat{\beta}_0 + \sum_{i=1}^k \hat{\beta}_i x_i$ |
| Second order | $y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \sum_{i< j} \sum_{j=2}^{k} \beta_{ij} x_i x_j + \varepsilon$ | $y = \hat{\beta}_0 + \sum_{i=1}^k \hat{\beta}_i x_i + \sum_{i=1}^k \hat{\beta}_{ii} x_i^2 + \sum_{i$ |

 Table 26.1
 Formulas for RSM first- and second-order models.

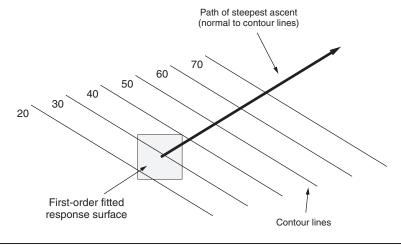


Figure 26.12 First-order fitted response surface model moving along the path of steepest ascent.

Steepest Ascent/Descent Experiments. The method of steepest ascent is a wellknown optimization technique that can be used to systematically climb a response surface in specific step sizes to seek out an optimum point. The method of steepest ascent applies to searching for the maximum point on the response surface. By contrast, the *method of steepest descent* applies when searching for the minimum point. The direction of the search (that is, steepest ascent) is normal to the contour lines, as shown in Figure 26.12, and is taken through the center of the operating region, as in Figure 26.13. Contour plots and lines of constant response are used to help the experimenter understand the response surface for the purpose of more rapidly converging on the optimum point. Because the response surface may be rather hilly or bumpy, multiple maximums or minimums or even saddle points might exist. For this reason, determination of the starting operating conditions and the step sizes of the variables is critically important. For example, if the starting operating conditions are somewhat off, the optimization technique may climb the wrong hill and reach a less than desirable maximum. Similarly, if the step sizes of the variables are too large, the technique may step over the hill, climb the wrong hill, and converge again on a less than desirable maximum.

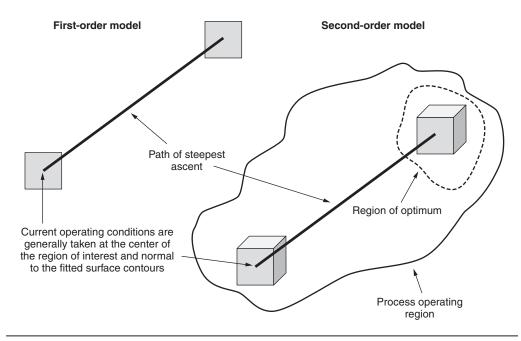


Figure 26.13 Comparison of the first-order and second-order fitted response surface models.

Conducting an RSM Analysis. The following steps may give the appearance that an RSM analysis is quick and easy to do. However, expect it to involve significant time, effort, and money:

- 1. Establish the operating region to be investigated.
- 2. Determine and run the experimental design.
- 3. Fit the data to an appropriate model. Start with a first-order approximation. Be sure to test for curvature.
- 4. Determine the path of steepest ascent, assuming it passes through $x_1 = x_2 = \ldots = x_k = 0$. Montgomery (2005a) notes that the "path of steepest ascent is proportional to the signs and magnitudes of the regression coefficients" and provides the following generalized algorithm for determining the path for *k* points:
 - a. "Choose a step size in one of the process variables, say Δx_j . Usually, we would select the variable we know the most about, or we would select the variable that has the largest absolute regression coefficient $|\hat{\beta}_i|$.
 - b. The step size in the other variables is

$$\Delta x_i = \frac{\hat{\beta}_i}{\hat{\beta}_i / \Delta x_i}; i = 1, 2, \dots, k; i \neq j.$$

c. Convert the Δx_i from the coded variables to the natural variables."

- 5. Apply the path of steepest ascent method to determine the vicinity of the optimum point.
- 6. Reset the operating conditions in the region of the optimum point and go to step 3.
- 7. Repeat the procedure (that is, go back to step 3) until no further costeffective gain in the response variable is evident.

Evolutionary Operations (EVOP)

Evolutionary operations, commonly referred to as EVOP, is a methodology devised by George Box in the late 1950s. The purpose of the methodology is to improve a process through systematic changes in the operating conditions of a given set of factors. An experimental design is established and conducted through a series of phases and cycles. The effects are tested for statistical significance against experimental error, when such error can be calculated. When a factor(s) is found to be significant, the operating conditions for that factor(s) are reset and the experiment conducted again. This process continues until no further gain is achieved. Hence, the concept of an evolution is established.

The following terminology is basic to the discussion of evolutionary operations:

- *Cycle.* Cycles are replicates of the design conducted within phases. Two or more cycles are necessary to compute experimental error.
- *Phase.* A phase comprises two or more cycles or replications of the experiment and is completed when a determination is made regarding the significance of a factor(s). When significance is demonstrated, the operating conditions for the significant factor(s) are reset and a new phase begins.

Conducting an Evolutionary Operations Analysis. Like the RSM, the steps of an EVOP also can be time-consuming:

- 1. Determine the process factors, operating conditions (that is, high, low levels), and response variable.
- 2. Establish the experimental design to be conducted.
- 3. Conduct two cycles before computing experimental error.
- 4. Compare the effects of the factors to the experimental error. If the effects fall within the range of the experimental error, conclude that they are not significant and conduct another cycle. If one or more effects fall outside the range of experimental error, reset the operating conditions and begin another phase.
- 5. Conclude the EVOP when no further gain is evident.

Taguchi Robustness Concepts

Robustness means resistance to the effect of variation of some factor. For example, if Brand A chocolate bar is very soft at 100° F and brittle at 40° F, and Brand B

maintains the same level of hardness at these temperature extremes, it could be said that Brand B is more robust to temperature changes in this range. If a painting process produces the same color on moist wood as dry wood, the color is robust to variation in moisture content. The changes in temperature and humidity are referred to as *noise*. Producing products that are robust to noise of various kinds is clearly desirable. The Japanese engineer Genichi Taguchi is credited with developing techniques for improving robustness of products and processes.

One approach to improving robustness is illustrated in Table 26.2. As usual, the average value for each run is calculated and labeled. In addition, the standard deviation of the values in the run is calculated and shown in the column labeled "S."

Now the experimenter can complete the usual main effects calculations to determine the levels of each of the factors that will optimize the response value *y*. In addition, the main effects calculations can be run using the values in the *S* column to find the levels of each of the factors that will minimize the *S*-value. If these two combinations of levels do not agree, then a compromise between optimizing the response and minimizing the variation must be made. One way to approach the compromising process is through what Taguchi called the *signal-tonoise* (*S*/*N*) *ratio*. If it is desirable to maximize *y*, the signal-to-noise ratio may be calculated for each run, using

S/N ratio =
$$\frac{\overline{y}}{S}$$

The main effects may then be calculated, using the S/N ratios to find the best levels for each factor. If, instead, it is desirable to make y as small as possible, the signal-to-noise ratio can be defined as

S/N ratio =
$$\frac{1}{\overline{y}S}$$

| Α | В | С | Replications | | | | \overline{y} | S |
|---|---|---|--------------|----|----|----|----------------|-----|
| _ | - | - | 34 | 29 | 38 | 25 | 31.5 | 5.7 |
| _ | - | + | 42 | 47 | 39 | 38 | 41.5 | 4.0 |
| _ | + | - | 54 | 41 | 48 | 43 | 46.5 | 5.8 |
| _ | + | + | 35 | 31 | 32 | 34 | 33.0 | 1.8 |
| + | - | - | 62 | 68 | 63 | 69 | 65.5 | 3.5 |
| + | - | + | 25 | 33 | 36 | 21 | 28.8 | 6.9 |
| + | + | + | 58 | 54 | 58 | 60 | 57.5 | 2.5 |
| + | + | _ | 39 | 35 | 42 | 45 | 40.3 | 4.3 |

Table 26.2 Example of using the signal-to-noise ratio.

If it is desirable to make *y* as close to some nominal value *N* as possible, the signal-to-noise ratio can be defined as

S/N ratio =
$$\frac{1}{|\overline{y} - N|S|}$$

Note that the signal-to-noise ratio is an attempt to find a useful compromise between two competing goals, optimizing *y* and minimizing *S*. It does not necessarily accomplish either of these goals, so it should be used with a bit of judgment.

Another technique Taguchi used for improving robustness is called the inner/ outer array design. In this procedure, the uncontrollable factors—those factors that the experimenter either can not or chooses not to control—are placed in separate columns next to the controllable factors, as shown in Table 26.3. Design factors are placed in the inner array, and noise or uncontrollable factors are placed in the outer array. In this example, hardness of steel and ambient temperature are the uncontrollable factors. These factors could conceivably be controlled by putting the machine in an environmental enclosure and putting a tighter specification on the steel, but the experimenter chooses not to do either of these. Instead, anticipated extremes of hardness and ambient temperature are used for the experiment to determine settings of the controllable variables that will minimize variation in the output quality characteristic.

When the first run of the design in Table 26.3 is executed, the feed, speed, and coolant temperature are all set at their low levels. One part is made with low-hardness steel and low ambient temperature, and the value of the quality characteristic is entered in the spot labeled "a." When all 32 values have been entered, the averages and standard deviations are calculated. In this inner/outer array

| | Inner ar | ray | | | Ou | ter array | , | | |
|------|----------|-----------------|-----------------|---|----|-----------|---|---|---|
| | | | Hardness | _ | _ | + | + | | |
| Feed | Speed | Coolant temp | Ambient temp | - | + | - | + | y | S |
| - | - | - | | а | | | | | |
| - | - | + | | | | | | | |
| - | + | _ | | | | | | | |
| - | + | + | | | | | | | |
| + | - | _ | | | | | | | |
| + | _ | + | | | | | | | |
| + | + | - | | | | | | | |
| + | + | + | | | | | | | |

 Table 26.3
 Example of Taguchi's inner and outer arrays.

approach, the design intentionally causes perturbations in the uncontrollable factors to find level combinations for the controllable factors that will minimize the variation in the quality characteristics under the anticipated hardness and ambient temperature variation. One might ask why hardness and ambient temperature are not merely added to the factor list, making five factors at two levels, which would require $2^5 = 32$ tests, exactly the number required in this example. That approach would establish the best settings for hardness and ambient temperature. But that would also require a tighter specification on hardness and ambient temperature. Instead, the inner/outer array design determines optimum levels for the control factors in the presence of variation in the outer array factors.

Mixture Experiments

A *mixture experiment* is an experiment design in which each experimental run is constrained such that when summed across the factors, the factor levels are constrained to sum to a constant. The typical applications involve chemical experiments in which the factors are liquids, or sometimes gases. In such a case, it is the proportion of each liquid ingredient, not its weight or volume, that is the essential issue.

Several key assumptions regarding mixture experiments should be noted. These include:

- The response depends on the proportion of the components in the mixture and not on the volume of the mixture itself.
- The errors are independent and identically distributed normal with a zero mean and constant variance.
- The response region is continuous over the region being investigated.

The basic constraints on most mixture problems where there are p components and x_i represents the proportions of each component are

$$x_1 + x_2 + \dots + x_p = \sum_{i=1}^p x_i = 1$$

 $0 \le x_i \le 1; i = 1, 2, \dots, p$

If both upper and lower bound are provided, then

$$L_i \le x_i \le U_i; i = 1, 2, ..., p$$

However, in some cases, only a lower bound will be specified:

$$L_i \le x_i \le 1; i = 1, 2, \dots, p$$

Figure 26.14 illustrates a three-component model. The vertices of the triangle represent a pure mixture while the blends occur along the edges. For higher-order models, pure mixtures would occur at the corner points. Figure 26.15 illustrates a three-component model with lower bounds. The triangle has just been rotated

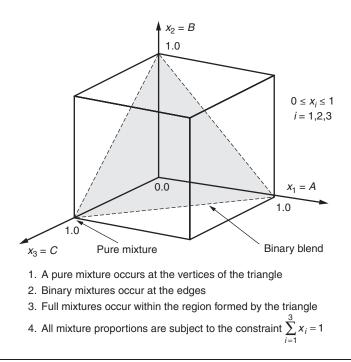


Figure 26.14 Example of the inference space for a three-component model.

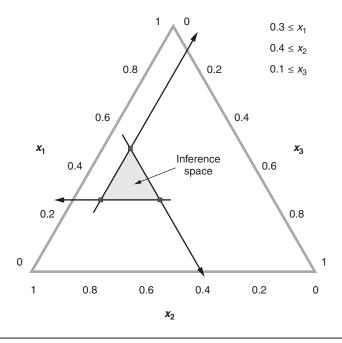


Figure 26.15 Example of a three-component model with lower bounds.

for convenience so that it appears to be on a flat surface. Notice that the feasible region or inference space is bounded on three sides by the constraints and is inscribed inside the original simplex region. When this occurs, it is easier to deal with pseudocomponents given by

$$x_i^* = \frac{x_i - l_i}{1 - \sum_{j=1}^p l_j}; \sum_{j=1}^p l_j < 1$$

Using pseudocomponents will allow the new values x_i^* to range over the values of 0 to 1, inclusively. The original components are obtained simply by applying translation:

$$x_i = l_i + \left(1 - \sum_{j=1}^p l_j\right) x_i^*$$

Pseudocomponents also allow us the use of two methods: the *simplex lattice design* method or the *simplex centroid design* method.

The simplex lattice design is often designated by its parameters p and m where p is the number of components and m is the number of equal spaces. Thus, the simplex lattice design will have m + 1 point values given by the formula

$$x_i = 0, \frac{1}{m}, \frac{2}{m}, \dots, 1; i = 1, 2, \dots, p$$

The simplex lattice design will generate

$$N = \frac{(p+m-1)!}{m!(p-1)!}$$

points whereas the simplex centroid design will generate $2^p - 1$. For example, with p = 3 and m = 2, the simplex lattice design will generate 6 points while the simplex centroid design will generate 7 points.

Although there are many mixture designs, there are four standard forms of models that are widely used. These include the linear, quadratic, full cubic, and special cubic designs.

Mixture designs are highly useful tools and should be included in the knowledge base of every Master Black Belt.

Split-Plot Designs

Jones and Nachtsheim (2009) provide a succinct definition of split-plot designs when they state:

... a split-plot experiment is a blocked experiment, where the blocks themselves serve as experimental units for a subset of the factors. Thus, there are two levels of experimental units. The blocks are referred to as whole plots, while the experimental units within plots are called split plots, split units, or subplots. Corresponding to the levels of experimental units are two levels of randomization. One randomization is conducted to determine the assignment of block-level treatments to the whole plots. Then, as always in a blocked experiment, a randomization of treatments to split-plot experimental units occurs within each block or whole plot.

A split-plot design is used when there are two or more factors and it is difficult to randomize one of the factors. There are typically three situations that give rise to the use of split-plot designs:

- Some of the factors are hard to vary while others are easy. Difficulty may arise from physical limitations, lack of features or capabilities, and so on. When we have this situation, the order in which the treatment combinations for the experiment are run is determined by the ordering of the hard-to-vary factors (that is, hardest to easiest).
- Experimental units are processed together as a batch in a treatment combination for one or more factors.
- Experimental units are processed in a continuous manner without resetting the factor setting for a particular treatment combination.

Split-plot designs provide us with the flexibility necessary to work around the realities of life.

R. A. Fisher developed the split-plot design in the 1920s. The concept of a splitplot design is rooted in agriculture and is illustrated in Example 26.2.

D-optimal Designs

Montgomery (2009) states that, "In general, a design that minimizes the variance of the model regression coefficients is called a D-optimal design." Consequently, the optimality of the D-optimal design is a function of the model provided. The D terminology is used because these designs are found by selecting runs in the design to maximize the determinant of *X*′*X* where *X* is obtained from the matrix form of the model $y = X\beta + \varepsilon$.

EXAMPLE 26.2

There are four cornfields as exhibited in Figure 26.16. Two irrigation methods to water the whole fields are available. We have two types of fertilizer and want to determine the effects on the growth of the corn.

The first randomization is for assigning an irrigation method to a whole plot or field. Remember, it must irrigate the whole plot since the capability to water smaller sections does not exist. The irrigation methods represent the A factor in Figure 26.16. The second randomization is for assigning a fertilizer type to the split plots within each whole plot or field.

The split-plot design can be easily modeled using Minitab.

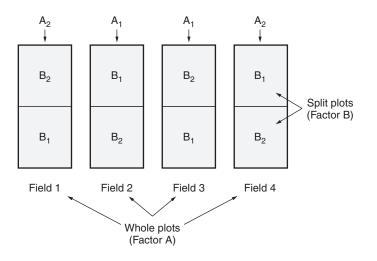


Figure 26.16 Split-plot design for Example 26.2. *Source:* Jones and Nachtsheim (2009).

D-optimal designs are generally generated using computer algorithms, and there is no guarantee a D-optimal design will be generated.

There are two categories of reasons for using D-optimal designs. These include:

- Factorial or fractional factorial designs require more runs than time and money permit.
- The design space is constrained such that there are certain factor settings that are either not feasible or impossible to run.

Consider an experiment with three factors. Factor A has five levels, factor B has two levels, and factor C has two levels. A full factorial would generate twenty runs. If resources were sufficient for only twelve runs, the D-optimal algorithm would be beneficial in selecting the twelve runs for the given model such that the variance of the model regression coefficients is minimized. Minitab has the ability to generate D-optimal designs.

Chapter 27

Automated Process Control (APC) and Statistical Process Control (SPC)

conjunction with SPC. (Understand)

Recognize when to use APC instead of or in

Body of Knowledge VI.D

Occasionally, the terms *automated process control* (APC) and *statistical process control* (SPC) become confused or used interchangeably in the minds of practitioners. Perhaps this is because most of SPC today is executed using software systems.

Nevertheless, they are vastly different. SPC deals with the statistical aspects of processes while APC systems are electromechanical in nature. In some cases, APC systems may be integrated with SPC software.

The ASQ *Glossary and Tables for Statistical Quality Control*, Fourth Edition (2005) defines SPC as "The use of statistical techniques such as control charts to reduce variations, increase knowledge about the process, and to steer the process in the desired way." On the other hand, Smith (2002) states that "The objective of an automatic process control system is to adjust the manipulated variable to maintain the controlled variable at its set point in spite of disturbances."

TERMINOLOGY

In order to understand the concept of automated process control, several basic definitions are required to provide perspective:

- *Controlled variable.* This variable must be maintained at a set or specified level. The control variable is also known as the *process variable* or simply the *measurement*. In Lean Six Sigma terminology, it is the "Y" variable.
- *Set point.* This is the specified value of the controlled variable. This variable is equivalent to the nominal value in a specification.
- *Manipulated variable*. This is the variable used to maintain the controlled variable at the set point. In Lean Six Sigma terminology, it is the "X" variable. There can be many manipulated variables. In

the event there are, the APC system becomes more complex. Examples of such variables include concentration, temperature, pressure, and flow rate.

- *Disturbance* or *upset*. These variables cause the controlled variable to deviate from the set point. In Lean Six Sigma terminology, these are *noise variables*.
- *Automated process control.* Automated process control is the application of automation to effect process control by maintaining manipulated variables at set point levels such that controlled variables are maintained at specified levels. Generally, automation allows for a greater number of manipulated variables to be controlled with greater speed and accuracy than possible by human intervention.

ADVANTAGES OF AUTOMATED PROCESS CONTROL

There are several advantages to the use of an automated process control system. Some of the top advantages include:

- Eliminates or reduces manual intervention, which is slower and more expensive
- Eliminates or reduces potential safety hazards
- Reduces measurement system error
- Achieves product quality levels that might be otherwise unattainable on a manual basis
- Accommodates the ability to monitor and react to numerous controlled variables simultaneously

APC systems are particularly useful in managing continuous processes due to the highly dynamic and interactive nature of them.

BASIC CONTROL SYSTEMS

For a control system to be a control system, it must comprise three basic components and operations:

- *Sensor/transmitter.* These components capture the measurement, convert it to a signal, and transmit it to the controller for interpretation. These are also called the primary and secondary elements. The basic operation performed is "measurement."
- *Controller*. The controller is considered the brain of the system since it compares the value of the signal to the set value (that is, nominal or desired value) and sends a signal to the final control element for action. Essentially, the controller makes a decision. The basic operation performed is "decide."

• *Final control element.* Upon receiving direction from the controller, the final control element will take action to adjust the appropriate "manipulated variable" (that is, input variable). Examples of final control elements include control values, electric motors, conveyors, and variable-speed pumps. When the manipulated variable is adjusted by the final control element, and that variable, in turn, affects the next measurement, the system is said to be a closed-loop feedback system. The basic operation of the final control element is "action."

Smith (2002) describes a closed-loop feedback control system quite succinctly when he states, "That is, based on the measurement, a decision is made, and based on this decision, an action is taken. The action taken must come back and affect the measurement."

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Appendix 1 ASQ Code of Ethics (May 2005)

FUNDAMENTAL PRINCIPLES

ASQ requires its members and certification holders to conduct themselves ethically by:

- I. Being honest and impartial in serving the public, their employers, customers, and clients.
- II. Striving to increase the competence and prestige of the quality profession, and
- III. Using their knowledge and skill for the enhancement of human welfare.

Members and certification holders are required to observe the tenets set forth below:

Relations with the Public

Article 1—Hold paramount the safety, health, and welfare of the public in the performance of their professional duties.

Relations with Employers, Customers, and Clients

Article 2—Perform services only in their areas of competence.

Article 3—Continue their professional development throughout their careers and provide opportunities for the professional and ethical development of others.

Article 4—Act in a professional manner in dealings with ASQ staff and each employer, customer, or client.

Article 5—Act as faithful agents or trustees and avoid conflict of interest and the appearance of conflicts of interest.

Relations with Peers

Article 6—Build their professional reputation on the merit of their services and not compete unfairly with others.

Article 7—Assure that credit for the work of others is given to those to whom it is due.

Appendix 2

ASQ Certified Master Black Belt (MBB) Body of Knowledge (2010)

MULTIPLE-CHOICE SECTION-100 QUESTIONS-21/2 HOURS

The topics in this Body of Knowledge (BOK) include descriptive details (subtext) that will be used by the Exam Development Committee as guidelines for writing test questions. This subtext is also designed to help candidates prepare for the exam by identifying specific content within each topic that may be tested. The subtext is not intended to limit the subject matter or be all-inclusive of what might be covered in an exam but is intended to clarify how the topics relate to a Master Black Belt's role. The descriptor in parentheses at the end of each entry refers to the maximum cognitive level at which the topic will be tested. A complete description of cognitive levels is provided at the end of this document.

I. Enterprise-Wide Planning and Deployment (25 questions)

A. Strategic plan development

Describe strategic planning tools and methods (hoshin kanri, SWOT, PEST, etc.) and their utilization in developing enterprise planning. (Apply)

- B. Strategic plan alignment
 - 1. **Strategic deployment goals** Describe how to develop strategic deployment goals. (Apply)
 - 2. **Project alignment with strategic plan** Describe how to align projects to the organizational strategic plan. (Apply)
 - 3. **Project alignment with business objectives** Describe how projects are aligned with business objectives. (Apply)
- C. *Deployment of six sigma systems* Describe the following key deployment elements. (Apply)
 - 1. Governance (quality councils or process leadership teams)
 - 2. Assessment (maturity models and organizational readiness)
 - 3. Resource planning (identify candidates and costs/benefits)
 - 4. Resource development (train and coach)

- 5. Execution (deliver on project results)
- 6. Measure and improve the system (drive improvement into the systems, multiphase planning)

D. Six sigma methodologies

Demonstrate an advanced understanding of the following methodologies, including their associated tools and techniques. (Apply)

- 1. DMAIC
- 2. DFSS
- 3. Lean
- 4. Business systems and process management

E. Opportunities for improvement

1. Project identification

Facilitate working sessions to identify new project opportunities that can be prioritized. (Apply)

2. Project qualification

Determine the elements of a well-defined project (i.e., business case), the process for approving these projects, and tools used in project definition (QFD, process maps, value stream maps, FMEA, CTx (critical to ... customer, ... design, ... quality), etc. (Apply)

3. Stakeholder engagement

Describe how to engage stakeholders. (Apply)

4. Intervention techniques

Describe techniques for intervening across levels to prevent potential project failures. (Apply)

5. Creativity and innovation tools

Use these tools to develop concept alternatives. (Apply)

F. Risk analysis of projects and the pipeline

1. Risk management

Use risk management and analysis tools to analyze organizational elements, to appraise portfolios and critical projects, and to identify potential problem areas. (Evaluate)

2. Pipeline creation

Create, manage, and prioritize a pipeline of potential projects for consideration. (Create)

3. Pipeline management

Create a selection process that provides a portfolio of active six sigma opportunities that are clearly aligned and prioritized to meet/ exceed strategic goals. (Create)

G. Organizational design

1. Systems thinking

Apply systems thinking to anticipate the effect that components of a system can have on other subsystems and adjacent systems. Analyze the impact of actions taken in one area of the organization and how those actions can affect other areas or the customer, and use appropriate tools to prevent unintended consequences. (Analyze)

2. Organizational maturity and culture

Describe the implications these factors can have on six sigma implementation, including potential barriers. (Understand)

3. Organizational culture change techniques

Describe techniques for changing an organizational culture, such as rewards and recognition, team competitiveness, communications of program successes, and appropriate cascading of goals throughout the organization. (Apply)

H. Organizational commitment

1. Techniques to gain commitment

Describe how to gain commitment from the organization's leadership for the six sigma effort. (Understand)

2. Necessary organizational structure for deployment Develop the inherent organizational structure needed for successful deployment. (Apply)

3. Communications with management

Describe elements of effective communications with management regarding organizational benefits, failures, and lessons learned. (Apply)

4. Change management

Describe the MBB role in change management and apply various techniques to overcome barriers to successful organizational deployment. (Apply)

I. Organizational finance and business performance metrics

1. Financial measures

Define and use financial measures, including revenue growth, market share, margin, cost of quality (COQ), net present value (NPV), return on investment (ROI), cost–benefit analysis, activity-based cost analysis, and breakeven time performance, etc. (Analyze)

2. Business performance measures

Describe various business performance measures, including balanced scorecard, key performance indicators (KPIs), and the financial impact of customer loyalty; and describe how they are used for project selection, deployment, and management. (Analyze)

3. Project cash flow

Develop a project cash flow stream. Describe the relation of time to cash flow and difficulties in forecasting cash flow. (Analyze)

4. Sarbanes-Oxley (SOX) Act

Understand the requirements for financial controls dictated by SOX. (Understand)

II. Cross-Functional Competencies (15 Questions)

A. Data gathering

Assess the appropriate collection of Voice of the Customer and Voice of the Process data, both internal and external, and develop a customer-focused strategy for capturing and assessing customer feedback on a regular basis. (Evaluate)

B. Internal organizational challenges

1. Organizational dynamics

Use knowledge of human and organizational dynamics to enhance project success and align cultural objectives with organizational objectives. (Apply)

2. Intervention styles

Use appropriate intervention, communications, and influence styles, and adapt those styles to specific situations (i.e., situational leadership). (Apply)

3. Interdepartmental conflicts

Address and resolve potential situations that could cause the program or a project to under-perform. (Apply)

C. Executive and team leadership roles

1. Executive leadership roles

Describe the roles and responsibilities of executive leaders in the deployment of six sigma in terms of providing resources, managing change, communicating ideas, etc. (Analyze)

2. Leadership for deployment

Create action plans to support optimal functioning of master black belts, black belts, green belts, champions, and other participants in the deployment effort. Design, coordinate, and participate in deployment activities, and ensure that project leaders and teams have the required knowledge, skills, abilities, and attitudes to support the organization's six sigma program. (Create)

III. Project Management (15 Questions)

A. Project execution

1. Cross-functional project assessment

Appraise interrelated projects for scope overlap and refinement and identify opportunities for leveraging concomitant projects. Identify and participate in the implementation of multi-disciplinary redesign and improvement projects. (Analyze)

2. Executive and mid-level management engagement

Formulate the positioning of multiple projects in terms of providing strategic advice to top management and affected mid-level managers. (Create)

3. Project prioritization

Prioritize projects in terms of their criticality to the organization. (Apply)

B. Project oversight and management

1. Project management principles

Oversee critical projects and evaluate them in terms of their scope, goals, time, cost, quality, human resources requirements, communications needs, and risks. Identify and balance competing project demands with regard to prioritization, project resources, customer requirements, etc. (Evaluate)

2. Measurement

Support and review the development of an overall measurement methodology to record the progress and ongoing status of projects and their overall impact on the organization. (Evaluate)

3. Monitoring

Apply appropriate monitoring and control methodologies to ensure that consistent methods are used in tracking tasks and milestones. (Apply)

4. Project status communication

Develop and maintain communication techniques that will keep critical stakeholders and communities apprised of project status, results, and accountability. (Create)

5. Supply/Demand management

Generate accurate project supply/demand projections, associated resource requirements analysis, and mitigate any issues. (Create)

6. Corrective action

Facilitate corrective actions and responses to customers about the corrective action and its impact. (Apply)

C. Project management infrastructure

1. Governance methods and tools

Develop governance documents, tracking tools, and other methodologies that will support project success. (Create)

2. Performance measurement

Design a system for measuring project and portfolio performance. (Create)

D. Project financial tools

1. Budgets and forecasts

Assess and explain budget implications, forecasting, measurement, monitoring, risk analysis, and prioritization for portfolio level projects. (Evaluate)

2. Costing concepts

Define the concepts of hard and soft dollars and use cost of poor quality tools, activity-based costing, and other methods to assess and prioritize portfolios. (Apply)

IV. Training Design and Delivery (10 Questions)

A. Training needs analysis

Assess the current level of knowledge and skills in each target group in relation to the skills and abilities that are needed. Conduct a gap analysis to determine the training needs for each target group. (Evaluate)

B. Training plans

Design training plans to close the knowledge and skills gaps. Refine the plans based on the number of people needing to be trained in a particular technique or skill, and whether multi-disciplinary or multilevel competency training is appropriate. (Create)

C. Training materials and curriculum development

1. Adult learning theory

Evaluate and select training materials and resources that adhere to adult learning theories. (Analyze)

2. Integration

Ensure that the training harmonizes and leverages other tools and approaches being used and that it is aligned with the organization's strategic objectives and culture. (Evaluate)

3. Training delivery

Monitor and measure training to ensure that it is delivered effectively and efficiently by qualified individuals. (Apply)

D. Training effectiveness evaluation

Develop an evaluation plan to assess and verify the acquisition of required knowledge and skills. (Create)

V. Mentoring Responsibilities (10 Questions)

A. Mentoring champions, change agents, and executives

1. Project reviews

Collaborate with executives and champions on reviewing projects, including timing, questions to ask, and setting expectations for project timing and completion. (Create)

2. **Project sizing**

Collaborate with executives and champions on sizing projects and selecting individuals and assignments for various projects. (Evaluate)

3. Communications

Coach executives and champions on the need for constancy of purpose and message, and the importance of using clear communication techniques and consistent messages. (Evaluate)

4. Feedback

Use constructive techniques to provide feedback to champions and executives. (Evaluate)

B. Mentoring black belts and green belts

1. Individuals

Develop a career progression ladder for black belts and green belts. Assess their progress and provide constructive feedback to enable them to work effectively on team projects. Use coaching, mentoring, and intervention skills as needed, including canceling or reassigning projects if necessary. (Evaluate)

2. Technical reviews

Create guidelines and expectations for project reviews, and perform them in a timely manner. Assist project leaders in selecting appropriate content for presentation to management. (Create)

3. Team facilitation and meeting management

Practice and teach meeting control, analyze team performance at various stages of team development, and support appropriate interventions for overcoming team challenges, including floundering, reviewing and diagnosing failing projects, etc. (Create)

C. Mentoring non-belt employees

Develop information that will help non-belt project participants to advance their understanding of six sigma and develop the necessary skills and knowledge to become green belts or black belts. (Create)

VI. Advanced Measurement Methods and Tools (25 Questions)

A. Measurement systems analysis (MSA)

1. Propagation of errors

Use this technique to evaluate measurement systems and calculated values. (Evaluate)

2. Attribute (discrete) measurement systems

Use various tools and methods (e.g., percent agreement, Kappa, Kendall, intra-class correlation coefficient [ICC]) to analyze and interpret discrete measurement systems data. (Evaluate)

3. Variables (continuous) measurement systems

Use various tools and methods (e.g., $\overline{X} - R$, $\overline{X} - s$, individual and moving range) to analyze and interpret continuous measurement systems data. (Evaluate)

4. **Process capability for non-normal data** Calculate capability using Weibull and other methods for non-normal data. (Apply)

B. Measuring and modeling relationships between variables

1. Autocorrelation and forecasting

Identify autocorrelated data, including time-series modeling (e.g., ARIMA) and forecasting. (Understand)

2. Multiple regression analysis

Apply and interpret multiple regression analysis, including using variance inflation factors (VIFs) to identify collinearity issues. (Apply)

3. Logistic regression analysis

Apply and interpret logistic regression analysis, including binary, ordinal, and nominal data considerations. (Apply)

4. Model fitting for non-linear parameters

Apply and interpret fits of models that are non-linear. (Apply)

5. General linear models (GLMs)

Apply and interpret GLMs using assumptions and assumptions testing. Compare and contrast GLMs with various other models, including ANOVA results, (crossed, nested, and mixed models) simple linear regression, multiple regression, ANCOVA, and continuous MSA. (Apply)

6. Components of variation

Select, calculate, and interpret components of variation and nested design studies. (Evaluate)

7. Simulation

Apply simulation tools such as Monte Carlo, dynamic process simulation, queuing theory, etc. (Apply)

8. Linear programming

Understand how linear programming principles, such as critical path analysis, can be used in modeling diverse types of problems (e.g., planning, routing, scheduling, assignment, design) to optimize system performance. (Understand)

9. Reliability modeling

Use reliability modeling and tools to enhance reliability of a product or process and reliability growth modeling. (Apply)

10. Qualitative analysis

Use appropriate qualitative analysis tools (affinity diagrams, force field analysis, etc.) and analyze the results. (Analyze)

C. Design of experiments (DOE)

1. Factor analysis

Apply and interpret factor relationship diagrams. (Apply)

2. Complex blocking structures

Recognize other designs for handling more complex blocking structures, including balanced incomplete block design (BIBD). (Understand)

3. Other DOE approaches

Recognize when to apply approaches such as response surface methodology (RSM), mixture experiments, evolutionary operations (EVOP), split-plot designs, Taguchi, D-optimal designs, etc. (Understand)

D. Automated process control (APC) and statistical process control (SPC)

Recognize when to use APC instead of or in conjunction with SPC. (Understand)

PERFORMANCE-BASED SECTION—ESSAY RESPONSE—21/2 HOURS

Candidates will be presented with a fictional multi-division organization that is reviewing multiple six sigma projects of various sizes. Each proposed project will be led by individual black belts and their teams. Candidates will have 2½ hours to write their response to opened-ended questions about these projects. Details will be presented in two sets of documents: Phase 1 will be Day 0 and will focus on project selection. Phase 2 will be presented as if 6–8 weeks have gone by since the start of the projects.

Phase 1—Candidates will be asked to prioritize projects and explain their reasoning in terms of project value and order. Responses will be scored on elements such as the rationale used for ranking the projects in a specific order, the content of presentations created in support of those choices, communications with staff at various levels in relation to those selections, etc., as outlined in the performancebased BOK descriptions below.

Phase 2—Candidates will be given updated information on each project and asked to respond to those conditions. Responses will be scored on elements such as analysis or evaluation of the projects based on their updates, insights on the cause of the problems and creative solutions, time and resource management, interpersonal skills for conflict resolution, and other skills required of high-level, multiproject leaders, as outlined in the performance-based BOK descriptions below.

PB-1. Enterprise-Wide Planning and Deployment

Apply project selection criteria to select and prioritize potential six sigma projects using strategic planning tools, immediate- and long-term business goals, executive-level directives, risk analysis results, etc. Develop and deliver formal presentations that support the project selection process, identify progress, explain its status at conclusion, etc.

PB-2. Cross-Functional Competencies

Use feedback and process data from various sources to identify or develop six sigma projects that will respond to customer needs, eliminate process barriers, or streamline processes, especially for managing projects that cross boundaries either within or between organizations. Use appropriate communication methods that are sensitive to specific audiences when explaining projects or solutions, encouraging participation, or resolving issues that arise between interorganizational groups

PB-3. Project Management

Develop and manage the scope, schedule, cost, and risk of six sigma projects using various project management tools to ensure proper monitoring, milestone achievement, and project success. Recognize when intervention steps must be taken to bring a project back on track or terminate it based on analysis of internal or external events.

PB-4. Training and Mentoring

Identify situations that require training or mentoring for all levels of participants in six sigma projects, from executive level champions to non-belt participants. Develop, review, and deliver information, training, or mentoring as needed for a variety of six sigma projects, based on needs analysis, participant requests, or recognition of situations that require intervention.

LEVELS OF COGNITION BASED ON BLOOM'S TAXONOMY—REVISED (2001)

In addition to *content* specifics, the subtext for each topic in this BOK also indicates the intended *complexity level* of the test questions for that topic. These levels are based on "Levels of Cognition" (from Bloom's Taxonomy—Revised, 2001) and are presented below in rank order, from least complex to most complex.

Remember

Recall or recognize terms, definitions, facts, ideas, materials, patterns, sequences, methods, principles, etc.

Understand

Read and understand descriptions, communications, reports, tables, diagrams, directions, regulations, etc.

Apply

Know when and how to use ideas, procedures, methods, formulas, principles, theories, etc.

Analyze

Break down information into its constituent parts and recognize their relationship to one another and how they are organized; identify sublevel factors or salient data from a complex scenario.

Evaluate

Make judgments about the value of proposed ideas, solutions, etc., by comparing the proposal to specific criteria or standards.

Create

Put parts or elements together in such a way as to reveal a pattern or structure not clearly there before; identify which data or information from a complex set is appropriate to examine further or from which supported conclusions can be drawn.

Appendix 3

Control Chart Combinations for Measurement Data

| | | Averages | | | Na | tural Process Lim | its |
|-----------------------|-------------------------------|------------------------------------------------------------|-------------------------------------------------------------------|----------------|-------------------------------|---------------------------------------------------------|---------------------------------------------------------|
| Chart | Center line | LCL | UCL | Chart | Center line | UNPL | LNPL |
| x | $\overline{\overline{X}}$ | $\overline{\overline{X}} - A_2 \overline{R}$ | $\overline{\overline{X}} + A_2\overline{R}$ | 1 | $\overline{\overline{X}}$ | $\overline{\overline{X}} - E_2\overline{R}$ | $\overline{\overline{X}} + E_2\overline{R}$ |
| R | R | $D_3\overline{R}$ | $D_4\overline{R}$ | R | R | $D_3\overline{R}$ | $D_4 \overline{R}$ |
| x | $\overline{\overline{X}}$ | $\overline{\overline{X}} - A_4 \widetilde{R}$ | $\overline{\overline{X}} + A_4 \widetilde{R}$ | 1 | $\overline{\overline{X}}$ | $\overline{\overline{X}} - E_5 \widetilde{R}$ | $\overline{\bar{X}} + E_5 \tilde{R}$ |
| R | Ñ | $D_5 \tilde{R}$ | $D_6 \tilde{R}$ | R | Ñ | $D_5 \tilde{R}$ | $D_6 \tilde{R}$ |
| x | $\overline{\overline{X}}$ | $\overline{\overline{X}} - A_1 \overline{\sigma}_{RMS}$ | $\overline{\overline{X}} + A_1 \overline{\sigma}_{RMS}$ | 1 | $\overline{\overline{X}}$ | $\overline{\overline{X}} - E_1 \overline{\sigma}_{RMS}$ | $\overline{\overline{X}} + E_1 \overline{\sigma}_{RMS}$ |
| σ_{RMS} | $\overline{\sigma}_{\rm RMS}$ | $B_3\overline{\sigma}_{RMS}$ | $B_4 \overline{\sigma}_{RMS}$ | σ_{RMS} | $\overline{\sigma}_{\rm RMS}$ | $B_3 \overline{\sigma}_{RMS}$ | $B_4 \overline{\sigma}_{RMS}$ |
| x | $\overline{\overline{X}}$ | $\overline{\overline{X}} - A_5 \overline{\sigma}_{RMS}$ | $\overline{\overline{X}} + A_5 \overline{\sigma}_{RMS}$ | 1 | $\overline{\overline{X}}$ | $\overline{\overline{X}} - E_4 \tilde{\sigma}_{RMS}$ | $\overline{\overline{X}} + E_4 \tilde{\sigma}_{RMS}$ |
| σ _{RMS} | $\tilde{\sigma}_{RMS}$ | $B_9 \tilde{\sigma}_{RMS}$ | $B_{10} \tilde{\sigma}_{RMS}$ | σ_{RMS} | $\tilde{\sigma}_{RMS}$ | $B_9 \tilde{\sigma}_{RMS}$ | $B_{10}	ilde{\sigma}_{RMS}$ |
| x | $\overline{\overline{X}}$ | $\overline{\overline{X}} - A_3\overline{s}$ | $\overline{\overline{X}} + A_3\overline{s}$ | 1 | $\overline{\overline{X}}$ | $\overline{\overline{X}} - E_3\overline{s}$ | $\overline{\overline{X}} + E_3\overline{s}$ |
| s | \overline{s} | $B_3\overline{s}$ | $B_4\overline{s}$ | s | \overline{s} | $B_3\overline{s}$ | $B_4\overline{s}$ |
| Ā | $\overline{\overline{X}}$ | $\overline{\overline{X}} - A_{10}\tilde{s}$ | $\overline{\overline{X}} + A_{10}\tilde{s}$ | 1 | $\overline{\overline{X}}$ | $\overline{\overline{X}} - E_6 \tilde{s}$ | $\overline{\overline{X}} + E_6 \tilde{S}$ |
| s | ŝ ¯_₹ | $B_9	ilde{s}$ | $B_{10}	ilde{s}$ | s | ŝ | $B_9	ilde{s}$ | $B_{10}\tilde{s}$ |
| X | $\overline{\overline{X}}$ | $\overline{\overline{X}} - A_7 \sqrt{\overline{s}^2}$ | $\overline{\overline{X}} + A_7 \sqrt{\overline{s}^2}$ | — | — | _ | _ |
| | | $\approx \overline{\overline{X}} - A\sqrt{\overline{s}^2}$ | $\approx \overline{X} + A\sqrt{\overline{s}^2}$ | | | | |
| s | $\sqrt{\overline{s}^2}$ | $B_7\sqrt{\overline{s}^2}$ | $B_8\sqrt{\overline{s}^2}$ | _ | — | _ | — |
| | | $\approx B_5\sqrt{\overline{s}^2}$ | $\approx B_6\sqrt{\overline{s}^2}$ | | | | |
| X | $\overline{\overline{X}}$ | $\overline{\overline{X}} - A_7 \sqrt{\overline{s}^2}$ | $\overline{\overline{X}} + A_7 \sqrt{\overline{s}^2}$ | _ | — | — | — |
| | | $\approx \overline{\overline{X}} - A\sqrt{\overline{S}^2}$ | $\approx \qquad \overline{\overline{X}} + A\sqrt{\overline{S}^2}$ | | | | |
| <i>S</i> ² | \overline{s}^2 | $B_{_{11}}\overline{s}^{_2}$ | $B_{_{12}}\overline{s}^{_2}$ | - | — | _ | — |
| Ñ | $\overline{\tilde{X}}$ | $\overline{\tilde{X}} - A_6 \overline{R}$ | $\overline{\tilde{X}} + A_6\overline{R}$ | 1 | $\overline{\tilde{X}}$ | $\overline{\tilde{X}} - E_2\overline{R}$ | $\overline{\tilde{X}} + E_2\overline{R}$ |
| R | R | $D_3\overline{R}$ | $D_4\overline{R}$ | R | R | $D_3\overline{R}$ | $D_4\overline{R}$ |
| <i>x</i> | $\overline{\tilde{X}}$ | $\overline{\tilde{X}} - A_9 \tilde{R}$ | $\overline{\tilde{X}} + A_9 \tilde{R}$ | 1 | $\overline{\tilde{X}}$ | $\overline{\tilde{X}} - E_5 \tilde{R}$ | $\overline{\tilde{X}} + E_5 \tilde{R}$ |
| R | Ñ | $D_5 \tilde{R}$ | $D_6 \tilde{R}$ | R | Ñ | $D_5 	ilde{R}$ | $D_6 \tilde{R}$ |

| | | Averages | | | Nat | tural Process Lim | its |
|----------------|----------------|------------------------|-------------------------|----------------|----------------|--------------------------------------|--------------------------------------|
| Chart | Center line | LCL | UCL | Chart | Center line | UNPL | LNPL |
| Ā | Nominal | Nominal – $A\sigma_x$ | Nominal + $A\sigma_x$ | 1 | Nominal | Nominal $-3\sigma_x$ | Nominal + $3\sigma_x$ |
| R | $d_2\sigma_x$ | $D_1\sigma_x$ | $D_2\sigma_x$ | R | $d_2\sigma_x$ | $D_1\sigma_x$ | $D_2\sigma_x$ |
| X | Nominal | Nominal – $A\sigma_x$ | Nominal + $A\sigma_x$ | 1 | Nominal | Nominal $-3\sigma_x$ | Nominal + $3\sigma_x$ |
| σ_{RMS} | $c_2\sigma_x$ | $B_1\sigma_X$ | $B_2\sigma_X$ | σ_{RMS} | $c_2\sigma_x$ | $B_1\sigma_X$ | $B_2\sigma_X$ |
| x | Nominal | Nominal – $A\sigma_x$ | Nominal + $A\sigma_x$ | 1 | Nominal | Nominal $-3\sigma_x$ | Nominal + $3\sigma_x$ |
| s | $C_4\sigma_x$ | $B_5\sigma_X$ | $B_6\sigma_X$ | s | $C_4\sigma_x$ | $B_5\sigma_X$ | $B_6\sigma_X$ |
| Ñ | Nominal | Nominal $-A_8\sigma_x$ | Nominal + $A_8\sigma_x$ | 1 | Nominal | Nominal $-3\sigma_x$ | Nominal + $3\sigma_x$ |
| R | $d_2\sigma_x$ | $D_1\sigma_x$ | $D_2\sigma_x$ | R | $d_2\sigma_x$ | $D_1\sigma_x$ | $D_2\sigma_x$ |
| _ | _ | _ | | 1 | X | $\overline{X} - E_2 m \overline{R}$ | $\overline{X} + E_2 m \overline{R}$ |
| _ | _ | — | — | R | mĀ | $D_3 m \overline{R}$ | $D_4 m \overline{R}$ |
| _ | _ | — | — | 1 | X | $\overline{X} - E_5 m \widetilde{R}$ | $\overline{X} + E_5 m \widetilde{R}$ |
| _ | _ | — | — | R | mĨ | D₅mĤ | $D_6 m \tilde{R}$ |

Notes:

When subgroups are involved, assume k subgroups of size n.

 \overline{R} is the average range of k subgroups of size n.

 \tilde{R} is the median range.

 $m\overline{R}$ is the average moving range.

 \tilde{mR} is the median range of all the subgroup ranges.

 $\sum_{i=1}^{n} (X_i - \overline{X})^2$

 σ_{RMS} is the average root mean squared deviation and computed as $\sqrt{\frac{r+1}{n}}$

"Nominal" may be construed as given, known, target, and historical or past values. See Appendix 6 for various ways of computing σ_X .

Appendix 4 Control Chart Constants

| 2.276 2.088 1.964 1.874 1.806 | 2.276 2.088 2.088 1.964 1.874 1.874 1.876 1.752 1.707 1.695 1.697 1.609 | 2.276 2.088 2.088 1.964 1.874 1.874 1.876 1.752 1.752 1.669 1.669 1.697 1.637 1.637 1.585 1.585 1.585 1.585 1.585 | 2.2.76 2.088 2.088 1.964 1.874 1.876 1.707 1.707 1.707 1.669 1.669 1.669 1.637 1.669 1.637 1.563 1.563 1.563 1.563 1.563 1.563 1.564 1.564 1.564 1.564 1.564 1.564 1.564 1.566 1.566 1.566 1.566 1.606 1.606 1.606 1.606 1.606 1.606 1.707 1.606 1.707 1.606 1.707 1.606 1.707 1.706 1.707 1.707 1.707 1.707 1.707 1.707 1.707 1.707 1.707 1.707 1.707 1.707 1.707 1.707 1.707 1.707 1.707 1.707 1.707 1.707 1.707 1.707 1.506 1.707 1.707 1.707 1.506 1.707 1.506 1.707 1.707 1.507 1.506 1.507 1.506 1.707 1.707 1.506 1.506 1.507 1.707 1.506 1.507 1.506 1.707 1.707 1.707 1.506 1.506 1.506 1.507 1.707 1.506 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.506 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 1.507 | 2.2.76 2.088 2.088 1.964 1.874 1.876 1.707 1.707 1.707 1.707 1.707 1.707 1.707 1.703 1.699 1.693 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.564 1.563 1.563 1.564 1.564 1.564 1.563 1.564 1.564 1.563 1.563 1.564 1.563 1.564 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 | 2.276 2.088 2.088 1.964 1.874 1.874 1.874 1.874 1.874 1.875 1.752 1.752 1.769 1.763 1.669 1.669 1.669 1.669 1.669 1.669 1.669 1.669 1.669 1.669 1.669 1.669 1.669 1.669 1.669 1.669 1.669 1.669 1.669 1.669 1.649 1.496 1.496 1.449 1.449 1.449 1.449 1.449 1.449 1.449 1.449 1.448 | 2.276 2.088 2.088 1.964 1.964 1.874 1.874 1.874 1.874 1.874 1.874 1.874 1.874 1.874 1.874 1.875 1.752 1.752 1.753 1.763 1.609 1.609 1.609 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.563 1.544 1.543 1.448 1.448 1.448 1.448 1.448 1.448 1.448 1.448 1.448 |
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| 0.509 | 0.350 | 0.350 0.350 0.350 0.350 0.308 0.308 0.308 0.276 | 0.350 0.350 0.350 0.308 0.308 0.276 | 0.350 | 0.350 | 0.350 |
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| _ | | | | | | |
| 0 272 | 0.337 0.337 0.308 0.308 0.285 0.266 | 0.275 0.337 0.308 0.285 0.285 0.285 0.285 0.285 0.235 | 0.237 0.337 0.308 0.266 0.266 0.249 0.235 0.235 0.235 0.235 0.235 0.233 0.233 | 0.377 0.337 0.308 0.285 0.285 0.285 0.285 0.285 0.249 0.249 0.235 0.235 0.235 0.232 0.212 0.203 0.212 0.203 0.187 0.187 | 0.037 0.337 0.308 0.285 0.285 0.249 0.249 0.249 0.249 0.249 0.249 0.249 0.249 0.249 0.249 0.212 0.212 0.212 0.194 0.187 0.180 0.173 | 0.0.37 0.337 0.308 0.2066 0.285 0.249 0.249 0.249 0.249 0.249 0.249 0.249 0.249 0.249 0.249 0.249 0.249 0.249 0.249 0.249 0.249 0.249 0.194 0.194 0.194 0.197 0.197 0.197 0.197 0.197 0.167 0.167 0.167 |
| 1 175 | 0.925 0.928 0.973 0.925 | 1.173 1.094 1.028 0.973 0.925 0.925 0.884 0.848 0.848 0.816 | 1.1.17 1.094 1.028 0.973 0.973 0.973 0.973 0.848 0.848 0.848 0.846 0.846 0.816 0.762 0.762 | 1173 1.094 1.025 0.973 0.973 0.884 0.884 0.884 0.848 0.848 0.848 0.848 0.717 0.717 0.717 | 11.5 1.094 1.028 0.973 0.973 0.984 0.884 0.848 0.848 0.848 0.848 0.788 0.788 0.778 0.778 0.778 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.773 0.7773 0.7773 0.7773 0.7773 0.7773 0.7773 0.7773 0.7773 0.7777 0.7773 0.7777 0.7773 0.7777 0.7773 0.7777 0.7777 0.7777 0.7777 0.7777 0.7777 0.7777 0.7777 0.7777 0.7777 0.7777 0.7777 0.7777 0.7777 0.7777 0.7777 0.7777 0.7777 0.7777 0.7777 0.7777 0.7777 0.7777 0.7777 0.7777 0.7777 0.7777 0.7777 0.7777 0.7777 0.7777 0.7777 0.7777 0.7777 0.7777 0.7777 0.7777 0.7777 0.7777 0.7777 0.7777 0.7777 0.77777 0.77777 0.77777 0.77777 0.77777 0.77777777 | 11.7 1.094 1.028 0.973 0.973 0.973 0.884 0.884 0.884 0.846 0.846 0.738 0.738 0.738 0.778 0.773 0.697 0.667 0.662 0.667 0.662 |
| 1 061 | 1.000 1.000 0.949 0.905 0.866 | 1.000 1.000 0.949 0.905 0.866 0.866 0.832 0.832 0.802 | 1.000 0.949 0.905 0.905 0.832 0.832 0.832 0.832 0.832 0.775 0.775 0.775 0.728 | 1.000 1.000 0.905 0.805 0.832 0.832 0.832 0.832 0.750 0.775 0.728 0.728 0.728 0.728 0.728 0.728 0.728 | 1.000 1.000 0.949 0.905 0.832 0.832 0.832 0.832 0.775 0.775 0.775 0.775 0.776 0.778 0.776 0.688 0.671 0.655 | 1.000 1.000 0.949 0.905 0.805 0.832 0.832 0.832 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.767 0.767 0.767 0.832 0.832 0.832 0.832 0.832 0.832 0.832 0.832 0.832 0.832 0.832 0.832 0.832 0.832 0.832 0.832 0.832 0.832 0.832 0.832 0.832 0.832 0.832 0.832 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.775 0.767 0.767 0.767 0.767 0.767 0.767 0.767 0.767 0.767 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6655 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.6555 0.65555 0.65555 0.6555 0.6555 0.65555 0.65555 0.6555 0.6555 |
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| u | å | B_{10} | B ₁₁ | B_{12} | D, | D_2 | D3 | D_4 | D_5 | De | Ē | E_2 | E ₃ | E_4 | E ₅ | E ₆ | u |
|--------|----------------------|----------|-------------------|------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------|---------|---------|----------|----------|-----------|------------|-----------|-----------------------|----------|----------------|----------------|----|
| 2 | | 3.864 | 0.000 | 5.243 | 0.000 | 3.686 | 0.000 | 3.267 | 0.000 | 3.863 | 5.317 | 2.660 | 3.760 | 6.289 | 3.145 | 4.444 | 2 |
| З | | 2.733 | 0.000 | 4.000 | 0.000 | 4.358 | 0.000 | 2.574 | 0.000 | 2.744 | 4.146 | 1.772 | 3.385 | 4.412 | 1.889 | 3.606 | с |
| 4 | | 2.351 | 0.000 | 3.449 | 0.000 | 4.698 | 0.000 | 2.282 | 0.000 | 2.375 | 3.760 | 1.457 | 3.256 | 4.115 | 1.517 | 3.378 | 4 |
| 5 | | 2.145 | 0.000 | 3.121 | 0.000 | 4.918 | 0.000 | 2.114 | 0.000 | 2.179 | 3.568 | 1.290 | 3.191 | 3.663 | 1.329 | 3.275 | 5 |
| 9 | 0.031 | 2.008 | 0.000 | 2.897 | 0.000 | 5.078 | 0.000 | 2.004 | 0.000 | 2.054 | 3.454 | 1.184 | 3.153 | 3.521 | 1.214 | 3.215 | 9 |
| 7 | 0.120 | 1.913 | 0.000 | 2.732 | 0.204 | 5.204 | 0.076 | 1.924 | 0.077 | 1.967 | 3.378 | 1.109 | 3.127 | 3.432 | 1.134 | 3.178 | 7 |
| 8 | 0.188 | 1.839 | 0.000 | 2.604 | 0.388 | 5.306 | 0.136 | 1.864 | 0.139 | 1.901 | 3.323 | 1.054 | 3.109 | 3.367 | 1.075 | 3.151 | 8 |
| 6 | 0.242 | 1.782 | 0.000 | 2.500 | 0.547 | 5.393 | 0.184 | 1.816 | 0.188 | 1.850 | 3.283 | 1.010 | 3.095 | 3.322 | 1.029 | 3.132 | 6 |
| 10 | 0.287 | 1.735 | 0.000 | 2.414 | 0.687 | 5.469 | 0.223 | 1.777 | 0.227 | 1.809 | 3.251 | 0.975 | 3.084 | 3.286 | 0.992 | 3.115 | 10 |
| 11 | 0.324 | 1.695 | 0.000 | 2.342 | 0.811 | 5.535 | 0.256 | 1.744 | 0.260 | 1.773 | 3.226 | 0.945 | 3.076 | 3.257 | 0.961 | 3.106 | : |
| 12 | 0.357 | 1.661 | 0.000 | 2.279 | 0.923 | 5.594 | 0.283 | 1.717 | 0.288 | 1.744 | 3.205 | 0.921 | 3.069 | 3.233 | 0.935 | 3.093 | 12 |
| 13 | 0.384 | 1.631 | 0.000 | 2.225 | 1.025 | 5.647 | 0.307 | 1.693 | 0.312 | 1.719 | 3.188 | 0.899 | 3.063 | 3.212 | 0.913 | 3.086 | 13 |
| 14 | 0.409 | 1.604 | 0.000 | 2.177 | 1.118 | 5.696 | 0.328 | 1.672 | 0.333 | 1.697 | 3.174 | 0.881 | 3.058 | 3.195 | 0.894 | 3.080 | 14 |
| 15 | 0.431 | 1.582 | 0.000 | 2.134 | 1.203 | 5.741 | 0.347 | 1.653 | 0.352 | 1.678 | 3.161 | 0.864 | 3.054 | 3.181 | 0.877 | 3.074 | 15 |
| 16 | | | 0.000 | 2.095 | 1.282 | 5.782 | 0.363 | 1.637 | 0.368 | 1.660 | 3.150 | 0.849 | 3.050 | | 0.862 | | 16 |
| 17 | | | 0.000 | 2.061 | 1.356 | 5.820 | 0.378 | 1.622 | 0.383 | 1.645 | 3.141 | 0.836 | 3.047 | | 0.848 | | 17 |
| 18 | | | 0.000 | 2.029 | 1.424 | 5.856 | 0.391 | 1.609 | 0.397 | 1.631 | 3.133 | 0.824 | 3.044 | | 0.835 | | 18 |
| 19 | | | 0.000 | 2.000 | 1.489 | 5.890 | 0.403 | 1.597 | 0.409 | 1.618 | 3.125 | 0.813 | 3.042 | | 0.824 | | 19 |
| 20 | | | 0.027 | 1.973 | 1.549 | 5.921 | 0.415 | 1.585 | 0.420 | 1.606 | 3.119 | 0.803 | 3.040 | | 0.814 | | 20 |
| 21 | | | 0.051 | 1.949 | 1.596 | 5.960 | 0.423 | 1.577 | 0.428 | 1.598 | 3.113 | 0.794 | 3.038 | | 0.804 | | 21 |
| 22 | | | 0.074 | 1.926 | 1.659 | 5.979 | 0.434 | 1.566 | 0.440 | 1.585 | 3.107 | 0.786 | 3.036 | | 0.796 | | 22 |
| 23 | | | 0.095 | 1.905 | 1.710 | 6.006 | 0.443 | 1.557 | 0.449 | 1.576 | 3.102 | 0.778 | 3.034 | | 0.787 | | 23 |
| 24 | | | 0.115 | 1.885 | 1.759 | 6.031 | 0.452 | 1.548 | 0.457 | 1.568 | 3.098 | 0.770 | 3.033 | | 0.780 | | 24 |
| 25 | | | 0.134 | 1.866 | 1.806 | 6.056 | 0.459 | 1.541 | 0.465 | 1.560 | 3.094 | 0.763 | 3.032 | | 0.773 | | 25 |
| Notes: | Notes: Only odd valu | | s of <i>n</i> are | es of <i>n</i> are given for As. As and As. Otherwise, the median for an even <i>n</i> is simply the average of the two middle values of | or A ₆ . A ₉ . | and Ao. | Otherwi | se the r | nedian f | or an eve | is si u ue | imply the | | e of the | two mido | le values | |

Notes: Only odd values of *n* are given for A_6 , A_8 , and A_9 . Otherwise, the median for an even *n* is simply the average of the two middle values of the subgroup. See Appendix 5 for values of A_7 , B_7 , and B_8 .

Appendix 5 Constants for A_7 , B_7 , and B_8

| n | k | A 7 | B 7 | B 8 |
|---|----|------------|------------|------------|
| 2 | 2 | 2.3937 | 0.0000 | 2.9409 |
| 2 | 3 | 2.3025 | 0.0000 | 2.8289 |
| 2 | 4 | 2.2568 | 0.0000 | 2.7727 |
| 2 | 5 | 2.2294 | 0.0000 | 2.7391 |
| 2 | 6 | 2.2112 | 0.0000 | 2.7167 |
| 2 | 7 | 2.1982 | 0.0000 | 2.7008 |
| 2 | 8 | 2.1885 | 0.0000 | 2.6888 |
| 2 | 9 | 2.1809 | 0.0000 | 2.6796 |
| 2 | 10 | 2.1749 | 0.0000 | 2.6722 |
| 2 | 11 | 2.1700 | 0.0000 | 2.6661 |
| 2 | 12 | 2.1659 | 0.0000 | 2.6611 |
| 2 | 13 | 2.1625 | 0.0000 | 2.6569 |
| 2 | 14 | 2.1595 | 0.0000 | 2.6532 |
| 2 | 15 | 2.1569 | 0.0000 | 2.6501 |
| 2 | 16 | 2.1547 | 0.0000 | 2.6473 |
| 2 | 17 | 2.1527 | 0.0000 | 2.6449 |
| 2 | 18 | 2.1510 | 0.0000 | 2.6427 |
| 2 | 19 | 2.1494 | 0.0000 | 2.6408 |
| 2 | 20 | 2.1480 | 0.0000 | 2.6391 |
| 2 | 21 | 2.1467 | 0.0000 | 2.6375 |
| 2 | 22 | 2.1456 | 0.0000 | 2.6361 |
| 2 | 23 | 2.1445 | 0.0000 | 2.6348 |
| 2 | 24 | 2.1435 | 0.0000 | 2.6336 |
| 2 | 25 | 2.1426 | 0.0000 | 2.6325 |
| 3 | 2 | 1.8426 | 0.0000 | 2.4213 |
| 3 | 3 | 1.8054 | 0.0000 | 2.3724 |
| 3 | 4 | 1.7869 | 0.0000 | 2.3480 |
| 3 | 5 | 1.7758 | 0.0000 | 2.3335 |

| n | k | A ₇ | B 7 | B 8 |
|---|----|-----------------------|------------|------------|
| 3 | 6 | 1.7685 | 0.0000 | 2.3238 |
| 3 | 7 | 1.7632 | 0.0000 | 2.3170 |
| 3 | 8 | 1.7593 | 0.0000 | 2.3118 |
| 3 | 9 | 1.7563 | 0.0000 | 2.3078 |
| 3 | 10 | 1.7538 | 0.0000 | 2.3046 |
| 3 | 11 | 1.7518 | 0.0000 | 2.3020 |
| 3 | 12 | 1.7502 | 0.0000 | 2.2998 |
| 3 | 13 | 1.7488 | 0.0000 | 2.2980 |
| 3 | 14 | 1.7476 | 0.0000 | 2.2964 |
| 3 | 15 | 1.7465 | 0.0000 | 2.2950 |
| 3 | 16 | 1.7456 | 0.0000 | 2.2938 |
| 3 | 17 | 1.7448 | 0.0000 | 2.2928 |
| 3 | 18 | 1.7441 | 0.0000 | 2.2918 |
| 3 | 19 | 1.7435 | 0.0000 | 2.2910 |
| 3 | 20 | 1.7429 | 0.0000 | 2.2902 |
| 3 | 21 | 1.7424 | 0.0000 | 2.2896 |
| 3 | 22 | 1.7419 | 0.0000 | 2.2889 |
| 3 | 23 | 1.7415 | 0.0000 | 2.2884 |
| 3 | 24 | 1.7411 | 0.0000 | 2.2879 |
| 3 | 25 | 1.7407 | 0.0000 | 2.2874 |
| 4 | 2 | 1.5635 | 0.0000 | 2.1762 |
| 4 | 3 | 1.5422 | 0.0000 | 2.1464 |
| 4 | 4 | 1.5315 | 0.0000 | 2.1316 |
| 4 | 5 | 1.5252 | 0.0000 | 2.1228 |
| 4 | 6 | 1.5210 | 0.0000 | 2.1169 |
| 4 | 7 | 1.5180 | 0.0000 | 2.1127 |
| 4 | 8 | 1.5157 | 0.0000 | 2.1096 |
| 4 | 9 | 1.5140 | 0.0000 | 2.1072 |

| n | k | A ₇ | B ₇ | B ₈ |
|---|----|-----------------------|-----------------------|----------------|
| 4 | 10 | 1.5125 | 0.0000 | 2.1052 |
| 4 | 11 | 1.5114 | 0.0000 | 2.1036 |
| 4 | 12 | 1.5105 | 0.0000 | 2.1023 |
| 4 | 13 | 1.5096 | 0.0000 | 2.1012 |
| 4 | 14 | 1.5090 | 0.0000 | 2.1002 |
| 4 | 15 | 1.5084 | 0.0000 | 2.0994 |
| 4 | 16 | 1.5078 | 0.0000 | 2.0987 |
| 4 | 17 | 1.5074 | 0.0000 | 2.0980 |
| 4 | 18 | 1.5070 | 0.0000 | 2.0974 |
| 4 | 19 | 1.5066 | 0.0000 | 2.0969 |
| 4 | 20 | 1.5063 | 0.0000 | 2.0965 |
| 4 | 21 | 1.5060 | 0.0000 | 2.0961 |
| 4 | 22 | 1.5057 | 0.0000 | 2.0957 |
| 4 | 23 | 1.5054 | 0.0000 | 2.0953 |
| 4 | 24 | 1.5052 | 0.0000 | 2.0950 |
| 4 | 25 | 1.4955 | 0.0000 | 2.0815 |
| 5 | 2 | 1.3841 | 0.0000 | 2.0258 |
| 5 | 3 | 1.3699 | 0.0000 | 2.0049 |
| 5 | 4 | 1.3628 | 0.0000 | 1.9945 |
| 5 | 5 | 1.3585 | 0.0000 | 1.9883 |
| 5 | 6 | 1.3557 | 0.0000 | 1.9842 |
| 5 | 7 | 1.3537 | 0.0000 | 1.9812 |
| 5 | 8 | 1.3522 | 0.0000 | 1.9790 |
| 5 | 9 | 1.3510 | 0.0000 | 1.9773 |
| 5 | 10 | 1.3501 | 0.0000 | 1.9759 |
| 5 | 11 | 1.3493 | 0.0000 | 1.9748 |
| 5 | 12 | 1.3486 | 0.0000 | 1.9739 |
| 5 | 13 | 1.3481 | 0.0000 | 1.9731 |
| 5 | 14 | 1.3476 | 0.0000 | 1.9724 |
| 5 | 15 | 1.3472 | 0.0000 | 1.9718 |
| 5 | 16 | 1.3469 | 0.0000 | 1.9713 |
| 5 | 17 | 1.3466 | 0.0000 | 1.9709 |
| 5 | 18 | 1.3463 | 0.0000 | 1.9705 |
| 5 | 19 | 1.3461 | 0.0000 | 1.9701 |
| 5 | 20 | 1.3458 | 0.0000 | 1.9698 |
| 5 | 21 | 1.3456 | 0.0000 | 1.9695 |
| 5 | 22 | 1.3455 | 0.0000 | 1.9692 |
| 5 | 23 | 1.3453 | 0.0000 | 1.9690 |
| 5 | 24 | 1.3451 | 0.0000 | 1.9687 |

| n | k | A ₇ | B 7 | B ₈ |
|---|----|-----------------------|------------|----------------|
| 5 | 25 | 1.3450 | 0.0000 | 1.9685 |
| 6 | 2 | 1.2557 | 0.0296 | 1.9215 |
| 6 | 3 | 1.2453 | 0.0294 | 1.9056 |
| 6 | 4 | 1.2401 | 0.0293 | 1.8977 |
| 6 | 5 | 1.2371 | 0.0292 | 1.8930 |
| 6 | 6 | 1.2350 | 0.0291 | 1.8899 |
| 6 | 7 | 1.2335 | 0.0291 | 1.8876 |
| 6 | 8 | 1.2324 | 0.0291 | 1.8859 |
| 6 | 9 | 1.2316 | 0.0291 | 1.8846 |
| 6 | 10 | 1.2309 | 0.0290 | 1.8836 |
| 6 | 11 | 1.2303 | 0.0290 | 1.8827 |
| 6 | 12 | 1.2299 | 0.0290 | 1.8820 |
| 6 | 13 | 1.2295 | 0.0290 | 1.8814 |
| 6 | 14 | 1.2291 | 0.0290 | 1.8809 |
| 6 | 15 | 1.2288 | 0.0290 | 1.8804 |
| 6 | 16 | 1.2286 | 0.0290 | 1.8800 |
| 6 | 17 | 1.2284 | 0.0290 | 1.8797 |
| 6 | 18 | 1.2282 | 0.0290 | 1.8794 |
| 6 | 19 | 1.2280 | 0.0290 | 1.8791 |
| 6 | 20 | 1.2278 | 0.0290 | 1.8789 |
| 6 | 21 | 1.2277 | 0.0290 | 1.8786 |
| 6 | 22 | 1.2275 | 0.0290 | 1.8784 |
| 6 | 23 | 1.2274 | 0.0290 | 1.8783 |
| 6 | 24 | 1.2273 | 0.0290 | 1.8781 |
| 6 | 25 | 1.2272 | 0.0289 | 1.8779 |
| 7 | 2 | 1.1577 | 0.1153 | 1.8438 |
| 7 | 3 | 1.1497 | 0.1145 | 1.8311 |
| 7 | 4 | 1.1458 | 0.1141 | 1.8247 |
| 7 | 5 | 1.1434 | 0.1138 | 1.8209 |
| 7 | 6 | 1.1418 | 0.1137 | 1.8184 |
| 7 | 7 | 1.1407 | 0.1136 | 1.8166 |
| 7 | 8 | 1.1398 | 0.1135 | 1.8153 |
| 7 | 9 | 1.1392 | 0.1134 | 1.8142 |
| 7 | 10 | 1.1386 | 0.1134 | 1.8134 |
| 7 | 11 | 1.1382 | 0.1133 | 1.8127 |
| 7 | 12 | 1.1378 | 0.1133 | 1.8121 |
| 7 | 13 | 1.1375 | 0.1133 | 1.8116 |
| 7 | 14 | 1.1373 | 0.1132 | 1.8112 |
| 7 | 15 | 1.1370 | 0.1132 | 1.8109 |

| n | k | A ₇ | B 7 | B 8 |
|---|----|-----------------------|------------|------------|
| 7 | 16 | 1.1369 | 0.1132 | 1.8105 |
| 7 | 17 | 1.1367 | 0.1132 | 1.8103 |
| 7 | 18 | 1.1365 | 0.1132 | 1.8100 |
| 7 | 19 | 1.1364 | 0.1132 | 1.8098 |
| 7 | 20 | 1.1363 | 0.1131 | 1.8096 |
| 7 | 21 | 1.1361 | 0.1131 | 1.8094 |
| 7 | 22 | 1.1360 | 0.1131 | 1.8093 |
| 7 | 23 | 1.1359 | 0.1131 | 1.8091 |
| 7 | 24 | 1.1359 | 0.1131 | 1.8090 |
| 7 | 25 | 1.1358 | 0.1131 | 1.8088 |
| 8 | 2 | 1.0798 | 0.1818 | 1.7830 |
| 8 | 3 | 1.0734 | 0.1808 | 1.7724 |
| 8 | 4 | 1.0702 | 0.1802 | 1.7671 |
| 8 | 5 | 1.0683 | 0.1799 | 1.7640 |
| 8 | 6 | 1.0670 | 0.1797 | 1.7619 |
| 8 | 7 | 1.0661 | 0.1795 | 1.7604 |
| 8 | 8 | 1.0654 | 0.1794 | 1.7593 |
| 8 | 9 | 1.0649 | 0.1793 | 1.7584 |
| 8 | 10 | 1.0645 | 0.1793 | 1.7577 |
| 8 | 11 | 1.0641 | 0.1792 | 1.7571 |
| 8 | 12 | 1.0638 | 0.1791 | 1.7567 |
| 8 | 13 | 1.0636 | 0.1791 | 1.7563 |
| 8 | 14 | 1.0634 | 0.1791 | 1.7559 |
| 8 | 15 | 1.0632 | 0.1790 | 1.7556 |
| 8 | 16 | 1.0630 | 0.1790 | 1.7554 |
| 8 | 17 | 1.0629 | 0.1790 | 1.7551 |
| 8 | 18 | 1.0628 | 0.1790 | 1.7549 |
| 8 | 19 | 1.0627 | 0.1790 | 1.7547 |
| 8 | 20 | 1.0626 | 0.1789 | 1.7546 |
| 8 | 21 | 1.0625 | 0.1789 | 1.7544 |
| 8 | 22 | 1.0624 | 0.1789 | 1.7543 |
| 8 | 23 | 1.0623 | 0.1789 | 1.7542 |
| 8 | 24 | 1.0622 | 0.1789 | 1.7541 |
| 8 | 25 | 1.0622 | 0.1789 | 1.7539 |
| 9 | 2 | 1.0157 | 0.2354 | 1.7337 |
| 9 | 3 | 1.0105 | 0.2342 | 1.7247 |
| 9 | 4 | 1.0078 | 0.2336 | 1.7202 |
| 9 | 5 | 1.0063 | 0.2332 | 1.7175 |
| 9 | 6 | 1.0052 | 0.2330 | 1.7157 |

| n | k | A 7 | B 7 | B ₈ | |
|----|----|------------|------------|-----------------------|--|
| 9 | 7 | 1.0045 | 0.2328 | 1.7145 | |
| 9 | 8 | 1.0039 | 0.2327 | 1.7135 | |
| 9 | 9 | 1.0035 | 0.2326 | 1.7128 | |
| 9 | 10 | 1.0031 | 0.2325 | 1.7122 | |
| 9 | 11 | 1.0028 | 0.2325 | 1.7117 | |
| 9 | 12 | 1.0026 | 0.2324 | 1.7113 | |
| 9 | 13 | 1.0024 | 0.2324 | 1.7109 | |
| 9 | 14 | 1.0022 | 0.2323 | 1.7106 | |
| 9 | 15 | 1.0021 | 0.2323 | 1.7104 | |
| 9 | 16 | 1.0020 | 0.2322 | 1.7102 | |
| 9 | 17 | 1.0018 | 0.2322 | 1.7100 | |
| 9 | 18 | 1.0017 | 0.2322 | 1.7098 | |
| 9 | 19 | 1.0016 | 0.2322 | 1.7096 | |
| 9 | 20 | 1.0016 | 0.2322 | 1.7095 | |
| 9 | 21 | 1.0015 | 0.2321 | 1.7094 | |
| 9 | 22 | 1.0014 | 0.2321 | 1.7093 | |
| 9 | 23 | 1.0014 | 0.2321 | 1.7091 | |
| 9 | 24 | 1.0013 | 0.2321 | 1.7091 | |
| 9 | 25 | 1.0013 | 0.2321 | 1.7090 | |
| 10 | 2 | 0.9619 | 0.2798 | 1.6927 | |
| 10 | 3 | 0.9575 | 0.2785 | 1.6849 | |
| 10 | 4 | 0.9553 | 0.2779 | 1.6810 | |
| 10 | 5 | 0.9540 | 0.2775 | 1.6787 | |
| 10 | 6 | 0.9531 | 0.2772 | 1.6771 | |
| 10 | 7 | 0.9525 | 0.2770 | 1.6760 | |
| 10 | 8 | 0.9520 | 0.2769 | 1.6752 | |
| 10 | 9 | 0.9516 | 0.2768 | 1.6745 | |
| 10 | 10 | 0.9513 | 0.2767 | 1.6740 | |
| 10 | 11 | 0.9511 | 0.2766 | 1.6736 | |
| 10 | 12 | 0.9509 | 0.2766 | 1.6732 | |
| 10 | 13 | 0.9507 | 0.2765 | 1.6729 | |
| 10 | 14 | 0.9506 | 0.2765 | 1.6727 | |
| 10 | 15 | 0.9504 | 0.2765 | 1.6725 | |
| 10 | 16 | 0.9503 | 0.2764 | 1.6723 | |
| 10 | 17 | 0.9502 | 0.2764 | 1.6721 | |
| 10 | 18 | 0.9501 | 0.2764 | 1.6719 | |
| 10 | 19 | 0.9501 | 0.2764 | 1.6718 | |
| 10 | 20 | 0.9500 | 0.2763 | 1.6717 | |
| 10 | 21 | 0.9499 | 0.2763 | 1.6716 | |

| n | k | A ₇ | B 7 | B 8 | |
|----|----|-----------------------|------------|------------|--|
| 10 | 22 | 0.9499 | 0.2763 | 1.6715 | |
| 10 | 23 | 0.9498 | 0.2763 | 1.6714 | |
| 10 | 24 | 0.9498 | 0.2763 | 1.6713 | |
| 10 | 25 | 0.9497 | 0.2763 | 1.6712 | |
| 11 | 2 | 0.9159 | 0.3173 | 1.6579 | |
| 11 | 3 | 0.9121 | 0.3160 | 1.6510 | |
| 11 | 4 | 0.9102 | 0.3153 | 1.6476 | |
| 11 | 5 | 0.9091 | 0.3149 | 1.6455 | |
| 11 | 6 | 0.9083 | 0.3147 | 1.6442 | |
| 11 | 7 | 0.9078 | 0.3145 | 1.6432 | |
| 11 | 8 | 0.9074 | 0.3143 | 1.6425 | |
| 11 | 9 | 0.9071 | 0.3142 | 1.6419 | |
| 11 | 10 | 0.9068 | 0.3141 | 1.6414 | |
| 11 | 11 | 0.9066 | 0.3141 | 1.6411 | |
| 11 | 12 | 0.9064 | 0.3140 | 1.6408 | |
| 11 | 13 | 0.9063 | 0.3140 | 1.6405 | |
| 11 | 14 | 0.9062 | 0.3139 | 1.6403 | |
| 11 | 15 | 0.9060 | 0.3139 | 1.6401 | |
| 11 | 16 | 0.9059 | 0.3139 | 1.6399 | |
| 11 | 17 | 0.9059 | 0.3138 | 1.6397 | |
| 11 | 18 | 0.9058 | 0.3138 | 1.6396 | |
| 11 | 19 | 0.9057 | 0.3138 | 1.6395 | |
| 11 | 20 | 0.9057 | 0.3138 | 1.6394 | |
| 11 | 21 | 0.9056 | 0.3137 | 1.6393 | |
| 11 | 22 | 0.9056 | 0.3137 | 1.6392 | |
| 11 | 23 | 0.9055 | 0.3137 | 1.6391 | |
| 11 | 24 | 0.9055 | 0.3137 | 1.6390 | |
| 11 | 25 | 0.9054 | 0.3137 | 1.6390 | |
| 12 | 2 | 0.8759 | 0.3495 | 1.6279 | |
| 12 | 3 | 0.8726 | 0.3482 | 1.6218 | |
| 12 | 4 | 0.8710 | 0.3475 | 1.6187 | |
| 12 | 5 | 0.8700 | 0.3472 | 1.6169 | |
| 12 | 6 | 0.8693 | 0.3469 | 1.6156 | |
| 12 | 7 | 0.8688 | 0.3467 | 1.6148 | |
| 12 | 8 | 0.8685 | 0.3466 | 1.6141 | |
| 12 | 9 | 0.8682 | 0.3465 | 1.6136 | |
| 12 | 10 | 0.8680 | 0.3464 | 1.6132 | |
| 12 | 11 | 0.8678 | 0.3463 | 1.6129 | |
| 12 | 12 | 0.8677 | 0.3462 | 1.6126 | |

| n | k | A ₇ | B 7 | B ₈ | |
|----|----|-----------------------|---------------|----------------|--|
| 12 | 13 | 0.8675 | 0.3462 | 1.6124 | |
| 12 | 14 | 0.8674 | 0.3461 | 1.6122 | |
| 12 | 15 | 0.8673 | 0.3461 | 1.6120 | |
| 12 | 16 | 0.8673 | 0.3461 | 1.6118 | |
| 12 | 17 | 0.8672 | 0.3460 | 1.6117 | |
| 12 | 18 | 0.8671 | 0.3460 | 1.6116 | |
| 12 | 19 | 0.8671 | 0.3460 | 1.6115 | |
| 12 | 20 | 0.8670 | 0.3460 | 1.6114 | |
| 12 | 21 | 0.8670 | 0.3460 | 1.6113 | |
| 12 | 22 | 0.8669 | 0.3459 | 1.6112 | |
| 12 | 23 | 0.8669 | 0.3459 | 1.6111 | |
| 12 | 24 | 0.8668 | 0.3459 | 1.6111 | |
| 12 | 25 | 0.8668 | 0.3459 | 1.6110 | |
| 13 | 2 | 0.8408 | 0.3776 | 1.6017 | |
| 13 | 3 | 0.8378 | 0.3763 | 1.5962 | |
| 13 | 4 | 0.8364 | 0.3756 | 1.5934 | |
| 13 | 5 | 0.8355 | 0.3753 | 1.5917 | |
| 13 | 6 | 0.8349 | 0.3750 | 1.5906 | |
| 13 | 7 | 0.8345 | 0.3748 | 1.5898 | |
| 13 | 8 | 0.8342 | 0.3747 | 1.5892 | |
| 13 | 9 | 0.8340 | 0.3746 | 1.5888 | |
| 13 | 10 | 0.8338 | 0.3745 | 1.5884 | |
| 13 | 11 | 0.8336 | 0.3744 | 1.5881 | |
| 13 | 12 | 0.8335 | 0.3743 | 1.5879 | |
| 13 | 13 | 0.8334 | 0.3743 | 1.5877 | |
| 13 | 14 | 0.8333 | 0.3743 | 1.5875 | |
| 13 | 15 | 0.8332 | 0.3742 | 1.5873 | |
| 13 | 16 | 0.8331 | 0.3742 | 1.5872 | |
| 13 | 17 | 0.8331 | 0.3742 | 1.5871 | |
| 13 | 18 | 0.8330 | 0.3741 | 1.5869 | |
| 13 | 19 | 0.8330 | 0.3741 | 1.5869 | |
| 13 | 20 | 0.8329 | 0.3741 | 1.5868 | |
| 13 | 21 | 0.8329 | 0.3741 | 1.5867 | |
| 13 | 22 | 0.8328 | 0.3741 | 1.5866 | |
| 13 | 23 | 0.8328 | 0.3740 | 1.5865 | |
| 13 | 24 | 0.8328 | 0.3740 1.5865 | | |
| 13 | 25 | 0.8327 | 0.3740 | 1.5864 | |
| 14 | 2 | 0.8095 | 0.4024 | 1.5785 | |
| 14 | 3 | 0.8069 | 0.4011 | 1.5735 | |

| n | k | A 7 | B 7 | B ₈ | | n | k | A ₇ | B 7 | B ₈ |
|----|----|------------|------------|-----------------------|---|----|----|-----------------------|------------|-----------------------|
| 14 | 4 | 0.8056 | 0.4004 | 1.5710 |] | 15 | 3 | 0.7792 | 0.4231 | 1.5532 |
| 14 | 5 | 0.8049 | 0.4001 | 1.5695 |] | 15 | 4 | 0.7781 | 0.4225 | 1.5509 |
| 14 | 6 | 0.8044 | 0.3998 | 1.5684 | | 15 | 5 | 0.7774 | 0.4221 | 1.5495 |
| 14 | 7 | 0.8040 | 0.3996 | 1.5677 |] | 15 | 6 | 0.7769 | 0.4219 | 1.5486 |
| 14 | 8 | 0.8037 | 0.3995 | 1.5672 | | 15 | 7 | 0.7766 | 0.4217 | 1.5479 |
| 14 | 9 | 0.8035 | 0.3994 | 1.5668 |] | 15 | 8 | 0.7763 | 0.4216 | 1.5475 |
| 14 | 10 | 0.8033 | 0.3993 | 1.5664 |] | 15 | 9 | 0.7761 | 0.4215 | 1.5471 |
| 14 | 11 | 0.8032 | 0.3992 | 1.5662 |] | 15 | 10 | 0.7760 | 0.4214 | 1.5468 |
| 14 | 12 | 0.8031 | 0.3992 | 1.5659 |] | 15 | 11 | 0.7759 | 0.4213 | 1.5465 |
| 14 | 13 | 0.8030 | 0.3991 | 1.5657 |] | 15 | 12 | 0.7758 | 0.4213 | 1.5463 |
| 14 | 14 | 0.8029 | 0.3991 | 1.5656 |] | 15 | 13 | 0.7757 | 0.4212 | 1.5461 |
| 14 | 15 | 0.8028 | 0.3990 | 1.5654 |] | 15 | 14 | 0.7756 | 0.4212 | 1.5460 |
| 14 | 16 | 0.8027 | 0.3990 | 1.5653 |] | 15 | 15 | 0.7755 | 0.4211 | 1.5458 |
| 14 | 17 | 0.8027 | 0.3990 | 1.5652 |] | 15 | 16 | 0.7755 | 0.4211 | 1.5457 |
| 14 | 18 | 0.8026 | 0.3989 | 1.5651 |] | 15 | 17 | 0.7754 | 0.4211 | 1.5456 |
| 14 | 19 | 0.8026 | 0.3989 | 1.5650 | | 15 | 18 | 0.7754 | 0.4210 | 1.5455 |
| 14 | 20 | 0.8026 | 0.3989 | 1.5649 |] | 15 | 19 | 0.7753 | 0.4210 | 1.5455 |
| 14 | 21 | 0.8025 | 0.3989 | 1.5649 |] | 15 | 20 | 0.7753 | 0.4210 | 1.5454 |
| 14 | 22 | 0.8025 | 0.3989 | 1.5648 |] | 15 | 21 | 0.7753 | 0.4210 | 1.5453 |
| 14 | 23 | 0.8025 | 0.3988 | 1.5647 |] | 15 | 22 | 0.7752 | 0.4210 | 1.5453 |
| 14 | 24 | 0.8024 | 0.3988 | 1.5647 | | 15 | 23 | 0.7752 | 0.4210 | 1.5452 |
| 14 | 25 | 0.8024 | 0.3988 | 1.5646 | 1 | 15 | 24 | 0.7752 | 0.4209 | 1.5452 |
| 15 | 2 | 0.7815 | 0.4244 | 1.5578 | 1 | 15 | 25 | 0.7752 | 0.4209 | 1.5452 |

Appendix 6 Factors for Estimating σ_X

| | 1 | | [| 1 | | 1 |
|----|------------|------------|-----------------------|-----------------------|-------|----|
| n | C 2 | C 4 | d ₂ | d ₃ | d_4 | n |
| 2 | 0.5642 | 0.7979 | 1.128 | 0.8525 | 0.954 | 2 |
| 3 | 0.7236 | 0.8862 | 1.693 | 0.8884 | 1.588 | 3 |
| 4 | 0.7979 | 0.9213 | 2.059 | 0.8798 | 1.978 | 4 |
| 5 | 0.8407 | 0.9400 | 2.326 | 0.8641 | 2.257 | 5 |
| 6 | 0.8686 | 0.9515 | 2.534 | 0.8480 | 2.472 | 6 |
| 7 | 0.8882 | 0.9594 | 2.704 | 0.8332 | 2.645 | 7 |
| 8 | 0.9027 | 0.9650 | 2.847 | 0.8198 | 2.791 | 8 |
| 9 | 0.9139 | 0.9693 | 2.970 | 0.8078 | 2.915 | 9 |
| 10 | 0.9227 | 0.9727 | 3.078 | 0.7971 | 3.024 | 10 |
| 11 | 0.9300 | 0.9754 | 3.173 | 0.7873 | 3.121 | 11 |
| 12 | 0.9359 | 0.9776 | 3.258 | 0.7785 | 3.207 | 12 |
| 13 | 0.9410 | 0.9794 | 3.336 | 0.7704 | 3.285 | 13 |
| 14 | 0.9453 | 0.9810 | 3.407 | 0.7630 | 3.356 | 14 |
| 15 | 0.9490 | 0.9823 | 3.472 | 0.7562 | 3.422 | 15 |
| 16 | 0.9523 | 0.9835 | 3.532 | 0.7499 | 3.482 | 16 |
| 17 | 0.9551 | 0.9845 | 3.588 | 0.7441 | 3.538 | 17 |
| 18 | 0.9576 | 0.9854 | 3.640 | 0.7386 | 3.591 | 18 |
| 19 | 0.9599 | 0.9862 | 3.689 | 0.7335 | 3.640 | 19 |
| 20 | 0.9619 | 0.9869 | 3.735 | 0.7287 | 3.686 | 20 |
| 21 | 0.9638 | 0.9876 | 3.778 | 0.7272 | 3.730 | 21 |
| 22 | 0.9655 | 0.9882 | 3.819 | 0.7199 | 3.771 | 22 |
| 23 | 0.9670 | 0.9887 | 3.858 | 0.7159 | 3.811 | 23 |
| 24 | 0.9684 | 0.9892 | 3.895 | 0.7121 | 3.847 | 24 |
| 25 | 0.9695 | 0.9896 | 3.931 | 0.7084 | 3.883 | 25 |

Note: σ_x may be estimated from *k* subgroups of size *n*:

| $\frac{\overline{\sigma}_{RMS}}{C_2}$ | $\frac{\overline{s}}{c_4}$ |
|---------------------------------------|-----------------------------|
| $\frac{\overline{R}}{d_{2}}$ | $\frac{\overline{mR}}{d_2}$ |
| $\frac{\tilde{R}}{d_{A}}$ | |

Appendix 7

Control Charts for Count Data

| Chart | Center line | LCL | UCL | Plot point |
|-------------|-------------------------------------------------------------------|-------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------|--------------------------------------|
| p | $\overline{\rho} = \frac{\sum_{i=1}^{k} D_i}{\sum_{i=1}^{k} n_i}$ | $LCL_{\rho} = \overline{\rho} - 3\sqrt{\frac{\overline{\rho}(1-\overline{\rho})}{n_{i}}}$ | $UCL_{p} = \overline{p} + 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n_{i}}}$ | $\boldsymbol{p}_i = \frac{D_i}{n_i}$ |
| $D_i = nun$ | nber of defective | (nonconforming) units in the | e i th subgroup | |
| np | $n\overline{p} = n \frac{\sum_{i=1}^{k} D_i}{\sum_{i=1}^{k} n_i}$ | $LCL_{np} = n\overline{p} - 3\sqrt{n\overline{p}(1-\overline{p})}$ | $UCL_{np} = n\overline{p} + 3\sqrt{n\overline{p}\left(1-\overline{p}\right)}$ | Di |
| С | $\overline{c} = \frac{\sum_{i=1}^{k} c_i}{k}$ | $LCL_c = \overline{c} - 3\sqrt{\overline{c}}$ | $UCL = \overline{c} + 3\sqrt{\overline{c}}$ | Ci |
| $C_i = nun$ | nber of defects (| nonconformities) in the <i>i</i> th su | bgroup | |
| u | $\overline{u} = \frac{\sum_{i=1}^{k} D_i}{\sum_{i=1}^{k} n_i}$ | $LCL_u = \overline{u} - 3\sqrt{\frac{\overline{u}}{n_i}}$ | $UCL_u = \overline{u} + 3\sqrt{\frac{\overline{u}}{n_i}}$ | $u_i = \frac{c_i}{n_i}$ |

Appendix 8 Binomial Distribution Table

| | | | | | | | Prob | ability o | f x occu | rrences | Probability of x occurrences in a sample of si | ď | size n | | | | | | | |
|----------|----------|---------------|---------------|---------------|--------|---------------------------------------------------------------------------------------------------|--------|---------------|----------|---------------|------------------------------------------------|--------|---------------|--------|----------------------|-----------------------------|---------------|--------|--------|----------|
| 0.05 | - | 0.10 | 0.15 | 0.20 | 0.25 | 0:30 | 0.35 | 0.40 | 0.45 | 0.50 | 0.55 | 09.0 | 0.65 | 0.70 | 0.75 | 0.80 | 0.85 | 0:00 | 0.95 | <i>u</i> |
| 0.9500 (| <u> </u> | 0.9000 | 0.8500 | 0.8000 | 0.7500 | 0.7000 | 0.6500 | 0.6000 | 0.5500 | | 0.5000 0.4500 | 0.4000 | 0.3500 | 0.3000 | 0.2500 | 0.2000 | 0.1500 | 0.1000 | 0.0500 | - |
| 0.0500 | | 0.1000 | | 0.1500 0.2000 | 0.2500 | 0.3000 | | 0.4000 | 0.4500 | 0.5000 | 0.3500 0.4000 0.4500 0.5000 0.5500 0.6000 | 0.6000 | 0.6500 | 0.7000 | 0.7000 0.7500 | 0.8000 | 0.8500 | 0.9000 | 0.9500 | |
| 0.9025 | | 0.8100 | 0.7225 | 0.7225 0.6400 | 0.5625 | 0.4900 | 0.4225 | 0.3600 | 0.3025 | | 0.2500 0.2025 | 0.1600 | 0.1225 | 0060.0 | 0.0625 | 0.0400 | 0.0225 | 0.0100 | 0.0025 | 2 |
| 0.0950 | | 0.1800 | 0.2550 | 0.3200 | 0.3750 | 0.4200 | 0.4550 | 0.4800 | 0.4950 | 0.5000 | 0.4950 | 0.4800 | 0.4550 | 0.4200 | 0.3750 | 0.3200 | 0.2550 | 0.1800 | 0.0950 | |
| 0.0025 | | 0.0100 | 0.0100 0.0225 | 0.0400 | 0.0625 | 0.0900 | 0.1225 | 0.1600 | 0.2025 | | 0.2500 0.3025 | | 0.3600 0.4225 | 0.4900 | 0.4900 0.5625 | 0.6400 | 0.7225 | 0.8100 | 0.9025 | |
| 0.8574 | | 0.7290 | 0.6141 | 0.5120 | 0.4219 | 0.3430 | 0.2746 | 0.2160 | 0.1664 | 0.1250 | 0.0911 | 0.0640 | 0.0429 | 0.0270 | 0.0156 | 0.0080 | 0.0034 | 0.0010 | 0.0001 | e |
| 0.1354 | | 0.2430 | 0.3251 | 0.3840 | 0.4219 | 0.4410 | 0.4436 | 0.4320 | 0.4084 | | 0.3750 0.3341 0.2880 | | | 0.1890 | 0.2389 0.1890 0.1406 | 0.0960 | 0.0574 | 0.0270 | 0.0071 | |
| 0.0071 | | 0.0270 | 0.0574 | 0.0960 | 0.1406 | 0.1890 | 0.2389 | 0.2880 | 0.3341 | 0.3750 | 0.4084 | 0.4320 | 0.4436 | 0.4410 | 0.4219 | 0.3840 | 0.3251 | 0.2430 | 0.1354 | |
| 0.0001 | | 0.0010 | | 0.0034 0.0080 | 0.0156 | 0.0270 | 0.0429 | 0.0640 | 0.0911 | 0.1250 | 0.1664 | 0.2160 | 0.2746 | 0.3430 | 0.4219 | 0.5120 | 0.6141 | 0.7290 | 0.8574 | |
| 0.8145 | | 0.6561 | 0.5220 | 0.4096 | 0.3164 | 0.2401 | 0.1785 | 0.1296 | 0.0915 | 0.0625 | 0.0410 | 0.0256 | 0.0150 | 0.0081 | 0.0039 | 0.0016 | 0.0005 | 0.0001 | 0.0000 | 4 |
| 0.1715 | | 0.2916 | 0.3685 | 0.4096 | 0.4219 | 0.4116 | 0.3845 | 0.3456 | 0.2995 | 0.2500 | 0.2005 | 0.1536 | 0.1115 | 0.0756 | 0.0469 | 0.0256 | 0.0115 | 0.0036 | 0.0005 | _ |
| 0.0135 | | 0.0486 | 0.0975 | 0.1536 | 0.2109 | 0.2646 | 0.3105 | 0.3456 | 0.3675 | 0.3750 | 0.3675 | 0.3456 | 0.3105 | 0.2646 | 0.2109 | 0.1536 | 0.0975 | 0.0486 | 0.0135 | |
| 0.0005 | | 0.0036 | 0.0115 | 0.0256 | 0.0469 | 0.0756 | 0.1115 | 0.1536 | 0.2005 | | 0.2500 0.2995 | 0.3456 | 0.3845 | 0.4116 | 0.4219 | 0.4096 | 0.3685 | 0.2916 | 0.1715 | |
| 0.0000 | - | 0.0001 | 0.0005 | 0.0016 | 0.0039 | 0.0081 | 0.0150 | 0.0256 | 0.0410 | 0.0625 | 0.0915 | 0.1296 | 0.1785 | 0.2401 | 0.3164 | 0.4096 | 0.5220 | 0.6561 | 0.8145 | |
| 0.7738 | | 0.5905 | 0.4437 | 0.3277 | 0.2373 | 0.1681 | 0.1160 | 0.1160 0.0778 | | 0.0503 0.0313 | 0.0185 | 0.0102 | | 0.0024 | 0.0010 | 0.0053 0.0024 0.0010 0.0003 | 0.0001 0.0000 | 0.0000 | 0.0000 | 5 |
| 0.2036 | | 0.3281 | 0.3915 | 0.4096 | 0.3955 | 0.3602 | 0.3124 | 0.2592 | 0.2059 | 0.1563 | 0.1128 | 0.0768 | 0.0488 | 0.0284 | 0.0146 | 0.0064 | 0.0022 | 0.0005 | 0.0000 | |
| 0.0214 | | 0.0729 | 0.1382 | 0.2048 | 0.2637 | 0.3087 | 0.3364 | 0.3456 | 0.3369 | 0.3125 | 0.2757 | 0.2304 | 0.1811 | 0.1323 | 0.1811 0.1323 0.0879 | 0.0512 | 0.0244 | 0.0081 | 0.0011 | |
| 0011 | | 0.0011 0.0081 | | 0.0244 0.0512 | 0.0879 | 0.1323 | 0.1811 | 0.2304 | 0.2757 | 0.3125 | 0.3369 | 0.3456 | 0.3364 | 0.3087 | 0.2637 | 0.2048 | 0.1382 | 0.0729 | 0.0214 | |
| 0.0000 | | 0.0005 | 0.0022 | 0.0064 | 0.0146 | 0.0284 | 0.0488 | 0.0768 | 0.1128 | 0.1563 | 0.2059 | 0.2592 | 0.3124 | 0.3602 | 0.3955 | 0.4096 | 0.3915 | 0.3281 | 0.2036 | |
| 0.0000 | - | 0.0000 | 0.0000 0.0001 | 0.0003 | | 0.0010 0.0024 0.0053 0.0102 0.0185 0.0313 0.0503 0.0778 0.1160 0.1681 0.2373 0.3277 0.4437 0.5905 | 0.0053 | 0.0102 | 0.0185 | 0.0313 | 0.0503 | 0.0778 | 0.1160 | 0.1681 | 0.2373 | 0.3277 | 0.4437 | 0 5905 | 0 7738 | |

| | | × | 0 | - | 2 | 3 | 4 | 5 | 9 | 0 | - | 2 | e | 4 | 5 | 9 | 7 | 0 | - | 2 | e | 4 | 5 | 9 | 7 | 8 |
|--------------------------------------------------------------|---------------------------------------------|--------|---------------|-----------------------------------------------------------------------|-----------------------------|-------------------------------------------------------------------------------------|---------------|--------------------------------------------------------------------------------------------|-----------------------------|--------------------------------------------------------------------------------------------|----------------------|-------------------------------------------------------------------------------------|-----------------------------|----------------------------------------------------------------|-----------|--------------------------------------------------|---------------|---------------------------------------------------------------------------------------------------|---------------|-----------------------------------------------------------------------|-------------------------------------------|----------------------|---------------|-----------------------------|--------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------|
| | | u | 9 | | | | | | | 2 | | | | | | | | 8 | - | | | | | | | |
| | | 0.95 | 0.0000 | 0.0000 | 0.0001 | 0.0021 | 0.0305 | 0.2321 | 0.7351 | 0.0000 | 0.0000 | 0.0000 | 0.0002 | 0.0036 | 0.0406 | 0.2573 | 0.6983 | 0.0000 | 0.0000 | 0.0000 0.0000 | 0.0000 | 0.0004 | 0.0054 | 0.0515 | 0.2793 | 0.6634 |
| | | 0.90 | 0.0000 | 0.0001 | 0.0012 | 0.0146 | 0.0984 | 0.3543 | 0.5314 | 0.0000 | 0.0000 | 0.0002 | 0.0026 | 0.0230 | 0.1240 | 0.3720 | 0.4783 | 0.0000 | 0.0000 | 0.0000 | 0.0004 | 0.0046 | 0.0331 | 0.1488 | 0.3826 | 0.4305 |
| | | 0.85 | 0.0000 | 0.0004 | 0.0055 | 0.0415 | 0.1762 | 0.3993 | 0.3771 | 0.0000 | 0.0001 | 0.0012 | 0.0109 | 0.0617 | 0.2097 | 0.3960 | 0.3206 | 0.0000 | 0.0000 | 0.0002 | 0.0026 | 0.0185 | 0.0839 | 0.2376 | 0.3847 | 0.2725 |
| | | 0.80 | 0.0001 | 0.0015 | 0.0154 0.0055 | 0.0819 | 0.2458 | 0.3932 | 0.2621 | 0.0000 | 0.0004 0.0001 | 0.0043 | 0.0287 0.0109 | 0.1147 0.0617 | 0.2753 | 0.3670 | 0.2097 | 0.0000 | 0.0001 0.0000 | 0.0011 | 0.0092 | 0.0459 | 0.1468 | 0.2936 0.2376 | 0.3355 | 0.1678 |
| | | 0.75 | 0.0002 | 0.0044 | | 0.1318 | 0.2966 | 0.3560 | | 0.0001 | | 0.0115 | | 0.1730 | 0.3115 | 0.3115 | 0.1335 | 0.0000 | 0.0004 | 0.0038 | 0.0231 | | | | 0.2670 | 0.1001 |
| | | 0.70 | 0.0007 | 0.0102 | 0.0595 | 0.1852 | 0.3241 | 0.3025 | 0.0754 0.1176 0.1780 | 0.0002 | 0.0084 0.0036 0.0013 | 0.0250 | 0.0972 0.0577 | 0.2269 | 0.3177 | 0.2471 | 0.0824 | 0.0001 | 0.0012 0.0004 | 0.0100 | 0.0467 | 0.1361 0.0865 | 0.2541 0.2076 | 0.2965 0.3115 | 0.1977 | 0.0576 |
| | еn | 0.65 | | 0.0205 (| 0.0951 (| .2355 (| 0.3280 (| 0.2437 0 | 0.0754 0 | 0.0006 (| 0.0084 0 | 0.0466 | | .2679 (| 0.2985 (| 0.1848 (| 0.0490 (| 0.0002 0 | 0.0033 (| 0.0217 0 | 0.0808 | 0.1875 0 | 0.2786 (| | 0.1373 0 | 0.0319 0 |
| - p) ^{n-k} | le of size <i>n</i> | 0.60 | 0.0041 0.0018 | 0369 0 | 0.1382 0.0951 0.0595 0.0330 | .2765 0 | 0.3110 0 | .1866 0 | 0.0467 0 | 0016 0 | 0.0172 0 | 0774 0 | 0.1935 0.1442 | .2903 0 | 0.2613 0 | 0.1306 0.1848 0.2471 0.3115 0.3670 0.3960 | 0.0280 0 | 0007 0 | 0.0079 0.0033 | .0413 0 | 0.1239 0.0808 0.0467 0.0231 0.0092 0.0026 | 0.2322 0 | 0.2787 0 | 0.2090 0.2587 | 0896 0 | 0.0168 |
| $\binom{1}{c}p^{k}(1-$ | a samp | 0.55 | | 06090 | | .3032 0 | | .1359 0 | .0277 0 | .0037 0 | .0320 0 | .1172 0 | | .2918 0 | 0.2140 0 | .0872 0 | | .0017 0 | .0164 0 | .0703 0 | 0.1719 0 | | 0.2568 0 | | .0548 0 | .0084 0 |
| $\Pr(X = x) = f(n; x, p) = \binom{n}{k} p^{k} (1 - p)^{n-k}$ | Probability of x occurrences in a sample of | 0.50 | 0.0156 0.0083 | 0.1866 0.1359 0.0938 0.0609 0.0369 0.0205 0.0102 0.0044 0.0015 0.0004 | 0.2344 0.1861 | .3125 0 | 0.2344 0.2780 | 0.0938 0 | 0.0041 0.0083 0.0156 0.0277 | 0078 0 | 0.0547 0.0320 | .1641 0 | 0.2903 0.2918 0.2734 0.2388 | .2734 0 | 0.1641 0 | 0547 0 | 0.0078 0.0152 | 0039 0 | 0.0313 0.0164 | 0.2090 0.1569 0.1094 0.0703 0.0413 0.0217 0.0100 0.0038 0.0011 0.0002 | 0.2188 0 | 0.2734 0.2627 | 0.2188 0 | 0.0413 0.0703 0.1094 0.1569 | 0.0313 0 | 0039 0 |
|) = f(n; > | x occurr | 0.45 | 0.0277 0 | .1359 0 | .2780 0 | .3032 0 | 0.1861 0 | 0609 0 | 0083 0 | 0152 0 | | .2140 0 | .2918 0 | .2388 0 | 0.1172 0 | 0320 0 | 0.0037 0 | 0084 0 | | .1569 0 | 0.2568 0.2188 | 0.2627 0 | 0.1719 0 | 0703 0 | 0164 0 | .0017 0 |
| r(X = x) | bility of | 0.40 | 0.0467 0 | .1866 0 | 0.3110 0.2780 | .2765 0 | 0.1382 0 | .0369 0 | .0041 0 | .0280 0 | 0.1306 0.0872 | .2613 0 | .2903 0 | .1935 0 | 0.0774 0 | .0172 0 | 0.0016 0 | .0168 0 | 0.0896 0.0548 | 2090 0 | | 0.2322 0 | 0.1239 0 | .0413 0 | 0 6200. | 0007_0 |
| | Proba | 0.35 | 0.0754 0 | .2437 0 | | 0.1852 0.2355 0.2765 0.3032 0.3125 0.3032 0.2765 0.2355 0.1852 0.1318 0.0819 0.0415 | 0.0951 0 | 0.0044 0.0102 0.0205 0.0369 0.0609 0.0938 0.1359 0.1866 0.2437 0.3025 0.3560 0.3932 0.3993 | .0018 0 | 0.0824 0.0490 0.0280 0.0152 0.0078 0.0037 0.0016 0.0006 0.0002 0.0001 0.0000 0.0000 0.0000 | 0.1848 0 | 0.3177 0.2985 0.2613 0.2140 0.1641 0.1172 0.0774 0.0466 0.0250 0.0115 0.0043 0.0012 | 0.2679 0 | 0.1442 0.1935 0.2388 0.2734 0.2918 0.2903 0.2679 0.2269 0.1730 | 0.0466 0 | 0.0013 0.0036 0.0084 0.0172 0.0320 0.0547 0.0872 | 0.0006 0 | 0.1001 0.0576 0.0319 0.0168 0.0084 0.0039 0.0017 0.0007 0.0002 0.0001 0.0000 0.0000 0.0000 0.0000 | 0.1373 0 | .2587 0 | 0.2541 0.2786 0.2787 | | 0.0808 0 | | 0.0004 0.0012 0.0033 0.0079 0.0164 0.0313 0.0548 0.0896 0.1373 0.1977 0.2670 0.3355 0.3847 | 0.0000 0.0001 0.0002 0.0007 0.0017 0.0039 0.0084 0.0168 0.0319 0.0576 0.1001 0.1678 0.2725 0.4305 0.6634 |
| | | 0.30 | 0.1176 0 | 0.3560 0.3025 0.2437 | 0.3241 0.3280 | .1852 0 | 0.0595 0 | .0102 0 | 0.0007 0.0018 | .0824 0 | 0.2471 0 | .3177 0 | 0.2269 0 | 0.0972 0 | 0.0250 0 | .0036 0 | 0.0002 0 | .0576 0 | 0.1977 0 | 0.2965 0.2587 | .2541 0 | 0.1361 0.1875 | 0.0467 0 | 0.0100 0.0217 | .0012 0 | .0001 |
| | | 0.25 | 0.1780 0 | .3560 0 | 0.2966 0 | 0.1318 0 | 0.0330 0 | 0044 0 | 0.0002 0 | 0.1335 0 | 0.3115 0 | 0.3115 0 | 0.1730 0 | 0.0577 0 | 0.0115 0 | 0013 0 | 0.0001 0 | 1001 0 | 0.2670 0 | 0.3115 0 | 0.2076 0 | 0.0865 0 | 0.0231 0 | 0.0038 0 | .0004 0 | 0000 |
| | | 0.20 | 0.2621 0 | 0.3932 0. | 0.2458 0. | | 0.0154 0. | | | | 0.3670 0. | | 0.1147 0. | | 0.0043 0. | | 0.0000 0. | | 0.3355 0. | | 0.1468 0 | | 0.0092 0. | 0.0011 0. | | |
| | | 0.15 | 0.3771 0. | .3993 0 | | 0.0146 0.0415 0.0819 | .0055 0 | 0.0000 0.0001 0.0004 0.0015 | 0.0000 0.0000 0.0001 | 0.4783 0.3206 0.2097 | | 0.0406 0.1240 0.2097 0.2753 | | 0.0026 0.0109 0.0287 | 0.0012 0. | 0.0000 0.0001 0.0004 | 0.0000 0. | 0.6634 0.4305 0.2725 0.1678 | 3847 0 | 0.1488 0.2376 0.2936 | .0839 0 | 0.0046 0.0185 0.0459 | .0026 0 | .0002 0 | 0.0000 0.0000 0.0000 0.0001 | 0.0000 0.0000 0.0000 0.0000 |
| | | 0.10 | 0.5314 0. | 0.3543 0.3993 | 0.0984 0.1762 | .0146 0. | 0.0012 0.0055 | 0001 0. | 0000 | 4783 0. | 0.3720 0.3960 | 1240 0. | 0.0230 0.0617 | 0026 0. | 0.0002 0. | 0000 | 0.0000 0. | 4305 0. | 0.3826 0.3847 | 1488 0. | 0.0331 0.0839 | .0046 0. | 0.0004 0.0026 | 0.0000 0.0002 | 0000 | 0000 |
| | | 0.05 (| 0.7351 0. | 0.2321 0. | 0.0305 0. | 0.0021 0. | 0.0001 0. | 0000 0. | 0.0000 0. | 0.6983 0. | 0.2573 0. | 0406 0. | 0.0036 0. | 0.0002 0. | 0.0000 0. | 0.0000 0. | 0.0000 0. | 6634 0. | 0.2793 0. | 0.0515 0. | 0.0054 0. | 0.0004 0. | 0.0000 0. | 0.0000 0. | 0000 0. | 0000 0. |
| | | | 0 0 | 1 0.2 | 2 0.0 | 3 0.0 | 4 0.(| 5 0.0 | 6 0.0 | 0 0.6 | 1 0.1 | 2 0.0 | 3 0.0 | 4 0.(| 5 0.(| 6 0.0 | 7 0.0 | 0 0.6 | 1 0. | 2 0.0 | 3 0.0 | 4 0.0 | 5 0.0 | 6 0.0 | 7 0.(| 8 0.0 |
| | | n x | 9 | | | | | | | 7 | | | | | | | | 8 | | | | | | | | |

| | | x | 0 | - | 2 | з | 4 | 5 | 9 | 7 | 8 | 6 | 0 | - | 2 | ю | 4 | 5 | 9 | 7 | 8 | 6 | 10 |
|----------------------------------------------------------------|-------------------------------------------------|--------|-----------------------------|-----------|-----------|-----------|---------------|---------------|-----------|-----------|---------------|----------------------|---------------|-----------|-----------|---------------|-----------|---------------|-----------|-----------|---------------|---------------|---------------------------------------------------------|
| | | u | 6 | | | | | | | | | | 10 | | | | | | | | | | |
| | | 0.95 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0006 | 0.0077 | 0.0629 | 0.2985 | 0.6302 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0001 | 0.0010 | 0.0105 | 0.0746 | 0.3151 | 0.5987 |
| | | 0.90 | 0.0000 | 0.0000 | 0.0000 | 0.0001 | 0.0008 | 0.0074 | 0.0446 | 0.1722 | 0.3874 | 0.3874 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0001 | 0.0015 | 0.0112 | 0.0574 | 0.1937 | 0.3874 | 0.3487 |
| | | 0.85 | 0.0000 | 0.0000 | 0.0000 | 0.0006 | 0.0050 | 0.0283 | 0.1069 | 0.2597 | 0.3679 | 0.2316 | 0.0000 | 0.0000 | 0.0000 | 0.0001 | 0.0012 | 0.0085 | 0.0401 | 0.1298 | 0.2759 | 0.3474 | |
| | | 0.80 | 0.0000 | 0.0000 | 0.0003 | 0.0028 | 0.0165 | 0.0661 | 0.1762 (| 0.3020 | 0.3020 | 0.1342 0 | 0.0000 | 0.0000 | 0.0001 | 0.0008 | 0.0055 | 0.0264 (| 0.0881 | 0.2013 | 0.3020 | 0.2684 | 0.1074 0.1969 |
| | | 0.75 | | 0.0001 0 | 0.0012 0 | 0.0087 0 | 0.0389 0 | 0.1168 0 | 0.2336 0 | 0.3003 (| 0.2253 (| | | 0.0000 0 | 0.0004 0 | | 0.0162 0 | 0.0584 0 | 0.1460 0 | 0.2503 (| 0.2816 0 | 0.1877 0 | 0.0563 (|
| | | 0.70 | 0.0000 0.0000 | 0.0004 0 | 0.0039 0 | 0.0210 | 0.0735 0 | 0.1715 0 | 0.2668 C | 0.2668 0 | 0.1556 0 | 0.0404 0.0751 | 0.0000 0.0000 | 0.0001 0 | 0.0014 0 | 0.0090 0.0031 | 0.0368 C | 0.1029 0 | 0.2001 0 | 0.2668 0 | 0.2335 0 | 0.1211 0 | 0.0282 0.0563 |
| | u e | 0.65 | 0.0001 0 | 0.0013 0 | 0.0098 0 | 0.0424 0 | 0.1181 C | 0.2194 0 | 0.2716 0 | 0.2162 0 | 0.1004 0 | 0207 0 | 0.0000 0 | 0.0005 0 | 0.0043 C | 0.0212 0 | 0.0689 0 | 0.1536 0 | 0.2377 0 | 0.2522 0 | 0.1757 0 | 0.0725 0 | 0.0135 0 |
| - p) ^{n-k} | le of size | 0.60 | 0.0003 0 | 0.0035 0 | 0.0212 0 | 0.0743 0 | 0.1672 0 | 0.2508 0 | 0.2508 0 | 0.1612 0 | 0.0605 0 | 0.0101 0.0207 | 0.0001 0 | 0.0016 0 | 0.0106 0 | 0.0425 0 | 0.1115 0 | 0.2007 0 | 0.2508 0 | 0.2150 0 | 0.1209 0 | 0.0403 0 | 0.0000 0.0000 0.0001 0.0003 0.0010 0.0025 0.0060 0.0135 |
| $\Pr(X = x) = f(n; x, p) = \binom{n}{k} p^{k} (1 - p)^{n - k}$ | Probability of <i>x</i> occurrences in a sample | 0.55 | 0008 0 | 0.0083 0 | 0.0407 0 | 0.1160 0 | | 0.2600 0 | 0.2119 0 | 0.1110 0 | 0.0339 0 | .0046 0 | | 0.0042 0 | 0.0229 0 | 0.0746 0 | 0.1596 0 | | 0.2384 0 | 0.1665 0 | 0.0763 0 | 0.0207 0 | 0025 0 |
| $(p) = \begin{pmatrix} r \\ h \end{pmatrix}$ | ences in | 0.50 | 0.0101 0.0046 0.0020 0.0008 | 0.0176 0 | 0.0703 0 | 0.1641 0 | 0.2461 0.2128 | 0.2461 0 | 0.1641 0 | 0.0703 0 | 0.0176 0 | 0.0020 0.0046 | 0.0010 0.0003 | 0.0098 0 | 0.0439 0 | 0.1172 0 | 0.2051 0 | 0.2461 0.2340 | 0.2051 0 | 0.1172 0 | 0.0439 0 | 0.0098 0 | .0010 |
| f=f(n;x) | r occurr | 0.45 | .0046 0 | 0.0339 0 | 0.1110 0 | 0.2119 0 | 0.2600 0 | 0.2128 0 | 0.1160 0 | 0.0407 0 | 0.0083 0 | | 0.0025 0 | 0.0207 0 | 0.0763 0 | 0.1665 0 | 0.2384 0 | 0.2340 0 | 0.1596 0 | 0.0746 0 | 0.0229 0 | 0.0042 0 | 0003 |
| r(X = x) | ility of y | 0.40 | .0101 0 | 0.0605 0 | 0.1612 0 | 0.2508 0 | 0.2508 0 | 0.1672 0 | 0.0743 0 | 0.0212 0 | 0.0035 0 | 0.0003 0.0008 | 0.0060 0 | 0.0403 0 | 0.1209 0 | 0.2150 0 | 0.2508 0 | 0.2007 0 | 0.1115 0 | 0.0425 0 | 0.0106 0 | 0.0016 0 | .0001 0 |
| <u></u> | Probat | 0.35 | 0.0207 0 | 0.1004 0 | 0.2162 0 | 0.2716 0 | 0.2194 0 | 0.1181 0 | 0.0424 0 | 0.0098 0 | 0.0013 0 | .0001 0 | 0.0135 0 | 0.0725 0 | 0.1757 0 | 0.2522 0 | 0.2377 0 | 0.1536 0 | 0.0689 0 | 0.0212 0 | 0.0043 0 | | 0000 |
| | | 0.30 | 0.0404 0 | 0.1556 0 | 0.2668 0 | 0.2668 0 | 0.1715 0 | 0.0735 0 | 0.0210 0 | 0.0039 0 | 0.0004 0 | 0.0000 0.0001 | 0.0282 0 | 0.1211 0 | 0.2335 0 | 0.2668 0 | 0.2001 0 | 0.1029 0 | 0.0368 0. | 0.0000 | 0.0014 0 | 0.0001 0.0005 | 0 0000 |
| | | 0.25 (| 0.0751 0. | 0.2253 0 | 0.3003 0 | 0.2336 0 | 0.1168 0. | 0.0389 0. | 0.0087 0. | 0.0012 0. | 0.0001 0. | 0.0000 0 | 0.0563 0. | 0.1877 0. | 0.2816 0. | 0.2503 0. | 0.1460 0. | 0.0584 0. | 0.0162 0. | 0.0031 0. | 0.0004 0. | 0.0000 0 | 0.0000 |
| | | 0.20 | 0.1342 0 | 0.3020 0 | 0.3020 0. | 0.1762 0. | 0.0661 0. | 0.0165 0. | 0.0028 0. | 0.0003 0. | 0.0000 0 | | 0.1074 0. | 0.2684 0. | 0.3020 0. | 0.2013 0. | 0.0881 0. | 0.0264 0. | 0.0055 0. | 0.0008 0. | 0.0001 0. | 0.0000 0 | 0.0000 |
| | | 0.15 (| 0.2316 0. | 0.3679 0. | 0.2597 0. | 0.1069 0. | | | 0.0006 0. | 0.0000 0. | 0 0000 | 0.0000 0.0000 0.0000 | 0.1969 0. | 0.3474 0. | 0.2759 0. | 0.1298 0. | 0.0401 0. | 0.0085 0. | 0.0012 0. | 0.0001 0. | 0000 | 00000 | |
| | | 0.10 (| 0.3874 0. | 0.3874 0. | 0.1722 0. | 0.0446 0. | 0.0074 0.0283 | 0.0008 0.0050 | 0.0001 0. | 0.0000 0. | 0.0000 0.0000 | 0000 | 0.3487 0. | 0.3874 0. | 0.1937 0. | 0.0574 0. | 0.0112 0. | 0.0015 0. | 0.0001 0. | 0.0000 0. | 0.0000 0.0000 | 0.0000 0.0000 | 0.0000 0.0000 |
| | | 0.05 0 | 0.6302 0. | 0.2985 0. | 0.0629 0. | 0.0077 0. | 0.0006 0. | 0.0000 0. | 0.0000 0. | 0.0000 0. | 0.0000 0. | 0.0000 0. | 0.5987 0. | 0.3151 0. | 0.0746 0. | 0.0105 0. | 0.0010 0. | 0.0001 0. | 0.0000 0. | 0.0000 0. | 0.0000 0. | 0.0000 0. | 0.0000 0. |
| | | | 0 0. | 1 0.5 | 2 0. | 3 0. | 4 0. | 5 0. | 6 0. | 7 0. | 8 0. | 9 0.0 | 0.0 | 1 | 2 0. | 3 0. | 4 0.0 | 5 0.0 | 6 0.0 | 7 0.(| 8 0. | 9 0. | 10 0. |
| | | n x | 6 | | | _ | | | | | | | 10 | | | _ | | | | | | | ,- |

Appendix 9 Cumulative Binomial Distribution Table

| $\frac{1}{x}$ | $\frac{1}{2}$ | $\frac{1}{x}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{x}(u)$ | | $\left(n \right) = \left(n \right) \left($ | (<i>n</i>) | | | | | | | | | | | |
|---------------------------------------------------------------|---------------|---------------|---------------|---------------|---------------|---------------|------------------|---------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------|--------------------------------------------------------------------------------------------|----------------|--------|--------|--------|--------|--------|--------|--------|---|---|
| | | | | | | | ц. | $r(X \leq x)$ | $(n; \lambda) = F(n; \lambda)$ | $(x,p) = \sum_{k=0}^{k}$ | $\int_{a}^{a} \left(x \right) b^{\alpha}$ | (1- <i>p</i>) | | | | | | | | | |
| | | | | | | | Pro | bability o | vf ≤ x occi | urrences | Probability of $\leq x$ occurrences in a sample of size n | ole of size | u | | | | | | | | |
| x 0.05 0.10 0.15 0.20 | 0.15 | | 0.2 | 0 | 0.25 | 0.30 | 0.35 | 0.40 | 0.45 | 0.50 | 0.55 | 0.60 | 0.65 | 0.70 | 0.75 | 0.80 | 0.85 | 0.90 | 0.95 | u | x |
| 0 0.9500 0.9000 0.8500 0.8000 | 0.9000 0.8500 | 0.8500 | 0.80 | 8 | 0.7500 | 0.7000 | 0.6500 | 0.6000 | 0.5500 | 0.5000 | 0.4500 | 0.4000 | 0.3500 | 0.3000 | 0.2500 | 0.2000 | 0.1500 | 0.1000 | 0.0500 | + | 0 |
| 1 1.0000 1.0000 1.0000 1.0000 | 1.0000 1.0000 | 1.0000 | | 8 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | | - |
| 0 0.9025 0.8100 0.7225 0.6400 | 0.8100 0.7225 | 0.7225 | | 8 | 0.5625 | 0.4900 | 0.4225 | 0.3600 | 0.3025 | 0.2500 | 0.2025 | 0.1600 | 0.1225 | 0.0900 | 0.0625 | 0.0400 | 0.0225 | 0.0100 | 0.0025 | 2 | 0 |
| 1 0.9975 0.9900 0.9775 0.9600 | 0.9900 0.9775 | 0.9775 | | 8 | 0.9375 | 0.9100 | 0.8775 | 0.8400 | 0.7975 | 0.7500 | 0.6975 | 0.6400 | 0.5775 | 0.5100 | 0.4375 | 0.3600 | 0.2775 | 0.1900 | 0.0975 | | - |
| 2 1.0000 1.0000 1.0000 1.0000 | 1.0000 1.0000 | 1.0000 | | 00 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | | 2 |
| 0 0.8574 0.7290 0.6141 0.5120 | 0.7290 0.6141 | 0.6141 | 0.51 | 20 | 0.4219 | 0.3430 | 0.2746 | 0.2160 | 0.1664 | 0.1250 | 0.0911 | 0.0640 | 0.0429 | 0.0270 | 0.0156 | 0.0080 | 0.0034 | 0.0010 | 0.0001 | 3 | 0 |
| 1 0.9928 0.9720 0.9393 0.8960 | 0.9720 0.9393 | 0.9393 | 0.89 | 60 | 0.8438 | 0.7840 | 0.7183 | 0.6480 | 0.5748 | 0.5000 | 0.4253 | 0.3520 | 0.2818 | 0.2160 | 0.1563 | 0.1040 | 0.0608 | 0.0280 | 0.0073 | | - |
| 2 0.9999 0.9990 0.9966 0.9920 | 0.9990 0.9966 | 0.9966 | | 20 | 0.9844 | 0.9730 | 0.9571 | 0.9360 | 0.9089 | 0.8750 | 0.8336 | 0.7840 | 0.7254 | 0.6570 | 0.5781 | 0.4880 | 0.3859 | 0.2710 | 0.1426 | | 0 |
| 3 1.0000 1.0000 1.0000 1.0 | 1.0000 1.0000 | 1.0000 | | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | | 3 |
| 0 0.8145 0.6561 0.5220 0.4 | 0.6561 0.5220 | | | 0.4096 | 0.3164 | 0.2401 | 0.1785 | 0.1296 | 0.0915 | 0.0625 | 0.0410 | 0.0256 | 0.0150 | 0.0081 | 0.0039 | 0.0016 | 0.0005 | 0.0001 | 0.0000 | 4 | 0 |
| 1 0.9860 0.9477 0.8905 0.8192 | 0.9477 0.8905 | 0.8905 | | 192 | 0.7383 | 0.6517 | 0.5630 | 0.4752 | 0.3910 | 0.3125 | 0.2415 | 0.1792 | 0.1265 | 0.0837 | 0.0508 | 0.0272 | 0.0120 | 0.0037 | 0.0005 | | - |
| 2 0.9995 0.9963 0.9880 0.9728 | 0.9963 0.9880 | 0.9880 | | 28 | 0.9492 | 0.9163 | 0.8735 | 0.8208 | 0.7585 | 0.6875 | 0.6090 | 0.5248 | 0.4370 | 0.3483 | 0.2617 | 0.1808 | 0.1095 | 0.0523 | 0.0140 | | 2 |
| 3 1.0000 0.9999 0.9995 0.9984 | 0.9999 0.9995 | 0.9995 | | 84 | 0.9961 | 0.9919 | 0.9850 | 0.9744 | 0.9590 | 0.9375 | 0.9085 | 0.8704 | 0.8215 | 0.7599 | 0.6836 | 0.5904 | 0.4780 | 0.3439 | 0.1855 | _ | e |
| 4 1.0000 1.0000 1.0000 1.0000 | 1.0000 1.0000 | 1.0000 | | 8 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | | 4 |
| 0 0.7738 0.5905 0.4437 0.3277 | 0.5905 0.4437 | 0.4437 | 0.32 | 1 | 0.2373 | 0.1681 | 0.1160 | 0.0778 | 0.0503 | 0.0313 | 0.0185 | 0.0102 | 0.0053 | 0.0024 | 0.0010 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 5 | 0 |
| 1 0.9774 0.9185 0.8352 0.7373 | 0.9185 0.8352 | 0.8352 | | 373 | 0.6328 | 0.5282 | 0.4284 | 0.3370 | 0.2562 | 0.1875 | 0.1312 | 0.0870 | 0.0540 | 0.0308 | 0.0156 | 0.0067 | 0.0022 | 0.0005 | 0.0000 | | ٦ |
| 2 0.9988 0.9914 0.9734 0.9421 | 0.9914 0.9734 | 0.9734 | 0.92 | 121 | 0.8965 | 0.8369 | 0.7648 | 0.6826 | 0.5931 | 0.5000 | 0.4069 | 0.3174 | 0.2352 | 0.1631 | 0.1035 | 0.0579 | 0.0266 | 0.0086 | 0.0012 | | 7 |
| 3 1.0000 0.9995 0.9978 0.9933 | 0.9995 0.9978 | 0.9978 | | 333 | 0.9844 | 0.9692 | 0.9460 | 0.9130 | 0.8688 | 0.8125 | 0.7438 | 0.6630 | 0.5716 | 0.4718 | 0.3672 | 0.2627 | 0.1648 | 0.0815 | 0.0226 | | e |
| 4 1.0000 1.0000 0.9999 0.9997 | 1.0000 0.9999 | 0.9999 | | 997 | 0.9990 | 0.9976 | 0.9947 | 0.9898 | 0.9815 | 0.9688 | 0.9497 | 0.9222 | 0.8840 | 0.8319 | 0.7627 | 0.6723 | 0.5563 | 0.4095 | 0.2262 | | 4 |
| 5 1.0000 1.0000 1.0000 1.00 | 1.0000 1.0000 | 1.0000 | | 1.0000 | 1.0000 | | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | | 5 |
| | | | | | | | | | | | | | | | | | | | | | |

| | | × | 0 | - | 2 | e | 4 | 5 | 9 | 0 | - | 0 | 3 | 4 | ß | 9 | 7 | 0 | 1 | 7 | з | 4 | 5 | 9 | 7 | ∞ |
|-----------------------------------------------------------------------------|--------------------------------|--------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| | | 2 | 9 | | | | | | | 7 | | | | | | | | 8 | | | | | | | | |
| | | 0.95 | 0.0000 | 0.0000 | 0.0001 | 0.0022 | 0.0328 | 0.2649 | 1.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0002 | 0.0038 | 0.0444 | 0.3017 | 1.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0004 | 0.0058 | 0.0572 | 0.3366 | 1.0000 |
| | | 06.0 | 0.0000 | 0.0001 | 0.0013 | 0.0159 | 0.1143 | 0.4686 | 1.0000 | 0.0000 | 0.0000 | 0.0002 | 0.0027 | 0.0257 | 0.1497 | 0.5217 | 1.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0004 | 0.0050 | 0.0381 | 0.1869 | 0.5695 | 1.0000 |
| | | 0.85 | 0.0000 | 0.0004 | 0.0059 | 0.0473 | 0.2235 | 0.6229 | 1.0000 | 0.0000 | 0.0001 | 0.0012 | 0.0121 | 0.0738 | 0.2834 | 0.6794 | 1.0000 | 0.0000 | 0.0000 | 0.0002 | 0.0029 | 0.0214 | 0.1052 | 0.3428 | 0.7275 | 1.0000 |
| | | 0.80 | 0.0001 | 0.0016 | 0.0170 | 0.0989 | 0.3446 | 0.7379 | 1.0000 | 0.0000 | 0.0004 | 0.0047 | 0.0333 | 0.1480 | 0.4233 | 0.7903 | 1.0000 | 0.0000 | 0.0001 | 0.0012 | 0.0104 | 0.0563 | 0.2031 | 0.4967 | 0.8322 | 1.0000 |
| | | 0.75 | 0.0002 0 | 0.0046 0 | 0.0376 (| 0.1694 0 | 0.4661 0 | 0.8220 0 | 1.0000 | 0.0001 0 | 0.0013 0 | 0.0129 0 | 0.0706 0 | 0.2436 (| 0.5551 0 | 0.8665 (| 1.0000 | 0.0000 0 | 0.0004 0 | 0.0042 0 | 0.0273 0 | 0.1138 0 | 0.3215 0 | 0.6329 0 | 0.8999 0 | 1.0000 |
| | | 0.70 | 0.0007 0 | 0.0109 0 | 0.0705 0 | 0.2557 0 | 0.5798 0 | 0.8824 0 | 1.0000 1 | 0.0002 0 | 0.0038 0 | 0.0288 0 | 0.1260 0 | 0.3529 0 | 0.6706 0 | 0.9176 0 | 1.0000 1 | 0.0001 0 | 0.0013 0 | 0.0113 0 | 0.0580 0 | 0.1941 0 | 0.4482 0 | 0.7447 0 | 0.9424 0 | 1.0000 1 |
| | | 0.65 (| 0.0018 0. | 0.0223 0. | 0.1174 0. | 0.3529 0. | 0.6809 0. | 0.9246 0. | 1.0000 1. | 0.0006 0. | 0.0090 0. | 0.0556 0. | 0.1998 0. | 0.4677 0. | 0.7662 0. | 0.9510 0. | 1.0000 1. | 0.0002 0. | 0.0036 0. | 0.0253 0. | 0.1061 0. | 0.2936 0. | 0.5722 0. | 0.8309 0. | 0.9681 0. | 1.0000 1. |
| $p)^{n-k}$ | of size n | 0.60 0 | 0.0041 0.0 | 0.0410 0.0 | 0.1792 0.1 | 0.4557 0.3 | 0.7667 0.6 | 0.9533 0.9 | 1.0000 1.0 | 0.0016 0.0 | 0.0188 0.0 | 0.0963 0.0 | 0.2898 0.1 | 0.5801 0.4 | 0.8414 0.7 | 0.9720 0.9 | 1.0000 1.0 | 0.0007 0.0 | 0.0085 0.0 | 0.0498 0.0 | 0.1737 0.1 | 0.4059 0.2 | 0.6846 0.5 | 0.8936 0.8 | 0.9832 0.9 | 1.0000 1.0 |
| $\Pr(X \le x) = F(n; x, p) = \sum_{k=0}^{x} \binom{n}{x} p^{k} (1-p)^{n-k}$ | ≤ x occurrences in a sample of | | | | | | | | | | | | | | | | | | | | | | | | | |
| $\sum_{k=0}^{n} \binom{n}{x}$ | ces in a s | 0.55 | 56 0.0083 | 94 0.0692 | 38 0.2553 | 33 0.5585 | 0.8364 | 44 0.9723 | 00001.0000 | 78 0.0037 | 25 0.0357 | 36 0.1529 | 0.3917 | 34 0.6836 | 75 0.8976 | 22 0.9848 | 00 1.0000 | 39 0.0017 | 52 0.0181 | 15 0.0885 | 33 0.2604 | 37 0.5230 | 55 0.7799 | 18 0.9368 | 31 0.9916 | 00 1.0000 |
| η; x , p) = | ccurrend | 0.50 | 7 0.0156 | 6 0.1094 | 5 0.3438 | 7 0.6563 | 8 0.8906 | 7 0.9844 | 0 1.0000 | 2 0.0078 | 4 0.0625 | 4 0.2266 | 3 0.5000 | 1 0.7734 | 3 0.9375 | 3 0.9922 | 0 1.0000 | 4 0.0039 | 2 0.0352 | 1 0.1445 | 0 0.3633 | 6 0.6367 | 5 0.8555 | 9 0.9648 | 3 0.9961 | 0 1.0000 |
| (x) = F(t) | of | 0.45 | 0.0277 | 0.1636 | 0.4415 | 0.7447 | 0.9308 | 0.9917 | 1.0000 | 0.0152 | 0.1024 | 0.3164 | 0.6083 | 0.8471 | 0.9643 | 0.9963 | 1.0000 | 3 0.0084 | 0.0632 | t 0.2201 | 0.4770 | 0.7396 | 0.9115 | 0.9819 | 0.9983 | 1.0000 |
| Pr(X ≤ | Probability | 0.40 | 0.0467 | 0.2333 | 0.5443 | 0.8208 | 0.9590 | 0.9959 | 1.0000 | 0.0280 | 0.1586 | 0.4199 | 0.7102 | 0.9037 | 0.9812 | 0.9984 | 1.0000 | 0.0168 | 0.1064 | 0.3154 | 0.5941 | 0.8263 | 0.9502 | 0.9915 | 0.9993 | 1.0000 |
| | Ē | 0.35 | 0.0754 | 0.3191 | 0.6471 | 0.8826 | 0.9777 | 0.9982 | 1.0000 | 0.0490 | 0.2338 | 0.5323 | 0.8002 | 0.9444 | 0.9910 | 0.9994 | 1.0000 | 0.0319 | 0.1691 | 0.4278 | 0.7064 | 0.8939 | 0.9747 | 0.9964 | 0.9998 | 1.0000 |
| | | 0:30 | 0.1176 | 0.4202 | 0.7443 | 0.9295 | 0.9891 | 0.9993 | 1.0000 | 0.0824 | 0.3294 | 0.6471 | 0.8740 | 0.9712 | 0.9962 | 0.9998 | 1.0000 | 0.0576 | 0.2553 | 0.5518 | 0.8059 | 0.9420 | 0.9887 | 0.9987 | 0.9999 | 1.0000 |
| | | 0.25 | 0.1780 | 0.5339 | 0.8306 | 0.9624 | 0.9954 | 0.9998 | 1.0000 | 0.1335 | 0.4449 | 0.7564 | 0.9294 | 0.9871 | 0.9987 | 0.9999 | 1.0000 | 0.1001 | 0.3671 | 0.6785 | 0.8862 | 0.9727 | 0.9958 | 0.9996 | 1.0000 | 1.0000 |
| | | 0.20 | 0.2621 | 0.6554 | 0.9011 | 0.9830 | 0.9984 | 0.9999 | 1.0000 | 0.2097 | 0.5767 | 0.8520 | 0.9667 | 0.9953 | 0.9996 | 1.0000 | 1.0000 | 0.1678 | 0.5033 | 0.7969 | 0.9437 | 0.9896 | 0.9988 | 0.9999 | 1.0000 | 1.0000 |
| | | 0.15 | 0.3771 | 0.7765 | 0.9527 | 0.9941 | 0.9996 | 1.0000 | 1.0000 | 0.3206 | 0.7166 | 0.9262 | 0.9879 | 0.9988 | 0.9999 | 1.0000 | 1.0000 | 0.2725 | 0.6572 | 0.8948 | 0.9786 | 0.9971 | 0.9998 | 1.0000 | 1.0000 | 1.0000 |
| | | 0.10 | 0.5314 | 0.8857 | 0.9842 | 0.9987 | 0.9999 | 1.0000 | 1.0000 | 0.4783 | 0.8503 | 0.9743 | 0.9973 | 0.9998 | 1.0000 | 1.0000 | 1.0000 | 0.4305 | 0.8131 | 0.9619 | 0.9950 | 0.9996 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| | | 0.05 | 0.7351 | 0.9672 | 0.9978 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 0.6983 | 0.9556 | 0.9962 | 0.9998 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.6634 | 0.9428 | 0.9942 | 0.9996 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| | | × | 0 0 | - | 2 | 3 | 4 1 | 5 1 | 9 | 0 | - | 5 | 3 | 4 1 | 5 | 6 1 | 7 1 | 0 | 1 | 2 | 3 0 | 4 | 5 | 6 1 | 7 1 | 8 |
| | | 2 | 9 | | | | | | | 7 | | | | | | | | 8 | | | | | | | | |

| | | × | 0 | - | 2 | 3 | 4 | 5 | 9 | 7 | 8 | 6 | 0 | - | 2 | 3 | 4 | 5 | 9 | 7 | 8 | 6 | 10 |
|-----------------------------------------------------------------------------|---------------------------------------------------------|---------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|---------------|
| | | " | 6 | | | | | | | | | | 10 | | | | | | | | | | |
| | | 0.95 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0006 | 0.0084 | 0.0712 | 0.3698 | 1.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0001 | 0.0010 | 0.0115 | 0.0861 | 0.4013 | 1.0000 |
| | | 06.0 | 0.0000 | 0.0000 | 0.0000 | 0.0001 | 0.0009 | 0.0083 | 0.0530 | 0.2252 | 0.6126 | 1.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0001 | 0.0016 | 0.0128 | 0.0702 | 0.2639 | 0.6513 | 1.0000 |
| | | 0.85 | 0.0000 | 0.0000 | 0.0000 | 0.0006 | 0.0056 | 0.0339 | 0.1409 | 0.4005 | 0.7684 | 1.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0001 | 0.0014 | 0.0099 | 0.0500 | 0.1798 | 0.4557 | 0.8031 | 1.0000 |
| | | 0.80 | 0.0000 (| 0.0000 0 | 0.0003 0 | 0.0031 0 | 0.0196 0 | 0.0856 (| 0.2618 0 | 0.5638 (| 0.8658 (| 1.0000 | 0.0000 | 0.0000 (| 0.0001 | 0.0009 (| 0.0064 0 | 0.0328 0 | 0.1209 (| 0.3222 (| 0.6242 0 | 0.8926 (| 1.0000 |
| | | 0.75 | 0.0000 | 0.0001 0 | 0.0013 0 | 0.0100 0 | 0.0489 0 | 0.1657 0 | 0.3993 0 | 0.6997 0 | 0.9249 (| 1.0000 | 0.0000 (| 0.0000 (| 0.0004 0 | 0.0035 0 | 0.0197 0 | 0.0781 0 | 0.2241 (| 0.4744 0 | 0.7560 0 | 0.9437 0 | 1.0000 |
| | | 0.70 | 0.0000 | 0.0004 C | 0.0043 C | 0.0253 C | 0.0988 C | 0.2703 C | 0.5372 C | 0.8040 C | 0.9596 C | 1.0000 1 | 0.0000 C | 0.0001 C | 0.0016 C | 0.0106 0 | 0.0473 0 | 0.1503 0 | 0.3504 0 | 0.6172 0 | 0.8507 0 | 0.9718 0 | |
| | | 0.65 | 0.0001 0 | 0.0014 0 | 0.0112 0 | 0.0536 0 | 0.1717 0 | 0.3911 0 | 0.6627 0 | 0.8789 0 | 0.9793 0 | 1.0000 1 | 0.0000 0 | 0.0005 0 | 0.0048 0 | 0.0260 0 | 0.0949 0 | 0.2485 0 | 0.4862 0 | 0.7384 0 | 0.9140 0 | 0.9865 0 | 1.0000 1.0000 |
| <i>p</i>) ^{<i>n-k</i>} | of size <i>n</i> | 0.60 0 | 0.0003 0. | 0.0038 0. | 0.0250 0. | 0.0994 0. | 0.2666 0. | 0.5174 0. | 0.7682 0. | 0.9295 0. | 0.9899 0. | 1.0000 1. | 0.0001 0. | 0.0017 0. | 0.0123 0. | 0.0548 0. | 0.1662 0. | 0.3669 0. | 0.6177 0. | 0.8327 0. | 0.9536 0. | 0.9940 0. | 1.0000 1. |
| $\Pr(X \le x) = F(n; x, p) = \sum_{k=0}^{x} \binom{n}{x} p^{k} (1-p)^{n-k}$ | Probability of $\leq x$ occurrences in a sample of size | 0.55 0 | 0.0008 0.0 | 0.0091 0.0 | 0.0498 0.0 | 0.1658 0.0 | 0.3786 0.2 | 0.6386 0.5 | 0.8505 0.7 | 0.9615 0.9 | 0.9954 0.9 | 1.0000 1.(| 0.0003 0.0 | 0.0045 0.0 | 0.0274 0.0 | 0.1020 0.0 | 0.2616 0.7 | 0.4956 0.3 | 0.7340 0.6 | 0.9004 0.8 | 0.9767 0.9 | 0.9975 0.9 | |
| $=\sum_{k=0}^{x} \binom{n}{x}$ | nces in a | 0.50 0. | | 0.0195 0.0 | 0.0898 0.0 | | 0.5000 0.3 | 0.7461 0.6 | 0.9102 0.8 | 0.9805 0.9 | 0.9980 0.9 | 1.0000 1.0 | 0.0010 0.0 | | 0.0547 0.0 | 0.1719 0.1 | 0.3770 0.2 | 0.6230 0.4 | 0.8281 0.7 | 0.9453 0.9 | | 0.9990 0.9 | 1.0000 1.0000 |
| (n;x,p) | occurrer | | 46 0.0020 | | | 14 0.2539 | | | | | | | | 33 0.0107 | | | | | | | 55 0.9893 | | 00 1.00 |
| ≤ X) = F | ty of ≤ x | 0.45 | 01 0.0046 | 0.0385 | 18 0.1495 | 26 0.3614 | 34 0.6214 | 0.8342 | 50 0.9502 | 32 0.9909 | 97 0.9992 | 1.0000 | 30 0.0025 | 34 0.0233 | 73 0.0996 | 23 0.2660 | 31 0.5044 | 38 0.7384 | 52 0.8980 | 77 0.9726 | 33 0.9955 | 0.9997 | 00 1.0000 |
| Pr(X ≤ | Probabili | 0.40 | 7 0.0101 | 1 0.0705 | 3 0.2318 | 9 0.4826 | 3 0.7334 | 4 0.9006 | 8 0.9750 | 6 0.9962 | 9 0.9997 | 0 1.0000 | 5 0.0060 | 0 0.0464 | 6 0.1673 | 8 0.3823 | 5 0.6331 | 1 0.8338 | 0 0.9452 | 2 0.9877 | 5 0.9983 | 0.9999 | 0 1.0000 |
| | | 0.35 | 0.0207 | 0.1211 | 0.3373 | 0.6089 | 0.8283 | 0.9464 | 0.9888 | 0.9986 | 0.9999 | 1.0000 | 0.0135 | 0.0860 | 0.2616 | 0.5138 | 0.7515 | 0.9051 | 0.9740 | 0.9952 | 0.9995 | 1.0000 | 1.0000 |
| | | 0:30 | 0.0404 | 0.1960 | 0.4628 | 0.7297 | 0.9012 | 0.9747 | 0.9957 | 0.9996 | 1.0000 | 1.0000 | 0.0282 | 0.1493 | 0.3828 | 0.6496 | 0.8497 | 0.9527 | 0.9894 | 0.9984 | 0.9999 | 1.0000 | 1.0000 |
| | | 0.25 | 0.0751 | 0.3003 | 0.6007 | 0.8343 | 0.9511 | 0.9900 | 0.9987 | 0.9999 | 1.0000 | 1.0000 | 0.0563 | 0.2440 | 0.5256 | 0.7759 | 0.9219 | 0.9803 | 0.9965 | 0.9996 | 1.0000 | 1.0000 | 1.0000 |
| | | 0.20 | 0.1342 | 0.4362 | 0.7382 | 0.9144 | 0.9804 | 0.9969 | 0.9997 | 1.0000 | 1.0000 | 1.0000 | 0.1074 | 0.3758 | 0.6778 | 0.8791 | 0.9672 | 0.9936 | 0.9991 | 0.9999 | 1.0000 | 1.0000 | 1.0000 |
| | | 0.15 | 0.2316 | 0.5995 | 0.8591 | 0.9661 | 0.9944 | 0.9994 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.1969 | 0.5443 | 0.8202 | 0.9500 | 0.9901 | 0.9986 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| | | 0.10 | 0.3874 | 0.7748 | 0.9470 | 0.9917 | 0.9991 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.3487 | 0.7361 | 0.9298 | 0.9872 | 0.9984 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| | | 0.05 | 0.6302 | 0.9288 | 0.9916 | 0.9994 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.5987 | 0.9139 | 0.9885 | 0.9990 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| | | × | 0 | - | 2 | 3 | 4 | 5 | 9 | 7 | 8 | 6 | 0 | - | 2 | 3 | 4 | 5 | 9 | 7 | 8 | 6 | 10 |
| | | u | 6 | | | | | | | | | | 10 | | | | | | | | | | |

Appendix 10 Poisson Distribution Table

| | | | | | | Pr(X | =x)=t | $f(x;\lambda) = $ | $\frac{\lambda^{x}}{x!}e^{-\lambda}$ | | | | | | |
|------|--------|--------|--------|--------|--------|--------|-----------|-------------------|--------------------------------------|--------|--------|--------|--------|--------|------|
| | | | | | | Probal | bility of | x occur | rences | | | | | | |
| λ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | λ |
| 0.05 | 0.9512 | 0.0476 | 0.0012 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.05 |
| 0.10 | 0.9048 | 0.0905 | 0.0045 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.10 |
| 0.15 | 0.8607 | 0.1291 | 0.0097 | 0.0005 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.15 |
| 0.20 | 0.8187 | 0.1637 | 0.0164 | 0.0011 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.20 |
| 0.25 | 0.7788 | 0.1947 | 0.0243 | 0.0020 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.25 |
| 0.30 | 0.7408 | 0.2222 | 0.0333 | 0.0033 | 0.0003 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.30 |
| 0.35 | 0.7047 | 0.2466 | 0.0432 | 0.0050 | 0.0004 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.35 |
| 0.40 | 0.6703 | 0.2681 | 0.0536 | 0.0072 | 0.0007 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.40 |
| 0.45 | 0.6376 | 0.2869 | 0.0646 | 0.0097 | 0.0011 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.45 |
| 0.50 | 0.6065 | 0.3033 | 0.0758 | 0.0126 | 0.0016 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.50 |
| 0.55 | 0.5769 | 0.3173 | 0.0873 | 0.0160 | 0.0022 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.55 |
| 0.60 | 0.5488 | 0.3293 | 0.0988 | 0.0198 | 0.0030 | 0.0004 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.60 |
| 0.65 | 0.5220 | 0.3393 | 0.1103 | 0.0239 | 0.0039 | 0.0005 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.65 |
| 0.70 | 0.4966 | 0.3476 | 0.1217 | 0.0284 | 0.0050 | 0.0007 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.70 |
| 0.75 | 0.4724 | 0.3543 | 0.1329 | 0.0332 | 0.0062 | 0.0009 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.75 |
| 0.80 | 0.4493 | 0.3595 | 0.1438 | 0.0383 | 0.0077 | 0.0012 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.80 |
| 0.85 | 0.4274 | 0.3633 | 0.1544 | 0.0437 | 0.0093 | 0.0016 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.85 |
| 0.90 | 0.4066 | 0.3659 | 0.1647 | 0.0494 | 0.0111 | 0.0020 | 0.0003 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.90 |
| 0.95 | 0.3867 | 0.3674 | 0.1745 | 0.0553 | 0.0131 | 0.0025 | 0.0004 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.95 |
| 1.00 | 0.3679 | 0.3679 | 0.1839 | 0.0613 | 0.0153 | 0.0031 | 0.0005 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.00 |
| 1.10 | 0.3329 | 0.3662 | 0.2014 | 0.0738 | 0.0203 | 0.0045 | 0.0008 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.10 |
| 1.20 | 0.3012 | 0.3614 | 0.2169 | 0.0867 | 0.0260 | 0.0062 | 0.0012 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.20 |
| 1.30 | 0.2725 | 0.3543 | 0.2303 | 0.0998 | 0.0324 | 0.0084 | 0.0018 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.30 |
| 1.40 | 0.2466 | 0.3452 | 0.2417 | 0.1128 | 0.0395 | 0.0111 | 0.0026 | 0.0005 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.40 |
| 1.50 | 0.2231 | 0.3347 | 0.2510 | 0.1255 | 0.0471 | 0.0141 | 0.0035 | 0.0008 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.50 |
| 1.60 | 0.2019 | 0.3230 | 0.2584 | 0.1378 | 0.0551 | 0.0176 | 0.0047 | 0.0011 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.60 |
| 1.70 | 0.1827 | 0.3106 | 0.2640 | 0.1496 | 0.0636 | 0.0216 | 0.0061 | 0.0015 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.70 |
| 1.80 | | | | 0.1607 | | | | | | | | | | | 1.80 |
| 1.90 | 0.1496 | 0.2842 | 0.2700 | 0.1710 | 0.0812 | 0.0309 | 0.0098 | 0.0027 | 0.0006 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.90 |
| 2.00 | 0.1353 | 0.2707 | 0.2707 | 0.1804 | 0.0902 | 0.0361 | 0.0120 | 0.0034 | 0.0009 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 2.00 |
| 2.10 | 0.1225 | 0.2572 | 0.2700 | 0.1890 | 0.0992 | 0.0417 | 0.0146 | 0.0044 | 0.0011 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 2.10 |
| 2.20 | 0.1108 | 0.2438 | 0.2681 | 0.1966 | 0.1082 | 0.0476 | 0.0174 | 0.0055 | 0.0015 | 0.0004 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 2.20 |
| 2.30 | 0.1003 | 0.2306 | 0.2652 | 0.2033 | 0.1169 | 0.0538 | 0.0206 | 0.0068 | 0.0019 | 0.0005 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 2.30 |
| 2.40 | 0.0907 | 0.2177 | 0.2613 | 0.2090 | 0.1254 | 0.0602 | 0.0241 | 0.0083 | 0.0025 | 0.0007 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 2.40 |
| 2.50 | 0.0821 | 0.2052 | 0.2565 | 0.2138 | 0.1336 | 0.0668 | 0.0278 | 0.0099 | 0.0031 | 0.0009 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 2.50 |
| 2.60 | 0.0743 | 0.1931 | 0.2510 | 0.2176 | 0.1414 | 0.0735 | 0.0319 | 0.0118 | 0.0038 | 0.0011 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 2.60 |
| 2.70 | 0.0672 | 0.1815 | 0.2450 | 0.2205 | 0.1488 | 0.0804 | 0.0362 | 0.0139 | 0.0047 | 0.0014 | 0.0004 | 0.0001 | 0.0000 | 0.0000 | 2.70 |

$$\Pr(X = x) = f(x; \lambda) = \frac{\lambda^{x}}{x!} e^{-\lambda}$$

| | | | | | Pr(X | = x) = f | $(x;\lambda) =$ | $\frac{\pi}{x!}e^{-\lambda}$ | | | | | |
|------|--------|--------|--------|--------|--------|-----------|-----------------|------------------------------|--------|--------|--------|--------|------|
| | | | | | Probal | oility of | x occur | rences | | | | | |
| λ | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | λ |
| 0.05 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.05 |
| 0.10 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.10 |
| 0.15 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.15 |
| 0.20 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.20 |
| 0.25 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.25 |
| 0.30 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.30 |
| 0.35 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.35 |
| 0.40 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.40 |
| 0.45 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.45 |
| 0.50 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.50 |
| 0.55 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.55 |
| 0.60 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.60 |
| 0.65 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.65 |
| 0.70 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.70 |
| 0.75 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.75 |
| 0.80 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.80 |
| 0.85 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.85 |
| 0.90 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.90 |
| 0.95 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.95 |
| 1.00 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.00 |
| 1.10 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.10 |
| 1.20 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.20 |
| 1.30 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.30 |
| 1.40 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.40 |
| 1.50 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.50 |
| 1.60 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.60 |
| 1.70 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.70 |
| 1.80 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.80 |
| 1.90 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.90 |
| 2.00 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 2.00 |
| 2.10 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 2.10 |
| 2.20 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 2.20 |
| 2.30 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 2.30 |
| 2.40 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 2.40 |
| 2.50 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 2.50 |
| 2.60 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 2.60 |
| 2.70 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 2.70 |

| Pr(X = | (x) = f(x) | $\lambda = \frac{\lambda^{x}}{x!} e^{-\lambda}$ |
|--------|------------|-------------------------------------------------|
| | | |

| | | | | | | Pr(X | =x)=t | $(x;\lambda) =$ | $\frac{\lambda^{x}}{x!}e^{-\lambda}$ | | | | | | |
|------|--------|--------|--------|--------|--------|--------|-----------|-----------------|--------------------------------------|--------|--------|--------|--------|--------|------|
| | | | | | | Probal | oility of | x occur | rences | | | | | | |
| λ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | λ |
| 2.80 | 0.0608 | 0.1703 | 0.2384 | 0.2225 | 0.1557 | 0.0872 | 0.0407 | 0.0163 | 0.0057 | 0.0018 | 0.0005 | 0.0001 | 0.0000 | 0.0000 | 2.80 |
| 2.90 | 0.0550 | 0.1596 | 0.2314 | 0.2237 | 0.1622 | 0.0940 | 0.0455 | 0.0188 | 0.0068 | 0.0022 | 0.0006 | 0.0002 | 0.0000 | 0.0000 | 2.90 |
| 3.00 | 0.0498 | 0.1494 | 0.2240 | 0.2240 | 0.1680 | 0.1008 | 0.0504 | 0.0216 | 0.0081 | 0.0027 | 0.0008 | 0.0002 | 0.0001 | 0.0000 | 3.00 |
| 3.10 | 0.0450 | 0.1397 | 0.2165 | 0.2237 | 0.1733 | 0.1075 | 0.0555 | 0.0246 | 0.0095 | 0.0033 | 0.0010 | 0.0003 | 0.0001 | 0.0000 | 3.10 |
| 3.20 | 0.0408 | 0.1304 | 0.2087 | 0.2226 | 0.1781 | 0.1140 | 0.0608 | 0.0278 | 0.0111 | 0.0040 | 0.0013 | 0.0004 | 0.0001 | 0.0000 | 3.20 |
| 3.30 | 0.0369 | 0.1217 | 0.2008 | 0.2209 | 0.1823 | 0.1203 | 0.0662 | 0.0312 | 0.0129 | 0.0047 | 0.0016 | 0.0005 | 0.0001 | 0.0000 | 3.30 |
| 3.40 | 0.0334 | 0.1135 | 0.1929 | 0.2186 | 0.1858 | 0.1264 | 0.0716 | 0.0348 | 0.0148 | 0.0056 | 0.0019 | 0.0006 | 0.0002 | 0.0000 | 3.40 |
| 3.50 | 0.0302 | 0.1057 | 0.1850 | 0.2158 | 0.1888 | 0.1322 | 0.0771 | 0.0385 | 0.0169 | 0.0066 | 0.0023 | 0.0007 | 0.0002 | 0.0001 | 3.50 |
| 3.60 | 0.0273 | 0.0984 | 0.1771 | 0.2125 | 0.1912 | 0.1377 | 0.0826 | 0.0425 | 0.0191 | 0.0076 | 0.0028 | 0.0009 | 0.0003 | 0.0001 | 3.60 |
| 3.70 | 0.0247 | 0.0915 | 0.1692 | 0.2087 | 0.1931 | 0.1429 | 0.0881 | 0.0466 | 0.0215 | 0.0089 | 0.0033 | 0.0011 | 0.0003 | 0.0001 | 3.70 |
| 3.80 | 0.0224 | 0.0850 | 0.1615 | 0.2046 | 0.1944 | 0.1477 | 0.0936 | 0.0508 | 0.0241 | 0.0102 | 0.0039 | 0.0013 | 0.0004 | 0.0001 | 3.80 |
| 3.90 | 0.0202 | 0.0789 | 0.1539 | 0.2001 | 0.1951 | 0.1522 | 0.0989 | 0.0551 | 0.0269 | 0.0116 | 0.0045 | 0.0016 | 0.0005 | 0.0002 | 3.90 |
| 4.00 | 0.0183 | 0.0733 | 0.1465 | 0.1954 | 0.1954 | 0.1563 | 0.1042 | 0.0595 | 0.0298 | 0.0132 | 0.0053 | 0.0019 | 0.0006 | 0.0002 | 4.00 |
| 4.10 | 0.0166 | 0.0679 | 0.1393 | 0.1904 | 0.1951 | 0.1600 | 0.1093 | 0.0640 | 0.0328 | 0.0150 | 0.0061 | 0.0023 | 0.0008 | 0.0002 | 4.10 |
| 4.20 | 0.0150 | 0.0630 | 0.1323 | 0.1852 | 0.1944 | 0.1633 | 0.1143 | 0.0686 | 0.0360 | 0.0168 | 0.0071 | 0.0027 | 0.0009 | 0.0003 | 4.20 |
| 4.30 | 0.0136 | 0.0583 | 0.1254 | 0.1798 | 0.1933 | 0.1662 | 0.1191 | 0.0732 | 0.0393 | 0.0188 | 0.0081 | 0.0032 | 0.0011 | 0.0004 | 4.30 |
| 4.40 | 0.0123 | 0.0540 | 0.1188 | 0.1743 | 0.1917 | 0.1687 | 0.1237 | 0.0778 | 0.0428 | 0.0209 | 0.0092 | 0.0037 | 0.0013 | 0.0005 | 4.40 |
| 4.50 | 0.0111 | 0.0500 | 0.1125 | 0.1687 | 0.1898 | 0.1708 | 0.1281 | 0.0824 | 0.0463 | 0.0232 | 0.0104 | 0.0043 | 0.0016 | 0.0006 | 4.50 |
| 4.60 | 0.0101 | 0.0462 | 0.1063 | 0.1631 | 0.1875 | 0.1725 | 0.1323 | 0.0869 | 0.0500 | 0.0255 | 0.0118 | 0.0049 | 0.0019 | 0.0007 | 4.60 |
| 4.70 | 0.0091 | 0.0427 | 0.1005 | 0.1574 | 0.1849 | 0.1738 | 0.1362 | 0.0914 | 0.0537 | 0.0281 | 0.0132 | 0.0056 | 0.0022 | 0.0008 | 4.70 |
| 4.80 | 0.0082 | 0.0395 | 0.0948 | 0.1517 | 0.1820 | 0.1747 | 0.1398 | 0.0959 | 0.0575 | 0.0307 | 0.0147 | 0.0064 | 0.0026 | 0.0009 | 4.80 |
| 4.90 | 0.0074 | 0.0365 | 0.0894 | 0.1460 | 0.1789 | 0.1753 | 0.1432 | 0.1002 | 0.0614 | 0.0334 | 0.0164 | 0.0073 | 0.0030 | 0.0011 | 4.90 |
| 5.00 | 0.0067 | 0.0337 | 0.0842 | 0.1404 | 0.1755 | 0.1755 | 0.1462 | 0.1044 | 0.0653 | 0.0363 | 0.0181 | 0.0082 | 0.0034 | 0.0013 | 5.00 |
| 5.10 | 0.0061 | 0.0311 | 0.0793 | 0.1348 | 0.1719 | 0.1753 | 0.1490 | 0.1086 | 0.0692 | 0.0392 | 0.0200 | 0.0093 | 0.0039 | 0.0015 | 5.10 |
| 5.20 | 0.0055 | 0.0287 | 0.0746 | 0.1293 | 0.1681 | 0.1748 | 0.1515 | 0.1125 | 0.0731 | 0.0423 | 0.0220 | 0.0104 | 0.0045 | 0.0018 | 5.20 |
| 5.30 | 0.0050 | 0.0265 | 0.0701 | 0.1239 | 0.1641 | 0.1740 | 0.1537 | 0.1163 | 0.0771 | 0.0454 | 0.0241 | 0.0116 | 0.0051 | 0.0021 | 5.30 |
| 5.40 | 0.0045 | 0.0244 | 0.0659 | 0.1185 | 0.1600 | 0.1728 | 0.1555 | 0.1200 | 0.0810 | 0.0486 | 0.0262 | 0.0129 | 0.0058 | 0.0024 | 5.40 |
| 5.50 | | | | | | | | | | | | | 0.0065 | | 5.50 |
| 5.60 | 0.0037 | 0.0207 | 0.0580 | 0.1082 | 0.1515 | 0.1697 | 0.1584 | 0.1267 | 0.0887 | 0.0552 | 0.0309 | 0.0157 | 0.0073 | 0.0032 | 5.60 |
| 5.70 | 0.0033 | 0.0191 | 0.0544 | 0.1033 | 0.1472 | 0.1678 | 0.1594 | 0.1298 | 0.0925 | 0.0586 | 0.0334 | 0.0173 | 0.0082 | 0.0036 | 5.70 |
| 5.80 | 0.0030 | 0.0176 | 0.0509 | 0.0985 | 0.1428 | 0.1656 | 0.1601 | 0.1326 | 0.0962 | 0.0620 | 0.0359 | 0.0190 | 0.0092 | 0.0041 | 5.80 |
| 5.90 | 0.0027 | 0.0162 | 0.0477 | 0.0938 | 0.1383 | 0.1632 | 0.1605 | 0.1353 | 0.0998 | 0.0654 | 0.0386 | 0.0207 | 0.0102 | 0.0046 | 5.90 |
| 6.00 | 0.0025 | 0.0149 | 0.0446 | 0.0892 | 0.1339 | 0.1606 | 0.1606 | 0.1377 | 0.1033 | 0.0688 | 0.0413 | 0.0225 | 0.0113 | 0.0052 | 6.00 |
| 6.10 | 0.0022 | 0.0137 | 0.0417 | 0.0848 | 0.1294 | 0.1579 | 0.1605 | 0.1399 | 0.1066 | 0.0723 | 0.0441 | 0.0244 | 0.0124 | 0.0058 | 6.10 |
| 6.20 | 0.0020 | 0.0126 | 0.0390 | 0.0806 | 0.1249 | 0.1549 | 0.1601 | 0.1418 | 0.1099 | 0.0757 | 0.0469 | 0.0265 | 0.0137 | 0.0065 | 6.20 |
| 6.30 | 0.0018 | 0.0116 | 0.0364 | 0.0765 | 0.1205 | 0.1519 | 0.1595 | 0.1435 | 0.1130 | 0.0791 | 0.0498 | 0.0285 | 0.0150 | 0.0073 | 6.30 |
| 6.40 | 0.0017 | 0.0106 | 0.0340 | 0.0726 | 0.1162 | 0.1487 | 0.1586 | 0.1450 | 0.1160 | 0.0825 | 0.0528 | 0.0307 | 0.0164 | 0.0081 | 6.40 |

$$\Pr(X = x) = f(x; \lambda) = \frac{\lambda^{x}}{x!} e^{-\lambda}$$

| | | | | | Pr(X | = x) = f | $(x;\lambda) = $ | $\frac{\lambda}{x!}e^{-\lambda}$ | | | | | |
|------|--------|--------|--------|--------|--------|-----------|------------------|----------------------------------|--------|--------|--------|--------|------|
| | | | | | Probal | oility of | x occur | rences | | | | | |
| λ | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | λ |
| 2.80 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 2.80 |
| 2.90 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 2.90 |
| 3.00 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 3.00 |
| 3.10 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 3.10 |
| 3.20 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 3.20 |
| 3.30 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 3.30 |
| 3.40 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 3.40 |
| 3.50 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 3.50 |
| 3.60 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 3.60 |
| 3.70 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 3.70 |
| 3.80 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 3.80 |
| 3.90 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 3.90 |
| 4.00 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 4.00 |
| 4.10 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 4.10 |
| 4.20 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 4.20 |
| 4.30 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 4.30 |
| 4.40 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 4.40 |
| 4.50 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 4.50 |
| 4.60 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 4.60 |
| 4.70 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 4.70 |
| 4.80 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 4.80 |
| 4.90 | 0.0004 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 4.90 |
| 5.00 | 0.0005 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 5.00 |
| 5.10 | 0.0006 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 5.10 |
| 5.20 | 0.0007 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 5.20 |
| 5.30 | 0.0008 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 5.30 |
| 5.40 | 0.0009 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 5.40 |
| 5.50 | 0.0011 | 0.0004 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 5.50 |
| 5.60 | 0.0013 | 0.0005 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 5.60 |
| 5.70 | 0.0015 | 0.0006 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 5.70 |
| 5.80 | 0.0017 | 0.0007 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 5.80 |
| 5.90 | 0.0019 | 0.0008 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 5.90 |
| 6.00 | 0.0022 | 0.0009 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 6.00 |
| 6.10 | 0.0025 | 0.0010 | 0.0004 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 6.10 |
| 6.20 | 0.0029 | 0.0012 | 0.0005 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 6.20 |
| 6.30 | 0.0033 | 0.0014 | 0.0005 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 6.30 |
| 6.40 | 0.0037 | 0.0016 | 0.0006 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 6.40 |

| Pr(X = | (x) = f(x) | $\lambda = \frac{\lambda^{x}}{x!} e^{-\lambda}$ |
|--------|------------|-------------------------------------------------|
| | | |

| $\Pr(X = x) = f(x; \lambda) = \frac{\lambda^x}{x!} e^{-\lambda}$ | | | | | | | | | | | | | | | |
|------------------------------------------------------------------|--------|--------|--------|--------|--------|--------|-----------|---------|--------|--------|--------|--------|--------|--------|-------|
| | | | | | | Probal | oility of | x occur | rences | | | | | | |
| λ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | λ |
| 6.50 | 0.0015 | 0.0098 | 0.0318 | 0.0688 | 0.1118 | 0.1454 | 0.1575 | 0.1462 | 0.1188 | 0.0858 | 0.0558 | 0.0330 | 0.0179 | 0.0089 | 6.50 |
| 6.60 | 0.0014 | 0.0090 | 0.0296 | 0.0652 | 0.1076 | 0.1420 | 0.1562 | 0.1472 | 0.1215 | 0.0891 | 0.0588 | 0.0353 | 0.0194 | 0.0099 | 6.60 |
| 6.70 | 0.0012 | 0.0082 | 0.0276 | 0.0617 | 0.1034 | 0.1385 | 0.1546 | 0.1480 | 0.1240 | 0.0923 | 0.0618 | 0.0377 | 0.0210 | 0.0108 | 6.70 |
| 6.80 | 0.0011 | 0.0076 | 0.0258 | 0.0584 | 0.0992 | 0.1349 | 0.1529 | 0.1486 | 0.1263 | 0.0954 | 0.0649 | 0.0401 | 0.0227 | 0.0119 | 6.80 |
| 6.90 | 0.0010 | 0.0070 | 0.0240 | 0.0552 | 0.0952 | 0.1314 | 0.1511 | 0.1489 | 0.1284 | 0.0985 | 0.0679 | 0.0426 | 0.0245 | 0.0130 | 6.90 |
| 7.00 | 0.0009 | 0.0064 | 0.0223 | 0.0521 | 0.0912 | 0.1277 | 0.1490 | 0.1490 | 0.1304 | 0.1014 | 0.0710 | 0.0452 | 0.0263 | 0.0142 | 7.00 |
| 7.10 | 0.0008 | 0.0059 | 0.0208 | 0.0492 | 0.0874 | 0.1241 | 0.1468 | 0.1489 | 0.1321 | 0.1042 | 0.0740 | 0.0478 | 0.0283 | 0.0154 | 7.10 |
| 7.20 | 0.0007 | 0.0054 | 0.0194 | 0.0464 | 0.0836 | 0.1204 | 0.1445 | 0.1486 | 0.1337 | 0.1070 | 0.0770 | 0.0504 | 0.0303 | 0.0168 | 7.20 |
| 7.30 | 0.0007 | 0.0049 | 0.0180 | 0.0438 | 0.0799 | 0.1167 | 0.1420 | 0.1481 | 0.1351 | 0.1096 | 0.0800 | 0.0531 | 0.0323 | 0.0181 | 7.30 |
| 7.40 | 0.0006 | 0.0045 | 0.0167 | 0.0413 | 0.0764 | 0.1130 | 0.1394 | 0.1474 | 0.1363 | 0.1121 | 0.0829 | 0.0558 | 0.0344 | 0.0196 | 7.40 |
| 7.50 | 0.0006 | 0.0041 | 0.0156 | 0.0389 | 0.0729 | 0.1094 | 0.1367 | 0.1465 | 0.1373 | 0.1144 | 0.0858 | 0.0585 | 0.0366 | 0.0211 | 7.50 |
| 7.60 | 0.0005 | 0.0038 | 0.0145 | 0.0366 | 0.0696 | 0.1057 | 0.1339 | 0.1454 | 0.1381 | 0.1167 | 0.0887 | 0.0613 | 0.0388 | 0.0227 | 7.60 |
| 7.70 | 0.0005 | 0.0035 | 0.0134 | 0.0345 | 0.0663 | 0.1021 | 0.1311 | 0.1442 | 0.1388 | 0.1187 | 0.0914 | 0.0640 | 0.0411 | 0.0243 | 7.70 |
| 7.80 | 0.0004 | 0.0032 | 0.0125 | 0.0324 | 0.0632 | 0.0986 | 0.1282 | 0.1428 | 0.1392 | 0.1207 | 0.0941 | 0.0667 | 0.0434 | 0.0260 | 7.80 |
| 7.90 | 0.0004 | 0.0029 | 0.0116 | 0.0305 | 0.0602 | 0.0951 | 0.1252 | 0.1413 | 0.1395 | 0.1224 | 0.0967 | 0.0695 | 0.0457 | 0.0278 | 7.90 |
| 8.00 | 0.0003 | 0.0027 | 0.0107 | 0.0286 | 0.0573 | 0.0916 | 0.1221 | 0.1396 | 0.1396 | 0.1241 | 0.0993 | 0.0722 | 0.0481 | 0.0296 | 8.00 |
| 8.10 | 0.0003 | 0.0025 | 0.0100 | 0.0269 | 0.0544 | 0.0882 | 0.1191 | 0.1378 | 0.1395 | 0.1256 | 0.1017 | 0.0749 | 0.0505 | 0.0315 | 8.10 |
| 8.20 | 0.0003 | 0.0023 | 0.0092 | 0.0252 | 0.0517 | 0.0849 | 0.1160 | 0.1358 | 0.1392 | 0.1269 | 0.1040 | 0.0776 | 0.0530 | 0.0334 | 8.20 |
| 8.30 | 0.0002 | 0.0021 | 0.0086 | 0.0237 | 0.0491 | 0.0816 | 0.1128 | 0.1338 | 0.1388 | 0.1280 | 0.1063 | 0.0802 | 0.0555 | 0.0354 | 8.30 |
| 8.40 | 0.0002 | 0.0019 | 0.0079 | 0.0222 | 0.0466 | 0.0784 | 0.1097 | 0.1317 | 0.1382 | 0.1290 | 0.1084 | 0.0828 | 0.0579 | 0.0374 | 8.40 |
| 8.50 | 0.0002 | 0.0017 | 0.0074 | 0.0208 | 0.0443 | 0.0752 | 0.1066 | 0.1294 | 0.1375 | 0.1299 | 0.1104 | 0.0853 | 0.0604 | 0.0395 | 8.50 |
| 8.60 | 0.0002 | 0.0016 | 0.0068 | 0.0195 | 0.0420 | 0.0722 | 0.1034 | 0.1271 | 0.1366 | 0.1306 | 0.1123 | 0.0878 | 0.0629 | 0.0416 | 8.60 |
| 8.70 | 0.0002 | 0.0014 | 0.0063 | 0.0183 | 0.0398 | 0.0692 | 0.1003 | 0.1247 | 0.1356 | 0.1311 | 0.1140 | 0.0902 | 0.0654 | 0.0438 | 8.70 |
| 8.80 | 0.0002 | 0.0013 | 0.0058 | 0.0171 | 0.0377 | 0.0663 | 0.0972 | 0.1222 | 0.1344 | 0.1315 | 0.1157 | 0.0925 | 0.0679 | 0.0459 | 8.80 |
| 8.90 | 0.0001 | 0.0012 | 0.0054 | 0.0160 | 0.0357 | 0.0635 | 0.0941 | 0.1197 | 0.1332 | 0.1317 | 0.1172 | 0.0948 | 0.0703 | 0.0481 | 8.90 |
| 9.00 | 0.0001 | 0.0011 | 0.0050 | 0.0150 | 0.0337 | 0.0607 | 0.0911 | 0.1171 | 0.1318 | 0.1318 | 0.1186 | 0.0970 | 0.0728 | 0.0504 | 9.00 |
| 9.10 | 0.0001 | 0.0010 | 0.0046 | 0.0140 | 0.0319 | 0.0581 | 0.0881 | 0.1145 | 0.1302 | 0.1317 | 0.1198 | 0.0991 | 0.0752 | 0.0526 | 9.10 |
| 9.20 | | | 0.0043 | | | | | | | | | | | | 9.20 |
| 9.30 | 0.0001 | 0.0009 | 0.0040 | 0.0123 | 0.0285 | 0.0530 | 0.0822 | 0.1091 | 0.1269 | 0.1311 | 0.1219 | 0.1031 | 0.0799 | 0.0572 | 9.30 |
| 9.40 | 0.0001 | 0.0008 | 0.0037 | 0.0115 | 0.0269 | 0.0506 | 0.0793 | 0.1064 | 0.1251 | 0.1306 | 0.1228 | 0.1049 | 0.0822 | 0.0594 | 9.40 |
| 9.50 | 0.0001 | 0.0007 | 0.0034 | 0.0107 | 0.0254 | 0.0483 | 0.0764 | 0.1037 | 0.1232 | 0.1300 | 0.1235 | 0.1067 | 0.0844 | 0.0617 | 9.50 |
| | 0.0001 | | | | | | | | | | | | | | 9.60 |
| 9.70 | 0.0001 | 0.0006 | 0.0029 | 0.0093 | 0.0226 | 0.0439 | 0.0709 | 0.0982 | 0.1191 | 0.1284 | 0.1245 | 0.1098 | 0.0888 | 0.0662 | 9.70 |
| 9.80 | 0.0001 | 0.0005 | 0.0027 | 0.0087 | 0.0213 | 0.0418 | 0.0682 | 0.0955 | 0.1170 | 0.1274 | 0.1249 | 0.1112 | 0.0908 | 0.0685 | 9.80 |
| 9.90 | 0.0001 | 0.0005 | 0.0025 | 0.0081 | 0.0201 | 0.0398 | 0.0656 | 0.0928 | 0.1148 | 0.1263 | 0.1250 | 0.1125 | 0.0928 | 0.0707 | 9.90 |
| 10.00 | 0.0000 | 0.0005 | 0.0023 | 0.0076 | 0.0189 | 0.0378 | 0.0631 | 0.0901 | 0.1126 | 0.1251 | 0.1251 | 0.1137 | 0.0948 | 0.0729 | 10.00 |

$$\Pr(X = x) = f(x; \lambda) = \frac{\lambda^{x}}{x!} e^{-\lambda}$$

| | | | | | Pr(X | =x)=f | $(x;\lambda) =$ | $\frac{1}{x!}e^{-\lambda}$ | | | | | |
|-------|--------|--------|--------|--------|--------|-----------|-----------------|----------------------------|--------|--------|--------|--------|-------|
| | | | | | Probal | oility of | x occur | rences | | | | | |
| λ | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | λ |
| 6.50 | 0.0041 | 0.0018 | 0.0007 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 6.50 |
| 6.60 | 0.0046 | 0.0020 | 0.0008 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 6.60 |
| 6.70 | 0.0052 | 0.0023 | 0.0010 | 0.0004 | 0.0001 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 6.70 |
| 6.80 | 0.0058 | 0.0026 | 0.0011 | 0.0004 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 6.80 |
| 6.90 | 0.0064 | 0.0029 | 0.0013 | 0.0005 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 6.90 |
| 7.00 | 0.0071 | 0.0033 | 0.0014 | 0.0006 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 7.00 |
| 7.10 | 0.0078 | 0.0037 | 0.0016 | 0.0007 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 7.10 |
| 7.20 | 0.0086 | 0.0041 | 0.0019 | 0.0008 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 7.20 |
| 7.30 | 0.0095 | 0.0046 | 0.0021 | 0.0009 | 0.0004 | 0.0001 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 7.30 |
| 7.40 | 0.0104 | 0.0051 | 0.0024 | 0.0010 | 0.0004 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 7.40 |
| 7.50 | 0.0113 | 0.0057 | 0.0026 | 0.0012 | 0.0005 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 7.50 |
| 7.60 | 0.0123 | 0.0062 | 0.0030 | 0.0013 | 0.0006 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 7.60 |
| 7.70 | 0.0134 | 0.0069 | 0.0033 | 0.0015 | 0.0006 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 7.70 |
| 7.80 | 0.0145 | 0.0075 | 0.0037 | 0.0017 | 0.0007 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 7.80 |
| 7.90 | 0.0157 | 0.0083 | 0.0041 | 0.0019 | 0.0008 | 0.0003 | 0.0001 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 7.90 |
| 8.00 | 0.0169 | 0.0090 | 0.0045 | 0.0021 | 0.0009 | 0.0004 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 8.00 |
| 8.10 | 0.0182 | 0.0098 | 0.0050 | 0.0024 | 0.0011 | 0.0005 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 8.10 |
| 8.20 | 0.0196 | 0.0107 | 0.0055 | 0.0026 | 0.0012 | 0.0005 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 8.20 |
| 8.30 | 0.0210 | 0.0116 | 0.0060 | 0.0029 | 0.0014 | 0.0006 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 8.30 |
| 8.40 | 0.0225 | 0.0126 | 0.0066 | 0.0033 | 0.0015 | 0.0007 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 8.40 |
| 8.50 | 0.0240 | 0.0136 | 0.0072 | 0.0036 | 0.0017 | 0.0008 | 0.0003 | 0.0001 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 8.50 |
| 8.60 | 0.0256 | 0.0147 | 0.0079 | 0.0040 | 0.0019 | 0.0009 | 0.0004 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 8.60 |
| 8.70 | 0.0272 | 0.0158 | 0.0086 | 0.0044 | 0.0021 | 0.0010 | 0.0004 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 8.70 |
| 8.80 | 0.0289 | 0.0169 | 0.0093 | 0.0048 | 0.0024 | 0.0011 | 0.0005 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 8.80 |
| 8.90 | 0.0306 | 0.0182 | 0.0101 | 0.0053 | 0.0026 | 0.0012 | 0.0005 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 8.90 |
| 9.00 | 0.0324 | 0.0194 | 0.0109 | 0.0058 | 0.0029 | 0.0014 | 0.0006 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 9.00 |
| 9.10 | 0.0342 | 0.0208 | 0.0118 | 0.0063 | 0.0032 | 0.0015 | 0.0007 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 9.10 |
| 9.20 | 0.0361 | 0.0221 | 0.0127 | 0.0069 | 0.0035 | 0.0017 | 0.0008 | 0.0003 | 0.0001 | 0.0001 | 0.0000 | 0.0000 | 9.20 |
| 9.30 | 0.0380 | 0.0235 | 0.0137 | 0.0075 | 0.0039 | 0.0019 | 0.0009 | 0.0004 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 9.30 |
| 9.40 | 0.0399 | 0.0250 | 0.0147 | 0.0081 | 0.0042 | 0.0021 | 0.0010 | 0.0004 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 9.40 |
| 9.50 | 0.0419 | 0.0265 | 0.0157 | 0.0088 | 0.0046 | 0.0023 | 0.0011 | 0.0005 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 9.50 |
| 9.60 | 0.0439 | 0.0281 | 0.0168 | 0.0095 | 0.0051 | 0.0026 | 0.0012 | 0.0006 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 9.60 |
| 9.70 | 0.0459 | 0.0297 | 0.0180 | 0.0103 | 0.0055 | 0.0028 | 0.0014 | 0.0006 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 9.70 |
| 9.80 | 0.0479 | 0.0313 | 0.0192 | 0.0111 | 0.0060 | 0.0031 | 0.0015 | 0.0007 | 0.0003 | 0.0001 | 0.0001 | 0.0000 | 9.80 |
| 9.90 | 0.0500 | 0.0330 | 0.0204 | 0.0119 | 0.0065 | 0.0034 | 0.0017 | 0.0008 | 0.0004 | 0.0002 | 0.0001 | 0.0000 | 9.90 |
| 10.00 | 0.0521 | 0.0347 | 0.0217 | 0.0128 | 0.0071 | 0.0037 | 0.0019 | 0.0009 | 0.0004 | 0.0002 | 0.0001 | 0.0000 | 10.00 |

$$\Pr(X = x) = f(x; \lambda) = \frac{\lambda^{x}}{x!} e^{-\lambda}$$

Appendix 11 Cumulative Poisson Distribution Table

| $\Pr(X \le x) = F(x; \lambda) = \sum_{x=0}^{n} \frac{\lambda^{x}}{x!} e^{-\lambda}$ | | | | | | | | | | | | | | | |
|-------------------------------------------------------------------------------------|--------|--------|--------|--------|--------|--------|------------|--------|---------|--------|--------|--------|--------|--------|------|
| | | | | | | Probab | ility of ≤ | x occu | rrences | | | | | | |
| λ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | λ |
| 0.05 | 0.9512 | 0.9988 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.05 |
| 0.10 | 0.9048 | 0.9953 | 0.9998 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.10 |
| 0.15 | 0.8607 | 0.9898 | 0.9995 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.15 |
| 0.20 | 0.8187 | 0.9825 | 0.9989 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.20 |
| 0.25 | 0.7788 | 0.9735 | 0.9978 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.25 |
| 0.30 | 0.7408 | 0.9631 | 0.9964 | 0.9997 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.30 |
| 0.35 | 0.7047 | 0.9513 | 0.9945 | 0.9995 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.35 |
| 0.40 | 0.6703 | 0.9384 | 0.9921 | 0.9992 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.40 |
| 0.45 | 0.6376 | 0.9246 | 0.9891 | 0.9988 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.45 |
| 0.50 | 0.6065 | 0.9098 | 0.9856 | 0.9982 | 0.9998 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.50 |
| 0.55 | 0.5769 | 0.8943 | 0.9815 | 0.9975 | 0.9997 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.55 |
| 0.60 | 0.5488 | 0.8781 | 0.9769 | 0.9966 | 0.9996 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.60 |
| 0.65 | 0.5220 | 0.8614 | 0.9717 | 0.9956 | 0.9994 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.65 |
| 0.70 | 0.4966 | 0.8442 | 0.9659 | 0.9942 | 0.9992 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.70 |
| 0.75 | 0.4724 | 0.8266 | 0.9595 | 0.9927 | 0.9989 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.75 |
| 0.80 | 0.4493 | 0.8088 | 0.9526 | 0.9909 | 0.9986 | 0.9998 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.80 |
| 0.85 | 0.4274 | 0.7907 | 0.9451 | 0.9889 | 0.9982 | 0.9997 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.85 |
| 0.90 | 0.4066 | 0.7725 | 0.9371 | 0.9865 | 0.9977 | 0.9997 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.90 |
| 0.95 | 0.3867 | 0.7541 | 0.9287 | 0.9839 | 0.9971 | 0.9995 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.95 |
| 1.00 | 0.3679 | 0.7358 | 0.9197 | 0.9810 | 0.9963 | 0.9994 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.00 |
| 1.10 | 0.3329 | 0.6990 | 0.9004 | 0.9743 | 0.9946 | 0.9990 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.10 |
| 1.20 | 0.3012 | 0.6626 | 0.8795 | 0.9662 | 0.9923 | 0.9985 | 0.9997 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.20 |
| 1.30 | 0.2725 | 0.6268 | 0.8571 | 0.9569 | 0.9893 | 0.9978 | 0.9996 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.30 |
| 1.40 | 0.2466 | 0.5918 | 0.8335 | 0.9463 | 0.9857 | 0.9968 | 0.9994 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.40 |
| 1.50 | 0.2231 | 0.5578 | 0.8088 | 0.9344 | 0.9814 | 0.9955 | 0.9991 | 0.9998 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.50 |
| 1.60 | 0.2019 | 0.5249 | 0.7834 | 0.9212 | 0.9763 | 0.9940 | 0.9987 | 0.9997 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.60 |
| 1.70 | 0.1827 | 0.4932 | 0.7572 | 0.9068 | 0.9704 | 0.9920 | 0.9981 | 0.9996 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.70 |
| 1.80 | 0.1653 | 0.4628 | 0.7306 | 0.8913 | 0.9636 | 0.9896 | 0.9974 | 0.9994 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.80 |
| 1.90 | 0.1496 | 0.4337 | 0.7037 | 0.8747 | 0.9559 | 0.9868 | 0.9966 | 0.9992 | 0.9998 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.90 |
| 2.00 | 0.1353 | 0.4060 | 0.6767 | 0.8571 | 0.9473 | 0.9834 | 0.9955 | 0.9989 | 0.9998 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 2.00 |
| 2.10 | 0.1225 | 0.3796 | 0.6496 | 0.8386 | 0.9379 | 0.9796 | 0.9941 | 0.9985 | 0.9997 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 2.10 |
| 2.20 | 0.1108 | 0.3546 | 0.6227 | 0.8194 | 0.9275 | 0.9751 | 0.9925 | 0.9980 | 0.9995 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 2.20 |
| 2.30 | 0.1003 | 0.3309 | 0.5960 | 0.7993 | 0.9162 | 0.9700 | 0.9906 | 0.9974 | 0.9994 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 2.30 |
| 2.40 | 0.0907 | 0.3084 | 0.5697 | 0.7787 | 0.9041 | 0.9643 | 0.9884 | 0.9967 | 0.9991 | 0.9998 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 2.40 |
| 2.50 | 0.0821 | 0.2873 | 0.5438 | 0.7576 | 0.8912 | 0.9580 | 0.9858 | 0.9958 | 0.9989 | 0.9997 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 2.50 |
| 2.60 | 0.0743 | 0.2674 | 0.5184 | 0.7360 | 0.8774 | 0.9510 | 0.9828 | 0.9947 | 0.9985 | 0.9996 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 2.60 |
| 2.70 | 0.0672 | 0.2487 | 0.4936 | 0.7141 | 0.8629 | 0.9433 | 0.9794 | 0.9934 | 0.9981 | 0.9995 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 2.70 |

| $\Pr(X \le x) = F(x; \lambda) = \sum_{x=0}^{n} \frac{\lambda}{x!} e^{-\lambda}$ | | | | | | | | | | | | | |
|---------------------------------------------------------------------------------|--------|--------|--------|--------|--------|------------|--------|---------|--------|--------|--------|--------|------|
| | | | | | Probab | ility of ≤ | x occu | rrences | | | | | |
| λ | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | λ |
| 0.05 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.05 |
| 0.10 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.10 |
| 0.15 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.15 |
| 0.20 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.20 |
| 0.25 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.25 |
| 0.30 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.30 |
| 0.35 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.35 |
| 0.40 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.40 |
| 0.45 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.45 |
| 0.50 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.50 |
| 0.55 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.55 |
| 0.60 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.60 |
| 0.65 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.65 |
| 0.70 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.70 |
| 0.75 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.75 |
| 0.80 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.80 |
| 0.85 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.85 |
| 0.90 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.90 |
| 0.95 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.95 |
| 1.00 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.00 |
| 1.10 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.10 |
| 1.20 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.20 |
| 1.30 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.30 |
| 1.40 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.40 |
| 1.50 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.50 |
| 1.60 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.60 |
| 1.70 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.70 |
| 1.80 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.80 |
| 1.90 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.90 |
| 2.00 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 2.00 |
| 2.10 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 2.10 |
| 2.20 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 2.20 |
| 2.30 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 2.30 |
| 2.40 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 2.40 |
| 2.50 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 2.50 |
| 2.60 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 2.60 |
| 2.70 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 2.70 |

| $\Pr(X \le x) = F(x; \lambda) = \sum_{x=0}^{n} \frac{\lambda^{x}}{x!} e^{-\lambda}$ |
|-------------------------------------------------------------------------------------|

| $\Pr(X \le x) = F(x; \lambda) = \sum_{x=0}^{n} \frac{\lambda^{x}}{x!} e^{-\lambda}$ | | | | | | | | | | | | | | | |
|-------------------------------------------------------------------------------------|--------|--------|--------|--------|--------|--------|------------|--------|---------|--------|--------|--------|--------|--------|------|
| | | | | | | Probab | ility of ≤ | x occu | rrences | | | | | | |
| λ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | λ |
| 2.80 | 0.0608 | 0.2311 | 0.4695 | 0.6919 | 0.8477 | 0.9349 | 0.9756 | 0.9919 | 0.9976 | 0.9993 | 0.9998 | 1.0000 | 1.0000 | 1.0000 | 2.80 |
| 2.90 | 0.0550 | 0.2146 | 0.4460 | 0.6696 | 0.8318 | 0.9258 | 0.9713 | 0.9901 | 0.9969 | 0.9991 | 0.9998 | 0.9999 | 1.0000 | 1.0000 | 2.90 |
| 3.00 | 0.0498 | 0.1991 | 0.4232 | 0.6472 | 0.8153 | 0.9161 | 0.9665 | 0.9881 | 0.9962 | 0.9989 | 0.9997 | 0.9999 | 1.0000 | 1.0000 | 3.00 |
| 3.10 | 0.0450 | 0.1847 | 0.4012 | 0.6248 | 0.7982 | 0.9057 | 0.9612 | 0.9858 | 0.9953 | 0.9986 | 0.9996 | 0.9999 | 1.0000 | 1.0000 | 3.10 |
| 3.20 | 0.0408 | 0.1712 | 0.3799 | 0.6025 | 0.7806 | 0.8946 | 0.9554 | 0.9832 | 0.9943 | 0.9982 | 0.9995 | 0.9999 | 1.0000 | 1.0000 | 3.20 |
| 3.30 | 0.0369 | 0.1586 | 0.3594 | 0.5803 | 0.7626 | 0.8829 | 0.9490 | 0.9802 | 0.9931 | 0.9978 | 0.9994 | 0.9998 | 1.0000 | 1.0000 | 3.30 |
| 3.40 | 0.0334 | 0.1468 | 0.3397 | 0.5584 | 0.7442 | 0.8705 | 0.9421 | 0.9769 | 0.9917 | 0.9973 | 0.9992 | 0.9998 | 0.9999 | 1.0000 | 3.40 |
| 3.50 | 0.0302 | 0.1359 | 0.3208 | 0.5366 | 0.7254 | 0.8576 | 0.9347 | 0.9733 | 0.9901 | 0.9967 | 0.9990 | 0.9997 | 0.9999 | 1.0000 | 3.50 |
| 3.60 | 0.0273 | 0.1257 | 0.3027 | 0.5152 | 0.7064 | 0.8441 | 0.9267 | 0.9692 | 0.9883 | 0.9960 | 0.9987 | 0.9996 | 0.9999 | 1.0000 | 3.60 |
| 3.70 | 0.0247 | 0.1162 | 0.2854 | 0.4942 | 0.6872 | 0.8301 | 0.9182 | 0.9648 | 0.9863 | 0.9952 | 0.9984 | 0.9995 | 0.9999 | 1.0000 | 3.70 |
| 3.80 | 0.0224 | 0.1074 | 0.2689 | 0.4735 | 0.6678 | 0.8156 | 0.9091 | 0.9599 | 0.9840 | 0.9942 | 0.9981 | 0.9994 | 0.9998 | 1.0000 | 3.80 |
| 3.90 | 0.0202 | 0.0992 | 0.2531 | 0.4532 | 0.6484 | 0.8006 | 0.8995 | 0.9546 | 0.9815 | 0.9931 | 0.9977 | 0.9993 | 0.9998 | 0.9999 | 3.90 |
| 4.00 | 0.0183 | 0.0916 | 0.2381 | 0.4335 | 0.6288 | 0.7851 | 0.8893 | 0.9489 | 0.9786 | 0.9919 | 0.9972 | 0.9991 | 0.9997 | 0.9999 | 4.00 |
| 4.10 | 0.0166 | 0.0845 | 0.2238 | 0.4142 | 0.6093 | 0.7693 | 0.8786 | 0.9427 | 0.9755 | 0.9905 | 0.9966 | 0.9989 | 0.9997 | 0.9999 | 4.10 |
| 4.20 | 0.0150 | 0.0780 | 0.2102 | 0.3954 | 0.5898 | 0.7531 | 0.8675 | 0.9361 | 0.9721 | 0.9889 | 0.9959 | 0.9986 | 0.9996 | 0.9999 | 4.20 |
| 4.30 | 0.0136 | 0.0719 | 0.1974 | 0.3772 | 0.5704 | 0.7367 | 0.8558 | 0.9290 | 0.9683 | 0.9871 | 0.9952 | 0.9983 | 0.9995 | 0.9998 | 4.30 |
| 4.40 | 0.0123 | 0.0663 | 0.1851 | 0.3594 | 0.5512 | 0.7199 | 0.8436 | 0.9214 | 0.9642 | 0.9851 | 0.9943 | 0.9980 | 0.9993 | 0.9998 | 4.40 |
| 4.50 | 0.0111 | 0.0611 | 0.1736 | 0.3423 | 0.5321 | 0.7029 | 0.8311 | 0.9134 | 0.9597 | 0.9829 | 0.9933 | 0.9976 | 0.9992 | 0.9997 | 4.50 |
| 4.60 | 0.0101 | 0.0563 | 0.1626 | 0.3257 | 0.5132 | 0.6858 | 0.8180 | 0.9049 | 0.9549 | 0.9805 | 0.9922 | 0.9971 | 0.9990 | 0.9997 | 4.60 |
| 4.70 | 0.0091 | 0.0518 | 0.1523 | 0.3097 | 0.4946 | 0.6684 | 0.8046 | 0.8960 | 0.9497 | 0.9778 | 0.9910 | 0.9966 | 0.9988 | 0.9996 | 4.70 |
| 4.80 | 0.0082 | 0.0477 | 0.1425 | 0.2942 | 0.4763 | 0.6510 | 0.7908 | 0.8867 | 0.9442 | 0.9749 | 0.9896 | 0.9960 | 0.9986 | 0.9995 | 4.80 |
| 4.90 | 0.0074 | 0.0439 | 0.1333 | 0.2793 | 0.4582 | 0.6335 | 0.7767 | 0.8769 | 0.9382 | 0.9717 | 0.9880 | 0.9953 | 0.9983 | 0.9994 | 4.90 |
| 5.00 | 0.0067 | 0.0404 | 0.1247 | 0.2650 | 0.4405 | 0.6160 | 0.7622 | 0.8666 | 0.9319 | 0.9682 | 0.9863 | 0.9945 | 0.9980 | 0.9993 | 5.00 |
| 5.10 | 0.0061 | 0.0372 | 0.1165 | 0.2513 | 0.4231 | 0.5984 | 0.7474 | 0.8560 | 0.9252 | 0.9644 | 0.9844 | 0.9937 | 0.9976 | 0.9992 | 5.10 |
| 5.20 | 0.0055 | 0.0342 | 0.1088 | 0.2381 | 0.4061 | 0.5809 | 0.7324 | 0.8449 | 0.9181 | 0.9603 | 0.9823 | 0.9927 | 0.9972 | 0.9990 | 5.20 |
| 5.30 | 0.0050 | 0.0314 | 0.1016 | 0.2254 | 0.3895 | 0.5635 | 0.7171 | 0.8335 | 0.9106 | 0.9559 | 0.9800 | 0.9916 | 0.9967 | 0.9988 | 5.30 |
| 5.40 | 0.0045 | 0.0289 | 0.0948 | 0.2133 | 0.3733 | 0.5461 | 0.7017 | 0.8217 | 0.9027 | 0.9512 | 0.9775 | 0.9904 | 0.9962 | 0.9986 | 5.40 |
| 5.50 | 0.0041 | 0.0266 | 0.0884 | 0.2017 | 0.3575 | 0.5289 | 0.6860 | 0.8095 | 0.8944 | 0.9462 | 0.9747 | 0.9890 | 0.9955 | 0.9983 | 5.50 |
| 5.60 | 0.0037 | 0.0244 | 0.0824 | 0.1906 | 0.3422 | 0.5119 | 0.6703 | 0.7970 | 0.8857 | 0.9409 | 0.9718 | 0.9875 | 0.9949 | 0.9980 | 5.60 |
| 5.70 | 0.0033 | 0.0224 | 0.0768 | 0.1800 | 0.3272 | 0.4950 | 0.6544 | 0.7841 | 0.8766 | 0.9352 | 0.9686 | 0.9859 | 0.9941 | 0.9977 | 5.70 |
| 5.80 | 0.0030 | 0.0206 | 0.0715 | 0.1700 | 0.3127 | 0.4783 | 0.6384 | 0.7710 | 0.8672 | 0.9292 | 0.9651 | 0.9841 | 0.9932 | 0.9973 | 5.80 |
| 5.90 | 0.0027 | 0.0189 | 0.0666 | 0.1604 | 0.2987 | 0.4619 | 0.6224 | 0.7576 | 0.8574 | 0.9228 | 0.9614 | 0.9821 | 0.9922 | 0.9969 | 5.90 |
| 6.00 | 0.0025 | 0.0174 | 0.0620 | 0.1512 | 0.2851 | 0.4457 | 0.6063 | 0.7440 | 0.8472 | 0.9161 | 0.9574 | 0.9799 | 0.9912 | 0.9964 | 6.00 |
| 6.10 | 0.0022 | 0.0159 | 0.0577 | 0.1425 | 0.2719 | 0.4298 | 0.5902 | 0.7301 | 0.8367 | 0.9090 | 0.9531 | 0.9776 | 0.9900 | 0.9958 | 6.10 |
| 6.20 | 0.0020 | 0.0146 | 0.0536 | 0.1342 | 0.2592 | 0.4141 | 0.5742 | 0.7160 | 0.8259 | 0.9016 | 0.9486 | 0.9750 | 0.9887 | 0.9952 | 6.20 |
| 6.30 | 0.0018 | 0.0134 | 0.0498 | 0.1264 | 0.2469 | 0.3988 | 0.5582 | 0.7017 | 0.8148 | 0.8939 | 0.9437 | 0.9723 | 0.9873 | 0.9945 | 6.30 |
| 6.40 | 0.0017 | 0.0123 | 0.0463 | 0.1189 | 0.2351 | 0.3837 | 0.5423 | 0.6873 | 0.8033 | 0.8858 | 0.9386 | 0.9693 | 0.9857 | 0.9937 | 6.40 |
| 6.50 | 0.0015 | 0.0113 | 0.0430 | 0.1118 | 0.2237 | 0.3690 | 0.5265 | 0.6728 | 0.7916 | 0.8774 | 0.9332 | 0.9661 | 0.9840 | 0.9929 | 6.50 |
| 6.60 | 0.0014 | 0.0103 | 0.0400 | 0.1052 | 0.2127 | 0.3547 | 0.5108 | 0.6581 | 0.7796 | 0.8686 | 0.9274 | 0.9627 | 0.9821 | 0.9920 | 6.60 |
| 6.70 | | 0.0095 | 0.0371 | 0.0988 | 0.2022 | 0.3406 | 0.4953 | 0.6433 | | 0.8596 | 0.9214 | 0.9591 | 0.9801 | 0.9909 | 6.70 |
| 50 | 1.0012 | | | | | | | | | | | | | | 50 |

| $\Pr(X \le x) = F(x; \lambda) =$ | $=\sum_{x=0}^{n}\frac{\lambda^{x}}{x!}e^{-\lambda}$ |
|----------------------------------|-----------------------------------------------------|
|----------------------------------|-----------------------------------------------------|

| | $\Pr(X \le x) = F(x; \lambda) = \sum_{x=0}^{\infty} \frac{\lambda}{x!} e^{-\lambda}$ | | | | | | | | | | | | |
|------|--------------------------------------------------------------------------------------|--------|--------|--------|--------|------------|--------|---------|--------|--------|--------|--------|------|
| | | | | | Probab | ility of ≤ | x occu | rrences | | | | | |
| λ | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | λ |
| 2.80 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 2.80 |
| 2.90 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 2.90 |
| 3.00 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 3.00 |
| 3.10 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 3.10 |
| 3.20 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 3.20 |
| 3.30 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 3.30 |
| 3.40 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 3.40 |
| 3.50 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 3.50 |
| 3.60 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 3.60 |
| 3.70 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 3.70 |
| 3.80 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 3.80 |
| 3.90 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 3.90 |
| 4.00 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 4.00 |
| 4.10 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 4.10 |
| 4.20 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 4.20 |
| 4.30 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 4.30 |
| 4.40 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 4.40 |
| 4.50 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 4.50 |
| 4.60 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 4.60 |
| 4.70 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 4.70 |
| 4.80 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 4.80 |
| 4.90 | 0.9998 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 4.90 |
| 5.00 | 0.9998 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 5.00 |
| 5.10 | 0.9997 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 5.10 |
| 5.20 | 0.9997 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 5.20 |
| 5.30 | 0.9996 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 5.30 |
| 5.40 | 0.9995 | 0.9998 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 5.40 |
| 5.50 | 0.9994 | 0.9998 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 5.50 |
| 5.60 | 0.9993 | 0.9998 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 5.60 |
| 5.70 | 0.9991 | 0.9997 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 5.70 |
| 5.80 | 0.9990 | 0.9996 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 5.80 |
| 5.90 | 0.9988 | 0.9996 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 5.90 |
| 6.00 | 0.9986 | 0.9995 | 0.9998 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 6.00 |
| 6.10 | 0.9984 | 0.9994 | 0.9998 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 6.10 |
| 6.20 | 0.9981 | 0.9993 | 0.9997 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 6.20 |
| 6.30 | 0.9978 | 0.9992 | 0.9997 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 6.30 |
| 6.40 | 0.9974 | 0.9990 | 0.9996 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 6.40 |
| 6.50 | 0.9970 | 0.9988 | 0.9996 | 0.9998 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 6.50 |
| 6.60 | 0.9966 | 0.9986 | 0.9995 | 0.9998 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 6.60 |
| 6.70 | 0.9961 | 0.9984 | 0.9994 | 0.9998 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 6.70 |

| $\Pr(X \le x) = F(x;\lambda) = \sum_{x=0}^{n} \frac{\lambda^{x}}{x!} e^{-\lambda}$ | |
|------------------------------------------------------------------------------------|--|
|------------------------------------------------------------------------------------|--|

| $\Pr(X \le x) = F(x; \lambda) = \sum_{x=0}^{n} \frac{\lambda^{x}}{x!} e^{-\lambda}$ | | | | | | | | | | | | | | | |
|-------------------------------------------------------------------------------------|--------|--------|--------|--------|--------|--------|------------|--------|---------|--------|--------|--------|--------|--------|-------|
| | | | | | | Probab | ility of ≤ | x occu | rrences | | | | | | |
| λ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | λ |
| 6.80 | 0.0011 | 0.0087 | 0.0344 | 0.0928 | 0.1920 | 0.3270 | 0.4799 | 0.6285 | 0.7548 | 0.8502 | 0.9151 | 0.9552 | 0.9779 | 0.9898 | 6.80 |
| 6.90 | 0.0010 | 0.0080 | 0.0320 | 0.0871 | 0.1823 | 0.3137 | 0.4647 | 0.6136 | 0.7420 | 0.8405 | 0.9084 | 0.9510 | 0.9755 | 0.9885 | 6.90 |
| 7.00 | 0.0009 | 0.0073 | 0.0296 | 0.0818 | 0.1730 | 0.3007 | 0.4497 | 0.5987 | 0.7291 | 0.8305 | 0.9015 | 0.9467 | 0.9730 | 0.9872 | 7.00 |
| 7.10 | 0.0008 | 0.0067 | 0.0275 | 0.0767 | 0.1641 | 0.2881 | 0.4349 | 0.5838 | 0.7160 | 0.8202 | 0.8942 | 0.9420 | 0.9703 | 0.9857 | 7.10 |
| 7.20 | 0.0007 | 0.0061 | 0.0255 | 0.0719 | 0.1555 | 0.2759 | 0.4204 | 0.5689 | 0.7027 | 0.8096 | 0.8867 | 0.9371 | 0.9673 | 0.9841 | 7.20 |
| 7.30 | 0.0007 | 0.0056 | 0.0236 | 0.0674 | 0.1473 | 0.2640 | 0.4060 | 0.5541 | 0.6892 | 0.7988 | 0.8788 | 0.9319 | 0.9642 | 0.9824 | 7.30 |
| 7.40 | 0.0006 | 0.0051 | 0.0219 | 0.0632 | 0.1395 | 0.2526 | 0.3920 | 0.5393 | 0.6757 | 0.7877 | 0.8707 | 0.9265 | 0.9609 | 0.9805 | 7.40 |
| 7.50 | 0.0006 | 0.0047 | 0.0203 | 0.0591 | 0.1321 | 0.2414 | 0.3782 | 0.5246 | 0.6620 | 0.7764 | 0.8622 | 0.9208 | 0.9573 | 0.9784 | 7.50 |
| 7.60 | 0.0005 | 0.0043 | 0.0188 | 0.0554 | 0.1249 | 0.2307 | 0.3646 | 0.5100 | 0.6482 | 0.7649 | 0.8535 | 0.9148 | 0.9536 | 0.9762 | 7.60 |
| 7.70 | 0.0005 | 0.0039 | 0.0174 | 0.0518 | 0.1181 | 0.2203 | 0.3514 | 0.4956 | 0.6343 | 0.7531 | 0.8445 | 0.9085 | 0.9496 | 0.9739 | 7.70 |
| 7.80 | 0.0004 | 0.0036 | 0.0161 | 0.0485 | 0.1117 | 0.2103 | 0.3384 | 0.4812 | 0.6204 | 0.7411 | 0.8352 | 0.9020 | 0.9454 | 0.9714 | 7.80 |
| 7.90 | 0.0004 | 0.0033 | 0.0149 | 0.0453 | 0.1055 | 0.2006 | 0.3257 | 0.4670 | 0.6065 | 0.7290 | 0.8257 | 0.8952 | 0.9409 | 0.9687 | 7.90 |
| 8.00 | 0.0003 | 0.0030 | 0.0138 | 0.0424 | 0.0996 | 0.1912 | 0.3134 | 0.4530 | 0.5925 | 0.7166 | 0.8159 | 0.8881 | 0.9362 | 0.9658 | 8.00 |
| 8.10 | 0.0003 | 0.0028 | 0.0127 | 0.0396 | 0.0940 | 0.1822 | 0.3013 | 0.4391 | 0.5786 | 0.7041 | 0.8058 | 0.8807 | 0.9313 | 0.9628 | 8.10 |
| 8.20 | 0.0003 | 0.0025 | 0.0118 | 0.0370 | 0.0887 | 0.1736 | 0.2896 | 0.4254 | 0.5647 | 0.6915 | 0.7955 | 0.8731 | 0.9261 | 0.9595 | 8.20 |
| 8.30 | 0.0002 | 0.0023 | 0.0109 | 0.0346 | 0.0837 | 0.1653 | 0.2781 | 0.4119 | 0.5507 | 0.6788 | 0.7850 | 0.8652 | 0.9207 | 0.9561 | 8.30 |
| 8.40 | 0.0002 | 0.0021 | 0.0100 | 0.0323 | 0.0789 | 0.1573 | 0.2670 | 0.3987 | 0.5369 | 0.6659 | 0.7743 | 0.8571 | 0.9150 | 0.9524 | 8.40 |
| 8.50 | 0.0002 | 0.0019 | 0.0093 | 0.0301 | 0.0744 | 0.1496 | 0.2562 | 0.3856 | 0.5231 | 0.6530 | 0.7634 | 0.8487 | 0.9091 | 0.9486 | 8.50 |
| 8.60 | 0.0002 | 0.0018 | 0.0086 | 0.0281 | 0.0701 | 0.1422 | 0.2457 | 0.3728 | 0.5094 | 0.6400 | 0.7522 | 0.8400 | 0.9029 | 0.9445 | 8.60 |
| 8.70 | 0.0002 | 0.0016 | 0.0079 | 0.0262 | 0.0660 | 0.1352 | 0.2355 | 0.3602 | 0.4958 | 0.6269 | 0.7409 | 0.8311 | 0.8965 | 0.9403 | 8.70 |
| 8.80 | 0.0002 | 0.0015 | 0.0073 | 0.0244 | 0.0621 | 0.1284 | 0.2256 | 0.3478 | 0.4823 | 0.6137 | 0.7294 | 0.8220 | 0.8898 | 0.9358 | 8.80 |
| 8.90 | 0.0001 | 0.0014 | 0.0068 | 0.0228 | 0.0584 | 0.1219 | 0.2160 | 0.3357 | 0.4689 | 0.6006 | 0.7178 | 0.8126 | 0.8829 | 0.9311 | 8.90 |
| 9.00 | 0.0001 | 0.0012 | 0.0062 | 0.0212 | 0.0550 | 0.1157 | 0.2068 | 0.3239 | 0.4557 | 0.5874 | 0.7060 | 0.8030 | 0.8758 | 0.9261 | 9.00 |
| 9.10 | 0.0001 | 0.0011 | 0.0058 | 0.0198 | 0.0517 | 0.1098 | 0.1978 | 0.3123 | 0.4426 | 0.5742 | 0.6941 | 0.7932 | 0.8684 | 0.9210 | 9.10 |
| 9.20 | 0.0001 | 0.0010 | 0.0053 | 0.0184 | 0.0486 | 0.1041 | 0.1892 | 0.3010 | 0.4296 | 0.5611 | 0.6820 | 0.7832 | 0.8607 | 0.9156 | 9.20 |
| 9.30 | 0.0001 | 0.0009 | 0.0049 | 0.0172 | 0.0456 | 0.0986 | 0.1808 | 0.2900 | 0.4168 | 0.5479 | 0.6699 | 0.7730 | 0.8529 | 0.9100 | 9.30 |
| 9.40 | 0.0001 | 0.0009 | 0.0045 | 0.0160 | 0.0429 | 0.0935 | 0.1727 | 0.2792 | 0.4042 | 0.5349 | 0.6576 | 0.7626 | 0.8448 | 0.9042 | 9.40 |
| 9.50 | 0.0001 | 0.0008 | 0.0042 | 0.0149 | 0.0403 | 0.0885 | 0.1649 | 0.2687 | 0.3918 | 0.5218 | 0.6453 | 0.7520 | 0.8364 | 0.8981 | 9.50 |
| 9.60 | 0.0001 | 0.0007 | 0.0038 | 0.0138 | 0.0378 | 0.0838 | 0.1574 | 0.2584 | 0.3796 | 0.5089 | 0.6329 | 0.7412 | 0.8279 | 0.8919 | 9.60 |
| 9.70 | 0.0001 | 0.0007 | 0.0035 | 0.0129 | 0.0355 | 0.0793 | 0.1502 | 0.2485 | 0.3676 | 0.4960 | 0.6205 | 0.7303 | 0.8191 | 0.8853 | 9.70 |
| 9.80 | 0.0001 | 0.0006 | 0.0033 | 0.0120 | 0.0333 | 0.0750 | 0.1433 | 0.2388 | 0.3558 | 0.4832 | 0.6080 | 0.7193 | 0.8101 | 0.8786 | 9.80 |
| 9.90 | 0.0001 | 0.0005 | 0.0030 | 0.0111 | 0.0312 | 0.0710 | 0.1366 | 0.2294 | 0.3442 | 0.4705 | 0.5955 | 0.7081 | 0.8009 | 0.8716 | 9.90 |
| 10.00 | 0.0000 | 0.0005 | 0.0028 | 0.0103 | 0.0293 | 0.0671 | 0.1301 | 0.2202 | 0.3328 | 0.4579 | 0.5830 | 0.6968 | 0.7916 | 0.8645 | 10.00 |

$$\Pr(X \le x) = F(x; \lambda) = \sum_{x=0}^{n} \frac{\lambda^{x}}{x!} e^{-\lambda}$$

| | $\Pr(X \le x) = F(x; \lambda) = \sum_{x=0}^{\infty} \frac{1}{x!} e^{-\lambda}$ | | | | | | | | | | | | |
|-------|--------------------------------------------------------------------------------|--------|--------|--------|--------|------------|--------|---------|--------|--------|--------|--------|-------|
| | | | | | Probab | ility of ≤ | x occu | rrences | | | | | |
| λ | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | λ |
| 6.80 | 0.9956 | 0.9982 | 0.9993 | 0.9997 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 6.80 |
| 6.90 | 0.9950 | 0.9979 | 0.9992 | 0.9997 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 6.90 |
| 7.00 | 0.9943 | 0.9976 | 0.9990 | 0.9996 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 7.00 |
| 7.10 | 0.9935 | 0.9972 | 0.9989 | 0.9996 | 0.9998 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 7.10 |
| 7.20 | 0.9927 | 0.9969 | 0.9987 | 0.9995 | 0.9998 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 7.20 |
| 7.30 | 0.9918 | 0.9964 | 0.9985 | 0.9994 | 0.9998 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 7.30 |
| 7.40 | 0.9908 | 0.9959 | 0.9983 | 0.9993 | 0.9997 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 7.40 |
| 7.50 | 0.9897 | 0.9954 | 0.9980 | 0.9992 | 0.9997 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 7.50 |
| 7.60 | 0.9886 | 0.9948 | 0.9978 | 0.9991 | 0.9996 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 7.60 |
| 7.70 | 0.9873 | 0.9941 | 0.9974 | 0.9989 | 0.9996 | 0.9998 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 7.70 |
| 7.80 | 0.9859 | 0.9934 | 0.9971 | 0.9988 | 0.9995 | 0.9998 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 7.80 |
| 7.90 | 0.9844 | 0.9926 | 0.9967 | 0.9986 | 0.9994 | 0.9998 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 7.90 |
| 8.00 | 0.9827 | 0.9918 | 0.9963 | 0.9984 | 0.9993 | 0.9997 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 8.00 |
| 8.10 | 0.9810 | 0.9908 | 0.9958 | 0.9982 | 0.9992 | 0.9997 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 8.10 |
| 8.20 | 0.9791 | 0.9898 | 0.9953 | 0.9979 | 0.9991 | 0.9997 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 8.20 |
| 8.30 | 0.9771 | 0.9887 | 0.9947 | 0.9977 | 0.9990 | 0.9996 | 0.9998 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 8.30 |
| 8.40 | 0.9749 | 0.9875 | 0.9941 | 0.9973 | 0.9989 | 0.9995 | 0.9998 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 8.40 |
| 8.50 | 0.9726 | 0.9862 | 0.9934 | 0.9970 | 0.9987 | 0.9995 | 0.9998 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 8.50 |
| 8.60 | 0.9701 | 0.9848 | 0.9926 | 0.9966 | 0.9985 | 0.9994 | 0.9998 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 8.60 |
| 8.70 | 0.9675 | 0.9832 | 0.9918 | 0.9962 | 0.9983 | 0.9993 | 0.9997 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 8.70 |
| 8.80 | 0.9647 | 0.9816 | 0.9909 | 0.9957 | 0.9981 | 0.9992 | 0.9997 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 8.80 |
| 8.90 | 0.9617 | 0.9798 | 0.9899 | 0.9952 | 0.9978 | 0.9991 | 0.9996 | 0.9998 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 8.90 |
| 9.00 | 0.9585 | 0.9780 | 0.9889 | 0.9947 | 0.9976 | 0.9989 | 0.9996 | 0.9998 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 9.00 |
| 9.10 | 0.9552 | 0.9760 | 0.9878 | 0.9941 | 0.9973 | 0.9988 | 0.9995 | 0.9998 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 9.10 |
| 9.20 | 0.9517 | 0.9738 | 0.9865 | 0.9934 | 0.9969 | 0.9986 | 0.9994 | 0.9998 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 9.20 |
| 9.30 | 0.9480 | 0.9715 | 0.9852 | 0.9927 | 0.9966 | 0.9985 | 0.9993 | 0.9997 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 9.30 |
| 9.40 | 0.9441 | 0.9691 | 0.9838 | 0.9919 | 0.9962 | 0.9983 | 0.9992 | 0.9997 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 9.40 |
| 9.50 | 0.9400 | 0.9665 | 0.9823 | 0.9911 | 0.9957 | 0.9980 | 0.9991 | 0.9996 | 0.9999 | 0.9999 | 1.0000 | 1.0000 | 9.50 |
| 9.60 | 0.9357 | 0.9638 | 0.9806 | 0.9902 | 0.9952 | 0.9978 | 0.9990 | 0.9996 | 0.9998 | 0.9999 | 1.0000 | 1.0000 | 9.60 |
| 9.70 | 0.9312 | 0.9609 | 0.9789 | 0.9892 | 0.9947 | 0.9975 | 0.9989 | 0.9995 | 0.9998 | 0.9999 | 1.0000 | 1.0000 | 9.70 |
| 9.80 | 0.9265 | 0.9579 | 0.9770 | 0.9881 | 0.9941 | 0.9972 | 0.9987 | 0.9995 | 0.9998 | 0.9999 | 1.0000 | 1.0000 | 9.80 |
| 9.90 | 0.9216 | 0.9546 | 0.9751 | 0.9870 | 0.9935 | 0.9969 | 0.9986 | 0.9994 | 0.9997 | 0.9999 | 1.0000 | 1.0000 | 9.90 |
| 10.00 | 0.9165 | 0.9513 | 0.9730 | 0.9857 | 0.9928 | 0.9965 | 0.9984 | 0.9993 | 0.9997 | 0.9999 | 1.0000 | 1.0000 | 10.00 |

Appendix 12

Standard Normal Distribution Table

| Pr(2 | $\Pr(Z \ge z) = 1 - \Phi(Z \le z) = \int_{z}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\mu^{2}/2} d\mu$ | | | | | | | | | | | |
|------|--------------------------------------------------------------------------------------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|-----|--|
| Z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 | Ζ | |
| 0.0 | 0.5000 | 0.4960 | 0.4920 | 0.4880 | 0.4840 | 0.4801 | 0.4761 | 0.4721 | 0.4681 | 0.4641 | 0.0 | |
| 0.1 | 0.4602 | 0.4562 | 0.4522 | 0.4483 | 0.4443 | 0.4404 | 0.4364 | 0.4325 | 0.4286 | 0.4247 | 0.1 | |
| 0.2 | 0.4207 | 0.4168 | 0.4129 | 0.4090 | 0.4052 | 0.4013 | 0.3974 | 0.3936 | 0.3897 | 0.3859 | 0.2 | |
| 0.3 | 0.3821 | 0.3783 | 0.3745 | 0.3707 | 0.3669 | 0.3632 | 0.3594 | 0.3557 | 0.3520 | 0.3483 | 0.3 | |
| 0.4 | 0.3446 | 0.3409 | 0.3372 | 0.3336 | 0.3300 | 0.3264 | 0.3228 | 0.3192 | 0.3156 | 0.3121 | 0.4 | |
| 0.5 | 0.3085 | 0.3050 | 0.3015 | 0.2981 | 0.2946 | 0.2912 | 0.2877 | 0.2843 | 0.2810 | 0.2776 | 0.5 | |
| 0.6 | 0.2743 | 0.2709 | 0.2676 | 0.2643 | 0.2611 | 0.2578 | 0.2546 | 0.2514 | 0.2483 | 0.2451 | 0.6 | |
| 0.7 | 0.2420 | 0.2389 | 0.2358 | 0.2327 | 0.2296 | 0.2266 | 0.2236 | 0.2206 | 0.2177 | 0.2148 | 0.7 | |
| 0.8 | 0.2119 | 0.2090 | 0.2061 | 0.2033 | 0.2005 | 0.1977 | 0.1949 | 0.1922 | 0.1894 | 0.1867 | 0.8 | |
| 0.9 | 0.1841 | 0.1814 | 0.1788 | 0.1762 | 0.1736 | 0.1711 | 0.1685 | 0.1660 | 0.1635 | 0.1611 | 0.9 | |
| 1.0 | 0.1587 | 0.1562 | 0.1539 | 0.1515 | 0.1492 | 0.1469 | 0.1446 | 0.1423 | 0.1401 | 0.1379 | 1.0 | |
| 1.1 | 0.1357 | 0.1335 | 0.1314 | 0.1292 | 0.1271 | 0.1251 | 0.1230 | 0.1210 | 0.1190 | 0.1170 | 1.1 | |
| 1.2 | 0.1151 | 0.1131 | 0.1112 | 0.1093 | 0.1075 | 0.1056 | 0.1038 | 0.1020 | 0.1003 | 0.0985 | 1.2 | |
| 1.3 | 0.0968 | 0.0951 | 0.0934 | 0.0918 | 0.0901 | 0.0885 | 0.0869 | 0.0853 | 0.0838 | 0.0823 | 1.3 | |
| 1.4 | 0.0808 | 0.0793 | 0.0778 | 0.0764 | 0.0749 | 0.0735 | 0.0721 | 0.0708 | 0.0694 | 0.0681 | 1.4 | |
| 1.5 | 0.0668 | 0.0655 | 0.0643 | 0.0630 | 0.0618 | 0.0606 | 0.0594 | 0.0582 | 0.0571 | 0.0559 | 1.5 | |
| 1.6 | 0.0548 | 0.0537 | 0.0526 | 0.0516 | 0.0505 | 0.0495 | 0.0485 | 0.0475 | 0.0465 | 0.0455 | 1.6 | |
| 1.7 | 0.0446 | 0.0436 | 0.0427 | 0.0418 | 0.0409 | 0.0401 | 0.0392 | 0.0384 | 0.0375 | 0.0367 | 1.7 | |
| 1.8 | 0.0359 | 0.0351 | 0.0344 | 0.0336 | 0.0329 | 0.0322 | 0.0314 | 0.0307 | 0.0301 | 0.0294 | 1.8 | |
| 1.9 | 0.0287 | 0.0281 | 0.0274 | 0.0268 | 0.0262 | 0.0256 | 0.0250 | 0.0244 | 0.0239 | 0.0233 | 1.9 | |
| 2.0 | 0.0228 | 0.0222 | 0.0217 | 0.0212 | 0.0207 | 0.0202 | 0.0197 | 0.0192 | 0.0188 | 0.0183 | 2.0 | |
| 2.1 | 0.0179 | 0.0174 | 0.0170 | 0.0166 | 0.0162 | 0.0158 | 0.0154 | 0.0150 | 0.0146 | 0.0143 | 2.1 | |
| 2.2 | 0.0139 | 0.0136 | 0.0132 | 0.0129 | 0.0125 | 0.0122 | 0.0119 | 0.0116 | 0.0113 | 0.0110 | 2.2 | |
| 2.3 | 0.0107 | 0.0104 | 0.0102 | 0.0099 | 0.0096 | 0.0094 | 0.0091 | 0.0089 | 0.0087 | 0.0084 | 2.3 | |

| Pr(2 | $Z \ge z) = 1 - 1$ | $\Phi(Z \le z)$ | $=\int_{z}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}}$ | ^{-μ²/2} dμ | | | | | 0 | $1 - \Phi(z)$ | α |
|------|--------------------|-----------------|-----------------------------------------------------------|---------------------|----------|----------|----------|----------|----------|---------------|-----|
| Z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 | Ζ |
| 2.4 | 0.0082 | 0.0080 | 0.0078 | 0.0075 | 0.0073 | 0.0071 | 0.0069 | 0.0068 | 0.0066 | 0.0064 | 2.4 |
| 2.5 | 0.0062 | 0.0060 | 0.0059 | 0.0057 | 0.0055 | 0.0054 | 0.0052 | 0.0051 | 0.0049 | 0.0048 | 2.5 |
| 2.6 | 0.0047 | 0.0045 | 0.0044 | 0.0043 | 0.0041 | 0.0040 | 0.0039 | 0.0038 | 0.0037 | 0.0036 | 2.6 |
| 2.7 | 0.0035 | 0.0034 | 0.0033 | 0.0032 | 0.0031 | 0.0030 | 0.0029 | 0.0028 | 0.0027 | 0.0026 | 2.7 |
| 2.8 | 0.0026 | 0.0025 | 0.0024 | 0.0023 | 0.0023 | 0.0022 | 0.0021 | 0.0021 | 0.0020 | 0.0019 | 2.8 |
| 2.9 | 0.0019 | 0.0018 | 0.0018 | 0.0017 | 0.0016 | 0.0016 | 0.0015 | 0.0015 | 0.0014 | 0.0014 | 2.9 |
| 3.0 | 0.0013 | 0.0013 | 0.0013 | 0.0012 | 0.0012 | 0.0011 | 0.0011 | 0.0011 | 0.0010 | 0.0010 | 3.0 |
| 3.1 | 0.00097 | 0.00094 | 0.00090 | 0.00087 | 0.00084 | 0.00082 | 0.00079 | 0.00076 | 0.00074 | 0.00071 | 3.1 |
| 3.2 | 0.00069 | 0.00066 | 0.00064 | 0.00062 | 0.00060 | 0.00058 | 0.00056 | 0.00054 | 0.00052 | 0.00050 | 3.2 |
| 3.3 | 0.00048 | 0.00047 | 0.00045 | 0.00043 | 0.00042 | 0.00040 | 0.00039 | 0.00038 | 0.00036 | 0.00035 | 3.3 |
| 3.4 | 0.00034 | 0.00032 | 0.00031 | 0.00030 | 0.00029 | 0.00028 | 0.00027 | 0.00026 | 0.00025 | 0.00024 | 3.4 |
| 3.5 | 0.00023 | 0.00022 | 0.00022 | 0.00021 | 0.00020 | 0.00019 | 0.00019 | 0.00018 | 0.00017 | 0.00017 | 3.5 |
| 3.6 | 0.00016 | 0.00015 | 0.00015 | 0.00014 | 0.00014 | 0.00013 | 0.00013 | 0.00012 | 0.00012 | 0.00011 | 3.6 |
| 3.7 | 0.00011 | 0.00010 | 0.00010 | 0.00010 | 0.00009 | 0.00009 | 0.00008 | 0.00008 | 0.00008 | 0.00008 | 3.7 |
| 3.8 | 0.00007 | 0.00007 | 0.00007 | 0.00006 | 0.00006 | 0.00006 | 0.00006 | 0.00005 | 0.00005 | 0.00005 | 3.8 |
| 3.9 | 0.00005 | 0.00005 | 0.00004 | 0.00004 | 0.00004 | 0.00004 | 0.00004 | 0.00004 | 0.00003 | 0.00003 | 3.9 |
| 4.0 | 0.00003 | 0.00003 | 0.00003 | 0.00003 | 0.00003 | 0.00003 | 0.00002 | 0.00002 | 0.00002 | 0.00002 | 4.0 |
| 4.1 | 0.00002 | 0.00002 | 0.00002 | 0.00002 | 0.00002 | 0.00002 | 0.00002 | 0.00002 | 0.00001 | 0.00001 | 4.1 |
| 4.2 | 0.00001 | 0.00001 | 0.00001 | 0.00001 | 0.00001 | 0.00001 | 0.00001 | 0.00001 | 0.00001 | 0.00001 | 4.2 |
| 4.3 | 0.00001 | 0.00001 | 0.00001 | 0.00001 | 0.00001 | 0.00001 | 0.00001 | 0.00001 | 0.00001 | 0.00001 | 4.3 |
| 4.4 | 0.000005 | 0.000005 | 0.000005 | 0.000005 | 0.000004 | 0.000004 | 0.000004 | 0.000004 | 0.000004 | 0.000004 | 4.4 |
| 4.5 | 0.000003 | 0.000003 | 0.000003 | 0.000003 | 0.000003 | 0.000003 | 0.000003 | 0.000002 | 0.000002 | 0.000002 | 4.5 |
| 4.6 | 0.000002 | 0.000002 | 0.000002 | 0.000002 | 0.000002 | 0.000002 | 0.000002 | 0.000002 | 0.000001 | 0.000001 | 4.6 |
| 4.7 | 0.000001 | 0.000001 | 0.000001 | 0.000001 | 0.000001 | 0.000001 | 0.000001 | 0.000001 | 0.000001 | 0.000001 | 4.7 |
| 4.8 | 0.000001 | 0.000001 | 0.000001 | 0.000001 | 0.000001 | 0.000001 | 0.000001 | 0.000001 | 0.000001 | 0.000001 | 4.8 |
| 4.9 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 4.9 |
| 5.0 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 5.0 |
| 5.1 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 5.1 |
| 5.2 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 5.2 |
| 5.3 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 5.3 |
| 5.4 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 5.4 |
| 5.5 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 5.5 |
| 5.6 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 5.6 |
| 5.7 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 5.7 |
| 5.8 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 5.8 |

| Pr(2 | $Z \ge z) = 1 - \Phi(Z \le z) = \int_{z}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\mu^{2}/2} d\mu$ | | | | | | $\frac{1-\Phi(z)}{Z_{\alpha}}$ | <i>α</i> | | | |
|---------------------------------------------------------|----------------------------------------------------------------------------------------------|----------|----------|----------|----------|----------|--------------------------------|----------|----------|----------|-----|
| Z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 | Ζ |
| 5.9 0.000000 0.000000 0.000000 0.000000 0.000000 | | | | | | | 0.000000 | 0.000000 | 0.000000 | 5.9 | |
| 6.0 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 6.0 |

Appendix 13

Cumulative Standard Normal Distribution Table

| Pr(2 | $\Pr(Z \le z) = \Phi(Z \le z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\mu^{2}/2} d\mu$ | | | | | | | | | | | |
|------|-----------------------------------------------------------------------------------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|-----|--|
| Ζ | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 | Z | |
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 | 0.0 | |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 | 0.1 | |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 | 0.2 | |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 | 0.3 | |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 | 0.4 | |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 | 0.5 | |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 | 0.6 | |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 | 0.7 | |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 | 0.8 | |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 | 0.9 | |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 | 1.0 | |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 | 1.1 | |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 | 1.2 | |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 | 1.3 | |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 | 1.4 | |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 | 1.5 | |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 | 1.6 | |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 | 1.7 | |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 | 1.8 | |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 | 1.9 | |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 | 2.0 | |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 | 2.1 | |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 | 2.2 | |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 | 2.3 | |

| Pr(2 | $Z \leq z) = \Phi(z)$ | $(Z \leq z) = \int_{-\infty}^{2}$ | $\int_{\infty}^{2} \frac{1}{\sqrt{2\pi}} e^{-\mu^2}$ | ^{/2} dµ | | | | | | Φ(Z) Z | |
|------|-----------------------|-----------------------------------|------------------------------------------------------|------------------|----------|----------|----------|----------|----------|-----------|-----|
| Ζ | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 | Ζ |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 | 2.4 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 | 2.5 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 | 2.6 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 | 2.7 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 | 2.8 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 | 2.9 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 | 3.0 |
| 3.1 | 0.99903 | 0.99906 | 0.99910 | 0.99913 | 0.99916 | 0.99918 | 0.99921 | 0.99924 | 0.99926 | 0.99929 | 3.1 |
| 3.2 | 0.99931 | 0.99934 | 0.99936 | 0.99938 | 0.99940 | 0.99942 | 0.99944 | 0.99946 | 0.99948 | 0.99950 | 3.2 |
| 3.3 | 0.99952 | 0.99953 | 0.99955 | 0.99957 | 0.99958 | 0.99960 | 0.99961 | 0.99962 | 0.99964 | 0.99965 | 3.3 |
| 3.4 | 0.99966 | 0.99968 | 0.99969 | 0.99970 | 0.99971 | 0.99972 | 0.99973 | 0.99974 | 0.99975 | 0.99976 | 3.4 |
| 3.5 | 0.99977 | 0.99978 | 0.99978 | 0.99979 | 0.99980 | 0.99981 | 0.99981 | 0.99982 | 0.99983 | 0.99983 | 3.5 |
| 3.6 | 0.99984 | 0.99985 | 0.99985 | 0.99986 | 0.99986 | 0.99987 | 0.99987 | 0.99988 | 0.99988 | 0.99989 | 3.6 |
| 3.7 | 0.99989 | 0.99990 | 0.99990 | 0.99990 | 0.99991 | 0.99991 | 0.99992 | 0.99992 | 0.99992 | 0.99992 | 3.7 |
| 3.8 | 0.99993 | 0.99993 | 0.99993 | 0.99994 | 0.99994 | 0.99994 | 0.99994 | 0.99995 | 0.99995 | 0.99995 | 3.8 |
| 3.9 | 0.99995 | 0.99995 | 0.99996 | 0.99996 | 0.99996 | 0.99996 | 0.99996 | 0.99996 | 0.99997 | 0.99997 | 3.9 |
| 4.0 | 0.99997 | 0.99997 | 0.99997 | 0.99997 | 0.99997 | 0.99997 | 0.99998 | 0.99998 | 0.99998 | 0.99998 | 4.0 |
| 4.1 | 0.99998 | 0.99998 | 0.99998 | 0.99998 | 0.99998 | 0.99998 | 0.99998 | 0.99998 | 0.99999 | 0.99999 | 4.1 |
| 4.2 | 0.99999 | 0.99999 | 0.99999 | 0.99999 | 0.99999 | 0.99999 | 0.99999 | 0.99999 | 0.99999 | 0.99999 | 4.2 |
| 4.3 | 0.99999 | 0.99999 | 0.99999 | 0.99999 | 0.99999 | 0.99999 | 0.99999 | 0.99999 | 0.99999 | 0.99999 | 4.3 |
| 4.4 | 0.999995 | 0.999995 | 0.999995 | 0.999995 | 0.999996 | 0.999996 | 0.999996 | 0.999996 | 0.999996 | 0.999996 | 4.4 |
| 4.5 | 0.999997 | 0.999997 | 0.999997 | 0.999997 | 0.999997 | 0.999997 | 0.999997 | 0.999998 | 0.999998 | 0.999998 | 4.5 |
| 4.6 | 0.999998 | 0.999998 | 0.999998 | 0.999998 | 0.999998 | 0.999998 | 0.999998 | 0.999998 | 0.999999 | 0.999999 | 4.6 |
| 4.7 | 0.999999 | 0.999999 | 0.999999 | 0.999999 | 0.999999 | 0.999999 | 0.999999 | 0.999999 | 0.999999 | 0.999999 | 4.7 |
| 4.8 | 0.999999 | 0.999999 | 0.999999 | 0.999999 | 0.999999 | 0.999999 | 0.999999 | 0.999999 | 0.999999 | 0.999999 | 4.8 |
| 4.9 | 1.000000 | 1.000000 | 1.000000 | 1.00000 | 1.00000 | 1.000000 | 1.00000 | 1.00000 | 1.00000 | 1.000000 | 4.9 |
| 5.0 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 5.0 |
| 5.1 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 5.1 |
| 5.2 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 5.2 |
| 5.3 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 5.3 |
| 5.4 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 5.4 |
| 5.5 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 5.5 |
| 5.6 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 5.6 |
| 5.7 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 5.7 |
| 5.8 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 5.8 |

| Pr(2 | $Z \leq z) = \Phi($ | $(Z \leq z) = \int_{-\infty}^{2}$ | $\int_{\infty}^{2} \frac{1}{\sqrt{2\pi}} e^{-\mu^2} dx$ | | | | | | | | |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------|-----------------------------------|---------------------------------------------------------|----------|----------|----------|----------|----------|----------|----------|-----|
| Z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 | Ζ |
| 5.9 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.0000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.0000000 1.00000000 | | | | | | | 1.000000 | 5.9 | | | |
| 6.0 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 6.0 |

Appendix 14 *t* Distribution Table

| | | | α | | | | | | | | | |
|----|-------|-------|--------|--------|--------|----|--|--|--|--|--|--|
| ν | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 | ν | | | | | | |
| 1 | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 | 1 | | | | | | |
| 2 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 | 2 | | | | | | |
| 3 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | 3 | | | | | | |
| 4 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 | 4 | | | | | | |
| 5 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 5 | | | | | | |
| 6 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 | 6 | | | | | | |
| 7 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 | 7 | | | | | | |
| 8 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 | 8 | | | | | | |
| 9 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 | 9 | | | | | | |
| 10 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 10 | | | | | | |
| 11 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | 11 | | | | | | |
| 12 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 12 | | | | | | |
| 13 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 | 13 | | | | | | |
| 14 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | 14 | | | | | | |
| 15 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | 15 | | | | | | |
| 16 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 | 16 | | | | | | |
| 17 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 | 17 | | | | | | |
| 18 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 | 18 | | | | | | |
| 19 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 | 19 | | | | | | |
| 20 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 | 20 | | | | | | |
| 21 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 | 21 | | | | | | |
| 22 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 | 22 | | | | | | |
| 23 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 | 23 | | | | | | |

| | | | α | | | | | | | | | |
|-----|-------|-------|-------|-------|-------|-----|--|--|--|--|--|--|
| ν | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 | ν | | | | | | |
| 24 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 | 24 | | | | | | |
| 25 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 | 25 | | | | | | |
| 26 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 | 26 | | | | | | |
| 27 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 | 27 | | | | | | |
| 28 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 | 28 | | | | | | |
| 29 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 | 29 | | | | | | |
| 30 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 | 30 | | | | | | |
| 40 | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 | 40 | | | | | | |
| 50 | 1.299 | 1.676 | 2.009 | 2.403 | 2.678 | 50 | | | | | | |
| 60 | 1.296 | 1.671 | 2.000 | 2.390 | 2.660 | 60 | | | | | | |
| 70 | 1.294 | 1.667 | 1.994 | 2.381 | 2.648 | 70 | | | | | | |
| 80 | 1.292 | 1.664 | 1.990 | 2.374 | 2.639 | 80 | | | | | | |
| 90 | 1.291 | 1.662 | 1.987 | 2.368 | 2.632 | 90 | | | | | | |
| 100 | 1.290 | 1.660 | 1.984 | 2.364 | 2.626 | 100 | | | | | | |
| 120 | 1.289 | 1.658 | 1.980 | 2.358 | 2.617 | 120 | | | | | | |
| ∞ | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | 8 | | | | | | |

Appendix 15 Chi-Square Distribution Table

| | $\chi^{2}_{a,v}$ | | | | | | | | | | | | | |
|----|---------------------------------------------------------------|---------|---------|---------|---------|--------|--------|--------|--------|--------|----|--|--|--|
| | | | 1 | 1 | 6 | χ | | | | r | | | | |
| ν | 0.995 0.990 0.975 0.950 0.900 0.100 0.050 0.025 0.010 0.005 | | | | | | | | | | | | | |
| 1 | 0.00004 | 0.00016 | 0.00098 | 0.00393 | 0.01579 | 2.7055 | 3.8415 | 5.0239 | 6.6349 | 7.8794 | 1 | | | |
| 2 | 0.010 | 0.020 | 0.051 | 0.103 | 0.211 | 4.605 | 5.991 | 7.378 | 9.210 | 10.597 | 2 | | | |
| 3 | 0.072 0.115 0.216 0.352 0.584 6.251 7.815 9.348 11.345 12.838 | | | | | | | | | | | | | |
| 4 | | | | | | | | | | | | | | |
| 5 | | | | | | | | | | | | | | |
| 6 | | | | | | | | | | | | | | |
| 7 | 0.989 | 1.239 | 1.690 | 2.167 | 2.833 | 12.017 | 14.067 | 16.013 | 18.475 | 20.278 | 7 | | | |
| 8 | 1.344 | 1.646 | 2.180 | 2.733 | 3.490 | 13.362 | 15.507 | 17.535 | 20.090 | 21.955 | 8 | | | |
| 9 | 1.735 | 2.088 | 2.700 | 3.325 | 4.168 | 14.684 | 16.919 | 19.023 | 21.666 | 23.589 | 9 | | | |
| 10 | 2.156 | 2.558 | 3.247 | 3.940 | 4.865 | 15.987 | 18.307 | 20.483 | 23.209 | 25.188 | 10 | | | |
| 11 | 2.603 | 3.053 | 3.816 | 4.575 | 5.578 | 17.275 | 19.675 | 21.920 | 24.725 | 26.757 | 11 | | | |
| 12 | 3.074 | 3.571 | 4.404 | 5.226 | 6.304 | 18.549 | 21.026 | 23.337 | 26.217 | 28.300 | 12 | | | |
| 13 | 3.565 | 4.107 | 5.009 | 5.892 | 7.042 | 19.812 | 22.362 | 24.736 | 27.688 | 29.819 | 13 | | | |
| 14 | 4.075 | 4.660 | 5.629 | 6.571 | 7.790 | 21.064 | 23.685 | 26.119 | 29.141 | 31.319 | 14 | | | |
| 15 | 4.601 | 5.229 | 6.262 | 7.261 | 8.547 | 22.307 | 24.996 | 27.488 | 30.578 | 32.801 | 15 | | | |
| 16 | 5.142 | 5.812 | 6.908 | 7.962 | 9.312 | 23.542 | 26.296 | 28.845 | 32.000 | 34.267 | 16 | | | |
| 17 | 5.697 | 6.408 | 7.564 | 8.672 | 10.085 | 24.769 | 27.587 | 30.191 | 33.409 | 35.718 | 17 | | | |
| 18 | 6.265 | 7.015 | 8.231 | 9.390 | 10.865 | 25.989 | 28.869 | 31.526 | 34.805 | 37.156 | 18 | | | |
| 19 | 6.844 | 7.633 | 8.907 | 10.117 | 11.651 | 27.204 | 30.144 | 32.852 | 36.191 | 38.582 | 19 | | | |
| 20 | 7.434 | 8.260 | 9.591 | 10.851 | 12.443 | 28.412 | 31.410 | 34.170 | 37.566 | 39.997 | 20 | | | |
| 21 | 8.034 | 8.897 | 10.283 | 11.591 | 13.240 | 29.615 | 32.671 | 35.479 | 38.932 | 41.401 | 21 | | | |
| 22 | 8.643 | 9.542 | 10.982 | 12.338 | 14.041 | 30.813 | 33.924 | 36.781 | 40.289 | 42.796 | 22 | | | |
| 23 | 9.260 | 10.196 | 11.689 | 13.091 | 14.848 | 32.007 | 35.172 | 38.076 | 41.638 | 44.181 | 23 | | | |
| 24 | 9.886 | 10.856 | 12.401 | 13.848 | 15.659 | 33.196 | 36.415 | 39.364 | 42.980 | 45.559 | 24 | | | |

| | \sim \sim ν 0.995 0.990 0.975 0.950 0.900 0.100 0.050 0.025 0.010 0.005 | | | | | | | | | | | | |
|-----|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------|--------|--------|--------|---------|---------|---------|---------|---------|-----|--|--|
| | | | | | | | | | | | | | |
| 25 | | | | | | | | | | | | | |
| | 26 11.160 12.198 13.844 15.379 17.292 35.563 38.885 41.923 45.642 48.290 27 11.000 10.070 10.015 10.015 10.015 10.015 | | | | | | | | | | | | |
| | 27 11.808 12.879 14.573 16.151 18.114 36.741 40.113 43.195 46.963 49.645 | | | | | | | | | | | | |
| 28 | 28 12.461 13.565 15.308 16.928 18.939 37.916 41.337 44.461 48.278 50.993 | | | | | | | | | | | | |
| 29 | 29 13.121 14.256 16.047 17.708 19.768 39.087 42.557 45.722 49.588 52.336 | | | | | | | | | | | | |
| 30 | 13.787 | 14.953 | 16.791 | 18.493 | 20.599 | 40.256 | 43.773 | 46.979 | 50.892 | 53.672 | 30 | | |
| 35 | 17.192 | 18.509 | 20.569 | 22.465 | 24.797 | 46.059 | 49.802 | 53.203 | 57.342 | 60.275 | 35 | | |
| 40 | 20.707 | 22.164 | 24.433 | 26.509 | 29.051 | 51.805 | 55.758 | 59.342 | 63.691 | 66.766 | 40 | | |
| 45 | 24.311 | 25.901 | 28.366 | 30.612 | 33.350 | 57.505 | 61.656 | 65.410 | 69.957 | 73.166 | 45 | | |
| 50 | 27.991 | 29.707 | 32.357 | 34.764 | 37.689 | 63.167 | 67.505 | 71.420 | 76.154 | 79.490 | 50 | | |
| 55 | 31.735 | 33.570 | 36.398 | 38.958 | 42.060 | 68.796 | 73.311 | 77.380 | 82.292 | 85.749 | 55 | | |
| 60 | 35.534 | 37.485 | 40.482 | 43.188 | 46.459 | 74.397 | 79.082 | 83.298 | 88.379 | 91.952 | 60 | | |
| 65 | 39.383 | 41.444 | 44.603 | 47.450 | 50.883 | 79.973 | 84.821 | 89.177 | 94.422 | 98.105 | 65 | | |
| 70 | 43.275 | 45.442 | 48.758 | 51.739 | 55.329 | 85.527 | 90.531 | 95.023 | 100.425 | 104.215 | 70 | | |
| 75 | 47.206 | 49.475 | 52.942 | 56.054 | 59.795 | 91.061 | 96.217 | 100.839 | 106.393 | 110.286 | 75 | | |
| 80 | 51.172 | 53.540 | 57.153 | 60.391 | 64.278 | 96.578 | 101.879 | 106.629 | 112.329 | 116.321 | 80 | | |
| 85 | 55.170 | 57.634 | 61.389 | 64.749 | 68.777 | 102.079 | 107.522 | 112.393 | 118.236 | 122.325 | 85 | | |
| 90 | 59.196 | 61.754 | 65.647 | 69.126 | 73.291 | 107.565 | 113.145 | 118.136 | 124.116 | 128.299 | 90 | | |
| 95 | 63.250 | 65.898 | 69.925 | 73.520 | 77.818 | 113.038 | 118.752 | 123.858 | 129.973 | 134.247 | 95 | | |
| 100 | 67.328 | 70.065 | 74.222 | 77.929 | 82.358 | 118.498 | 124.342 | 129.561 | 135.807 | 140.169 | 100 | | |

Appendix 16 *F*(0.99) Distribution Table

| | | | | | | | | 7 | <i>F</i> _{1-a} | 4 | 1-α | | | | | | | | |
|----------------------------------------|-------------------|------|------|------|------|------|------|-------|-------------------------|------------------------|----------|--------|------|------|------|------|------|------|----------|
| | | | | | | | De | grees | of free | | or the r | numera | ator | | | | | | <u> </u> |
| | (v ₂) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | (v ₁) 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | (v2) |
| | 1 | 0.00 | 0.01 | 0.03 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 | 0.09 | 0.10 | 0.10 | 0.11 | 0.11 | 0.11 | 0.12 | 0.12 | 0.12 | 1 |
| | 2 | 0.00 | 0.01 | 0.03 | 0.06 | 0.08 | 0.09 | 0.10 | 0.12 | 0.12 | 0.13 | 0.14 | 0.14 | 0.15 | 0.15 | 0.16 | 0.16 | 0.16 | 2 |
| | 3 | 0.00 | 0.01 | 0.03 | 0.06 | 0.08 | 0.10 | 0.12 | 0.13 | 0.14 | 0.15 | 0.16 | 0.17 | 0.17 | 0.18 | 0.18 | 0.19 | 0.19 | 3 |
| | 4 | 0.00 | 0.01 | 0.03 | 0.06 | 0.09 | 0.11 | 0.13 | 0.14 | 0.16 | 0.17 | 0.18 | 0.18 | 0.19 | 0.20 | 0.20 | 0.21 | 0.21 | 4 |
| | 5 | 0.00 | 0.01 | 0.04 | 0.06 | 0.09 | 0.11 | 0.13 | 0.15 | 0.17 | 0.18 | 0.19 | 0.20 | 0.21 | 0.21 | 0.22 | 0.23 | 0.23 | 5 |
| | 6 | 0.00 | 0.01 | 0.04 | 0.07 | 0.09 | 0.12 | 0.14 | 0.16 | 0.17 | 0.19 | 0.20 | 0.21 | 0.22 | 0.22 | 0.23 | 0.24 | 0.24 | 6 |
| | 7 | 0.00 | 0.01 | 0.04 | 0.07 | 0.10 | 0.12 | 0.14 | 0.16 | 0.18 | 0.19 | 0.20 | 0.22 | 0.23 | 0.23 | 0.24 | 0.25 | 0.25 | 7 |
| | 8 | 0.00 | 0.01 | 0.04 | 0.07 | 0.10 | 0.12 | 0.15 | 0.17 | 0.18 | 0.20 | 0.21 | 0.22 | 0.23 | 0.24 | 0.25 | 0.26 | 0.26 | 8 |
| | 9 | 0.00 | 0.01 | 0.04 | 0.07 | 0.10 | 0.13 | 0.15 | 0.17 | 0.19 | 0.20 | 0.22 | 0.23 | 0.24 | 0.25 | 0.26 | 0.26 | 0.27 | 9 |
| | 10 | 0.00 | 0.01 | 0.04 | 0.07 | 0.10 | 0.13 | 0.15 | 0.17 | 0.19 | 0.21 | 0.22 | 0.23 | 0.24 | 0.25 | 0.26 | 0.27 | 0.28 | 10 |
| | 11 | 0.00 | 0.01 | 0.04 | 0.07 | 0.10 | 0.13 | 0.15 | 0.17 | 0.19 | 0.21 | 0.22 | 0.24 | 0.25 | 0.26 | 0.27 | 0.28 | 0.28 | 11 |
| | 12 | 0.00 | 0.01 | 0.04 | 0.07 | 0.10 | 0.13 | 0.15 | 0.18 | 0.20 | 0.21 | 0.23 | 0.24 | 0.25 | 0.26 | 0.27 | 0.28 | 0.29 | 12 |
| ator | 13 | 0.00 | 0.01 | 0.04 | 0.07 | 0.10 | 0.13 | 0.16 | 0.18 | 0.20 | 0.22 | 0.23 | 0.24 | 0.26 | 0.27 | 0.28 | 0.29 | 0.29 | 13 |
| min | 14 | 0.00 | 0.01 | 0.04 | 0.07 | 0.10 | 0.13 | 0.16 | 0.18 | 0.20 | 0.22 | 0.23 | 0.25 | 0.26 | 0.27 | 0.28 | 0.29 | 0.30 | 14 |
| denc | 15 | 0.00 | 0.01 | 0.04 | 0.07 | 0.10 | 0.13 | 0.16 | 0.18 | 0.20 | 0.22 | 0.24 | 0.25 | 0.26 | 0.27 | 0.28 | 0.29 | 0.30 | 15 |
| the | 16 | 0.00 | 0.01 | 0.04 | 0.07 | 0.10 | 0.13 | 0.16 | 0.18 | 0.20 | 0.22 | 0.24 | 0.25 | 0.26 | 0.28 | 0.29 | 0.30 | 0.31 | 16 |
| for | 17 | 0.00 | 0.01 | 0.04 | 0.07 | 0.10 | 0.13 | 0.16 | 0.18 | 0.20 | 0.22 | 0.24 | 0.25 | 0.27 | 0.28 | 0.29 | 0.30 | 0.31 | 17 |
| mog | 18 | 0.00 | 0.01 | 0.04 | 0.07 | 0.10 | 0.13 | 0.16 | 0.18 | 0.21 | 0.22 | 0.24 | 0.26 | 0.27 | 0.28 | 0.29 | 0.30 | 0.31 | 18 |
| free | 19 | 0.00 | 0.01 | 0.04 | 0.07 | 0.10 | 0.13 | 0.16 | 0.19 | 0.21 | 0.23 | 0.24 | 0.26 | 0.27 | 0.28 | 0.29 | 0.30 | 0.31 | 19 |
| s of 1 | 20 | 0.00 | 0.01 | 0.04 | 0.07 | 0.10 | 0.14 | 0.16 | 0.19 | 0.21 | 0.23 | 0.24 | 0.26 | 0.27 | 0.29 | 0.30 | 0.31 | 0.32 | 20 |
| Degrees of freedom for the denominator | 21 | 0.00 | 0.01 | 0.04 | 0.07 | 0.10 | 0.14 | 0.16 | 0.19 | 0.21 | 0.23 | 0.25 | 0.26 | 0.27 | 0.29 | 0.30 | 0.31 | 0.32 | 21 |
| Deg | 22 | 0.00 | 0.01 | 0.04 | 0.07 | 0.11 | 0.14 | 0.16 | 0.19 | 0.21 | 0.23 | 0.25 | 0.26 | 0.28 | 0.29 | 0.30 | 0.31 | 0.32 | 22 |
| | 23 | 0.00 | 0.01 | 0.04 | 0.07 | 0.11 | 0.14 | 0.16 | 0.19 | 0.21 | 0.23 | 0.25 | 0.26 | 0.28 | 0.29 | 0.30 | 0.31 | 0.32 | 23 |
| | 24 | 0.00 | 0.01 | 0.04 | 0.07 | 0.11 | 0.14 | 0.16 | 0.19 | 0.21 | 0.23 | 0.25 | 0.26 | 0.28 | 0.29 | 0.30 | 0.31 | 0.32 | 24 |
| | 25 | 0.00 | 0.01 | 0.04 | 0.07 | 0.11 | 0.14 | 0.17 | 0.19 | 0.21 | 0.23 | 0.25 | 0.27 | 0.28 | 0.29 | 0.31 | 0.32 | 0.33 | 25 |
| | 26 | 0.00 | 0.01 | 0.04 | 0.07 | 0.11 | 0.14 | 0.17 | 0.19 | 0.21 | 0.23 | 0.25 | 0.27 | 0.28 | 0.29 | 0.31 | 0.32 | 0.33 | 26 |
| | 27 | 0.00 | 0.01 | 0.04 | 0.07 | 0.11 | 0.14 | 0.17 | 0.19 | 0.21 | 0.23 | 0.25 | 0.27 | 0.28 | 0.30 | 0.31 | 0.32 | 0.33 | 27 |
| | 28 | 0.00 | 0.01 | 0.04 | 0.07 | 0.11 | 0.14 | 0.17 | 0.19 | 0.21 | 0.23 | 0.25 | 0.27 | 0.28 | 0.30 | 0.31 | 0.32 | 0.33 | 28 |
| | 29 | 0.00 | 0.01 | 0.04 | 0.07 | 0.11 | 0.14 | 0.17 | 0.19 | 0.21 | 0.23 | 0.25 | 0.27 | 0.28 | 0.30 | 0.31 | 0.32 | 0.33 | 29 |
| | 30 | 0.00 | 0.01 | 0.04 | 0.07 | 0.11 | 0.14 | 0.17 | 0.19 | 0.22 | 0.24 | 0.25 | 0.27 | 0.29 | 0.30 | 0.31 | 0.32 | 0.33 | 30 |
| | 40 | 0.00 | 0.01 | 0.04 | 0.07 | 0.11 | 0.14 | 0.17 | 0.20 | 0.22 | 0.24 | 0.26 | 0.28 | 0.29 | 0.31 | 0.32 | 0.33 | 0.34 | 40 |
| | 60 | 0.00 | 0.01 | 0.04 | 0.07 | 0.11 | 0.14 | 0.17 | 0.20 | 0.22 | 0.24 | 0.26 | 0.28 | 0.30 | 0.31 | 0.33 | 0.34 | 0.35 | 60 |
| | 100 | 0.00 | 0.01 | 0.04 | 0.07 | 0.11 | 0.14 | 0.17 | 0.20 | 0.23 | 0.25 | 0.27 | 0.29 | 0.31 | 0.32 | 0.34 | 0.35 | 0.36 | 100 |
| | 00 | 0.00 | 0.01 | 0.04 | 0.07 | 0.11 | 0.15 | 0.18 | 0.21 | 0.23 | 0.26 | 0.28 | 0.30 | 0.32 | 0.33 | 0.35 | 0.36 | 0.38 | 00 |



| | | | | | | | _ | F_{1-a} | ,v ₁ ,v ₂ | | | _ | | | | | | | |
|-----------------------------------------|------|------|------|------|------|------|---------|-----------|---------------------------------|----------|-------|------|------|------|------|------|------|-------------------|--------------------|
| | | | | | | De | grees (| of free | dom fo | or the r | umera | ator | | | | | | | |
| | | | | | | | | | (v ₁) | | | | | | | | | | |
| (v ₂) | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 40 | 60 | 100 | 8 | (v ₂) | |
| 1 | 0.12 | 0.12 | 0.12 | 0.12 | 0.13 | 0.13 | 0.13 | 0.13 | 0.13 | 0.13 | 0.13 | 0.13 | 0.13 | 0.14 | 0.14 | 0.15 | 0.15 | 1 | |
| 2 | 0.17 | 0.17 | 0.17 | 0.17 | 0.17 | 0.18 | 0.18 | 0.18 | 0.18 | 0.18 | 0.18 | 0.18 | 0.19 | 0.19 | 0.20 | 0.21 | 0.22 | 2 | |
| 3 | 0.20 | 0.20 | 0.20 | 0.21 | 0.21 | 0.21 | 0.21 | 0.21 | 0.22 | 0.22 | 0.22 | 0.22 | 0.22 | 0.23 | 0.24 | 0.25 | 0.26 | 3 | |
| 4 | 0.22 | 0.22 | 0.23 | 0.23 | 0.23 | 0.23 | 0.24 | 0.24 | 0.24 | 0.24 | 0.25 | 0.25 | 0.25 | 0.26 | 0.27 | 0.28 | 0.30 | 4 | |
| 5 | 0.24 | 0.24 | 0.24 | 0.25 | 0.25 | 0.25 | 0.26 | 0.26 | 0.26 | 0.26 | 0.27 | 0.27 | 0.27 | 0.28 | 0.30 | 0.31 | 0.33 | 5 | |
| 6 | 0.25 | 0.25 | 0.26 | 0.26 | 0.27 | 0.27 | 0.27 | 0.28 | 0.28 | 0.28 | 0.28 | 0.29 | 0.29 | 0.30 | 0.32 | 0.33 | 0.36 | 6 | |
| 7 | 0.26 | 0.27 | 0.27 | 0.27 | 0.28 | 0.28 | 0.29 | 0.29 | 0.29 | 0.30 | 0.30 | 0.30 | 0.30 | 0.32 | 0.34 | 0.35 | 0.38 | 7 | |
| 8 | 0.27 | 0.28 | 0.28 | 0.29 | 0.29 | 0.29 | 0.30 | 0.30 | 0.30 | 0.31 | 0.31 | 0.31 | 0.32 | 0.33 | 0.35 | 0.37 | 0.40 | 8 | |
| 9 | 0.28 | 0.28 | 0.29 | 0.29 | 0.30 | 0.30 | 0.31 | 0.31 | 0.31 | 0.32 | 0.32 | 0.32 | 0.33 | 0.35 | 0.37 | 0.39 | 0.42 | 9 | |
| 10 | 0.29 | 0.29 | 0.30 | 0.30 | 0.31 | 0.31 | 0.32 | 0.32 | 0.32 | 0.33 | 0.33 | 0.33 | 0.34 | 0.36 | 0.38 | 0.40 | 0.43 | 10 | |
| 11 | 0.29 | 0.30 | 0.30 | 0.31 | 0.31 | 0.32 | 0.32 | 0.33 | 0.33 | 0.33 | 0.34 | 0.34 | 0.34 | 0.37 | 0.39 | 0.41 | 0.44 | 11 | |
| 12 | 0.30 | 0.30 | 0.31 | 0.32 | 0.32 | 0.33 | 0.33 | 0.33 | 0.34 | 0.34 | 0.35 | 0.35 | 0.35 | 0.38 | 0.40 | 0.42 | 0.46 | 12 | |
| 13 | 0.30 | 0.31 | 0.31 | 0.32 | 0.33 | 0.33 | 0.34 | 0.34 | 0.34 | 0.35 | 0.35 | 0.36 | 0.36 | 0.38 | 0.41 | 0.43 | 0.47 | 13 | Deg |
| 14 | 0.31 | 0.31 | 0.32 | 0.33 | 0.33 | 0.34 | 0.34 | 0.35 | 0.35 | 0.35 | 0.36 | 0.36 | 0.36 | 0.39 | 0.42 | 0.44 | 0.48 | 14 | Degrees |
| 15 | 0.31 | 0.32 | 0.32 | 0.33 | 0.34 | 0.34 | 0.35 | 0.35 | 0.36 | 0.36 | 0.36 | 0.37 | 0.37 | 0.40 | 0.43 | 0.45 | 0.49 | 15 | 오 |
| 16 | 0.31 | 0.32 | 0.33 | 0.33 | 0.34 | 0.35 | 0.35 | 0.36 | 0.36 | 0.36 | 0.37 | 0.37 | 0.38 | 0.40 | 0.43 | 0.46 | 0.50 | 16 | freedom |
| 17 | 0.32 | 0.32 | 0.33 | 0.34 | 0.34 | 0.35 | 0.35 | 0.36 | 0.36 | 0.37 | 0.37 | 0.38 | 0.38 | 0.41 | 0.44 | 0.46 | 0.51 | 17 | dom |
| 18 | 0.32 | 0.33 | 0.33 | 0.34 | 0.35 | 0.35 | 0.36 | 0.36 | 0.37 | 0.37 | 0.38 | 0.38 | 0.38 | 0.41 | 0.44 | 0.47 | 0.52 | 18 | for the denominato |
| 19 | 0.32 | 0.33 | 0.34 | 0.34 | 0.35 | 0.36 | 0.36 | 0.37 | 0.37 | 0.38 | 0.38 | 0.38 | 0.39 | 0.42 | 0.45 | 0.48 | 0.52 | 19 | theo |
| 20 | 0.32 | 0.33 | 0.34 | 0.35 | 0.35 | 0.36 | 0.37 | 0.37 | 0.38 | 0.38 | 0.38 | 0.39 | 0.39 | 0.42 | 0.45 | 0.48 | 0.53 | 20 | denc |
| 21 | 0.33 | 0.34 | 0.34 | 0.35 | 0.36 | 0.36 | 0.37 | 0.37 | 0.38 | 0.38 | 0.39 | 0.39 | 0.40 | 0.43 | 0.46 | 0.49 | 0.54 | 21 | min |
| 22 | 0.33 | 0.34 | 0.35 | 0.35 | 0.36 | 0.37 | 0.37 | 0.38 | 0.38 | 0.39 | 0.39 | 0.40 | 0.40 | 0.43 | 0.46 | 0.49 | 0.55 | 22 | ator |
| 23 | 0.33 | 0.34 | 0.35 | 0.35 | 0.36 | 0.37 | 0.37 | 0.38 | 0.38 | 0.39 | 0.39 | 0.40 | 0.40 | 0.43 | 0.47 | 0.50 | 0.55 | 23 | |
| 24 | 0.33 | 0.34 | 0.35 | 0.36 | 0.36 | 0.37 | 0.38 | 0.38 | 0.39 | 0.39 | 0.40 | 0.40 | 0.41 | 0.44 | 0.47 | 0.50 | 0.56 | 24 | |
| 25 | 0.34 | 0.34 | 0.35 | 0.36 | 0.37 | 0.37 | 0.38 | 0.38 | 0.39 | 0.39 | 0.40 | 0.40 | 0.41 | 0.44 | 0.48 | 0.51 | 0.56 | 25 | |
| 26 | 0.34 | 0.35 | 0.35 | 0.36 | 0.37 | 0.37 | 0.38 | 0.39 | 0.39 | 0.40 | 0.40 | 0.41 | 0.41 | 0.44 | 0.48 | 0.51 | 0.57 | 26 | |
| 27 | 0.34 | 0.35 | 0.36 | 0.36 | 0.37 | 0.38 | 0.38 | 0.39 | 0.39 | 0.40 | 0.40 | 0.41 | 0.41 | 0.45 | 0.48 | 0.52 | 0.57 | 27 | |
| 28 | 0.34 | 0.35 | 0.36 | 0.36 | 0.37 | 0.38 | 0.38 | 0.39 | 0.40 | 0.40 | 0.41 | 0.41 | 0.41 | 0.45 | 0.49 | 0.52 | 0.58 | 28 | |
| 29 | 0.34 | 0.35 | 0.36 | 0.37 | 0.37 | 0.38 | 0.39 | 0.39 | 0.40 | 0.40 | 0.41 | 0.41 | 0.42 | 0.45 | 0.49 | 0.52 | 0.58 | 29 | |
| 30 | 0.34 | 0.35 | 0.36 | 0.37 | 0.37 | 0.38 | 0.39 | 0.39 | 0.40 | 0.40 | 0.41 | 0.41 | 0.42 | 0.45 | 0.49 | 0.53 | 0.59 | 30 | |
| 40 | 0.35 | 0.36 | 0.37 | 0.38 | 0.39 | 0.39 | 0.40 | 0.41 | 0.41 | 0.42 | 0.42 | 0.43 | 0.43 | 0.47 | 0.52 | 0.56 | 0.63 | 40 | |
| 60 | 0.36 | 0.37 | 0.38 | 0.39 | 0.40 | 0.41 | 0.42 | 0.42 | 0.43 | 0.44 | 0.44 | 0.45 | 0.45 | 0.50 | 0.54 | 0.59 | 0.68 | 60 | |
| 100 | 0.37 | 0.38 | 0.39 | 0.40 | 0.41 | 0.42 | 0.43 | 0.44 | 0.44 | 0.45 | 0.46 | 0.46 | 0.47 | 0.52 | 0.57 | 0.63 | 0.74 | 100 | |
| ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~ | 0.39 | 0.40 | 0.41 | 0.42 | 0.43 | 0.44 | 0.45 | 0.46 | 0.47 | 0.48 | 0.48 | 0.49 | 0.50 | 0.55 | 0.62 | 0.70 | 1.00 | ∞ | |

Appendix 17 *F*(0.975) Distribution Table

| | | | | | | | | _ | F _{1-a} | ł | 1-α | | | | | | | | |
|----------------------------------------|-------------------|------|------|------------------|-----------|------------------|--------------|-------|------------------|-------------------|----------|--------|------|------|------|------|------|------|-------------------|
| | | | | | | | De | grees | of free | dom fo | or the r | numera | ator | | | | | | |
| | () | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | (v ₁) | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | () |
| | (v ₂) | 0.00 | 2 | 3 0.06 | 4 0.08 | 5 0.10 | 0 .11 | 0.12 | o 0.13 | 9 0.14 | 0.14 | 0.15 | 0.15 | 0.16 | 0.16 | 0.16 | 0.16 | 0.17 | (v ₂) |
| | 2 | 0.00 | 0.00 | 0.00 | 0.00 | 0.12 | 0.14 | 0.12 | 0.17 | 0.17 | 0.14 | 0.19 | 0.10 | 0.20 | 0.21 | 0.21 | 0.21 | 0.22 | 2 |
| | 3 | 0.00 | 0.03 | 0.06 | 0.10 | 0.13 | 0.15 | 0.17 | 0.18 | 0.20 | 0.21 | 0.22 | 0.22 | 0.23 | 0.24 | 0.24 | 0.25 | 0.25 | 3 |
| | 4 | 0.00 | 0.03 | 0.07 | 0.10 | 0.14 | 0.16 | 0.18 | 0.20 | 0.21 | 0.22 | 0.23 | 0.24 | 0.25 | 0.26 | 0.26 | 0.27 | 0.27 | 4 |
| | 5 | 0.00 | 0.03 | 0.07 | 0.11 | 0.14 | 0.17 | 0.19 | 0.21 | 0.22 | 0.24 | 0.25 | 0.26 | 0.27 | 0.27 | 0.28 | 0.29 | 0.29 | 5 |
| | 6 | 0.00 | 0.03 | 0.07 | 0.11 | 0.14 | 0.17 | 0.20 | 0.21 | 0.23 | 0.25 | 0.26 | 0.27 | 0.28 | 0.29 | 0.29 | 0.30 | 0.31 | 6 |
| | 7 | 0.00 | 0.03 | 0.07 | 0.11 | 0.15 | 0.18 | 0.20 | 0.22 | 0.24 | 0.25 | 0.27 | 0.28 | 0.29 | 0.30 | 0.30 | 0.31 | 0.32 | 7 |
| | 8 | 0.00 | 0.03 | 0.07 | 0.11 | 0.15 | 0.18 | 0.20 | 0.23 | 0.24 | 0.26 | 0.27 | 0.28 | 0.30 | 0.30 | 0.31 | 0.32 | 0.33 | 8 |
| | 9 | 0.00 | 0.03 | 0.07 | 0.11 | 0.15 | 0.18 | 0.21 | 0.23 | 0.25 | 0.26 | 0.28 | 0.29 | 0.30 | 0.31 | 0.32 | 0.33 | 0.34 | 9 |
| | 10 | 0.00 | 0.03 | 0.07 | 0.11 | 0.15 | 0.18 | 0.21 | 0.23 | 0.25 | 0.27 | 0.28 | 0.30 | 0.31 | 0.32 | 0.33 | 0.33 | 0.34 | 10 |
| | 11 | 0.00 | 0.03 | 0.07 | 0.11 | 0.15 | 0.18 | 0.21 | 0.24 | 0.26 | 0.27 | 0.29 | 0.30 | 0.31 | 0.32 | 0.33 | 0.34 | 0.35 | 11 |
| | 12 | 0.00 | 0.03 | 0.07 | 0.11 | 0.15 | 0.19 | 0.21 | 0.24 | 0.26 | 0.28 | 0.29 | 0.31 | 0.32 | 0.33 | 0.34 | 0.35 | 0.35 | 12 |
| ator | 13 | 0.00 | 0.03 | 0.07 | 0.11 | 0.15 | 0.19 | 0.22 | 0.24 | 0.26 | 0.28 | 0.29 | 0.31 | 0.32 | 0.33 | 0.34 | 0.35 | 0.36 | 13 |
| min | 14 | 0.00 | 0.03 | 0.07 | 0.12 | 0.15 | 0.19 | 0.22 | 0.24 | 0.26 | 0.28 | 0.30 | 0.31 | 0.32 | 0.34 | 0.35 | 0.35 | 0.36 | 14 |
| denc | 15 | 0.00 | 0.03 | 0.07 | 0.12 | 0.16 | 0.19 | 0.22 | 0.24 | 0.27 | 0.28 | 0.30 | 0.31 | 0.33 | 0.34 | 0.35 | 0.36 | 0.37 | 15 |
| Degrees of freedom for the denominator | 16 | 0.00 | 0.03 | 0.07 | 0.12 | 0.16 | 0.19 | 0.22 | 0.25 | 0.27 | 0.29 | 0.30 | 0.32 | 0.33 | 0.34 | 0.35 | 0.36 | 0.37 | 16 |
| for | 17 | 0.00 | 0.03 | 0.07 | 0.12 | 0.16 | 0.19 | 0.22 | 0.25 | 0.27 | 0.29 | 0.30 | 0.32 | 0.33 | 0.34 | 0.36 | 0.37 | 0.37 | 17 |
| dom | 18 | 0.00 | 0.03 | 0.07 | 0.12 | 0.16 | 0.19 | 0.22 | 0.25 | 0.27 | 0.29 | 0.31 | 0.32 | 0.34 | 0.35 | 0.36 | 0.37 | 0.38 | 18 |
| free | 19 | 0.00 | 0.03 | 0.07 | 0.12 | 0.16 | 0.19 | 0.22 | 0.25 | 0.27 | 0.29 | 0.31 | 0.32 | 0.34 | 0.35 | 0.36 | 0.37 | 0.38 | 19 |
| s of | 20 | 0.00 | 0.03 | 0.07 | 0.12 | 0.16 | 0.19 | 0.22 | 0.25 | 0.27 | 0.29 | 0.31 | 0.33 | 0.34 | 0.35 | 0.36 | 0.37 | 0.38 | 20 |
| gree | 21 | 0.00 | 0.03 | 0.07 | 0.12 | 0.16 | 0.19 | 0.22 | 0.25 | 0.27 | 0.29 | 0.31 | 0.33 | 0.34 | 0.35 | 0.36 | 0.38 | 0.38 | 21 |
| De | 22 | 0.00 | 0.03 | 0.07 | 0.12 | 0.16 | 0.19 | 0.23 | 0.25 | 0.27 | 0.30 | 0.31 | 0.33 | 0.34 | 0.36 | 0.37 | 0.38 | 0.39 | 22 |
| | 23 | 0.00 | 0.03 | 0.07 | 0.12 | 0.16 | 0.19 | 0.23 | 0.25 | 0.28 | 0.30 | 0.31 | 0.33 | 0.34 | 0.36 | 0.37 | 0.38 | 0.39 | 23 |
| | 24 | 0.00 | 0.03 | 0.07 | 0.12 | 0.16 | 0.20 | 0.23 | 0.25 | 0.28 | 0.30 | 0.32 | 0.33 | 0.35 | 0.36 | 0.37 | 0.38 | 0.39 | 24 |
| | 25 | 0.00 | 0.03 | 0.07 | | 0.16 | | 0.23 | 0.25 | | 0.30 | 0.32 | 0.33 | 0.35 | 0.36 | 0.37 | 0.38 | 0.39 | 25 |
| | 26 | 0.00 | 0.03 | 0.07 | 0.12 | 0.16 | 0.20 | 0.23 | 0.25 | 0.28 | 0.30 | 0.32 | 0.33 | 0.35 | 0.36 | 0.37 | 0.38 | 0.39 | 26 |
| | 27 | 0.00 | 0.03 | 0.07 | 0.12 | 0.16 | 0.20 | 0.23 | 0.26 | 0.28 | 0.30 | 0.32 | 0.33 | 0.35 | 0.36 | 0.37 | 0.39 | 0.40 | 27 |
| | 28 | 0.00 | 0.03 | 0.07 | 0.12 | 0.16 | 0.20 | 0.23 | 0.26 | 0.28 | 0.30 | 0.32 | 0.34 | 0.35 | 0.36 | 0.38 | 0.39 | 0.40 | 28 |
| | 29 | 0.00 | 0.03 | 0.07 | 0.12 | 0.16 | 0.20 | 0.23 | 0.26 | 0.28 | 0.30 | 0.32 | 0.34 | 0.35 | 0.36 | 0.38 | 0.39 | 0.40 | 29 |
| | 30 | 0.00 | 0.03 | 0.07 | 0.12 | 0.16 | 0.20 | 0.23 | 0.26 | 0.28 | 0.30 | 0.32 | 0.34 | 0.35 | 0.37 | 0.38 | 0.39 | 0.40 | 30 |
| | 40 60 | 0.00 | 0.03 | 0.07 | 0.12 | 0.16 | 0.20 | 0.23 | 0.26 | 0.29 | 0.31 | 0.33 | 0.34 | 0.36 | 0.37 | 0.39 | 0.40 | 0.41 | 40 60 |
| | 100 | 0.00 | 0.03 | 0.07 | 0.12 | 0.16 | 0.20 | 0.24 | 0.26 | 0.29 | 0.31 | 0.33 | 0.35 | 0.37 | 0.38 | 0.40 | 0.41 | 0.42 | 100 |
| | ∞ | 0.00 | 0.03 | 0.07 | 0.12 | 0.10 | 0.20 | 0.24 | 0.27 | 0.29 | 0.32 | 0.34 | 0.30 | 0.37 | 0.39 | 0.40 | 0.42 | 0.43 | ∞ |



| | | | | | | | | F_{1-a} | ,v ₁ ,v ₂ | | | | | | | | | | |
|-------------------|------|------|------|------|------|------|---------|-----------|---------------------------------|----------|--------|------|------|------|------|------|------|-------------------|-----------------------------|
| | | | | | | Deg | grees (| of free | dom fo | or the r | numera | tor | | | | | | | |
| | | | | | | | | | (ν_1) | | | | | | | | | | |
| (v ₂) | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 40 | 60 | 100 | ∞ | (v ₂) | |
| 1 | 0.17 | 0.17 | 0.17 | 0.17 | 0.17 | 0.17 | 0.17 | 0.18 | 0.18 | 0.18 | 0.18 | 0.18 | 0.18 | 0.18 | 0.19 | 0.19 | 0.20 | 1 | |
| 2 | 0.22 | 0.22 | 0.22 | 0.23 | 0.23 | 0.23 | 0.23 | 0.23 | 0.23 | 0.24 | 0.24 | 0.24 | 0.24 | 0.25 | 0.25 | 0.26 | 0.27 | 2 | |
| 3 | 0.25 | 0.26 | 0.26 | 0.26 | 0.26 | 0.27 | 0.27 | 0.27 | 0.27 | 0.27 | 0.28 | 0.28 | 0.28 | 0.29 | 0.30 | 0.31 | 0.32 | 3 | |
| 4 | 0.28 | 0.28 | 0.28 | 0.29 | 0.29 | 0.29 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.31 | 0.31 | 0.32 | 0.33 | 0.34 | 0.36 | 4 | |
| 5 | 0.30 | 0.30 | 0.30 | 0.31 | 0.31 | 0.31 | 0.32 | 0.32 | 0.32 | 0.32 | 0.33 | 0.33 | 0.33 | 0.34 | 0.36 | 0.37 | 0.39 | 5 | |
| 6 | 0.31 | 0.32 | 0.32 | 0.32 | 0.33 | 0.33 | 0.33 | 0.34 | 0.34 | 0.34 | 0.34 | 0.35 | 0.35 | 0.36 | 0.38 | 0.39 | 0.42 | 6 | |
| 7 | 0.32 | 0.33 | 0.33 | 0.34 | 0.34 | 0.34 | 0.35 | 0.35 | 0.35 | 0.36 | 0.36 | 0.36 | 0.36 | 0.38 | 0.40 | 0.41 | 0.44 | 7 | |
| 8 | 0.33 | 0.34 | 0.34 | 0.35 | 0.35 | 0.36 | 0.36 | 0.36 | 0.37 | 0.37 | 0.37 | 0.37 | 0.38 | 0.40 | 0.41 | 0.43 | 0.46 | 8 | |
| 9 | 0.34 | 0.35 | 0.35 | 0.36 | 0.36 | 0.37 | 0.37 | 0.37 | 0.38 | 0.38 | 0.38 | 0.39 | 0.39 | 0.41 | 0.43 | 0.45 | 0.47 | 9 | |
| 10 | 0.35 | 0.35 | 0.36 | 0.37 | 0.37 | 0.37 | 0.38 | 0.38 | 0.39 | 0.39 | 0.39 | 0.40 | 0.40 | 0.42 | 0.44 | 0.46 | 0.49 | 10 | |
| 11 | 0.36 | 0.36 | 0.37 | 0.37 | 0.38 | 0.38 | 0.39 | 0.39 | 0.39 | 0.40 | 0.40 | 0.40 | 0.41 | 0.43 | 0.45 | 0.47 | 0.50 | 11 | |
| 12 | 0.36 | 0.37 | 0.37 | 0.38 | 0.38 | 0.39 | 0.39 | 0.40 | 0.40 | 0.41 | 0.41 | 0.41 | 0.41 | 0.44 | 0.46 | 0.48 | 0.51 | 12 | |
| 13 | 0.37 | 0.37 | 0.38 | 0.38 | 0.39 | 0.40 | 0.40 | 0.40 | 0.41 | 0.41 | 0.42 | 0.42 | 0.42 | 0.44 | 0.47 | 0.49 | 0.53 | 13 | Deg |
| 14 | 0.37 | 0.38 | 0.38 | 0.39 | 0.40 | 0.40 | 0.41 | 0.41 | 0.41 | 0.42 | 0.42 | 0.42 | 0.43 | 0.45 | 0.48 | 0.50 | 0.54 | 14 | Degrees |
| 15 | 0.37 | 0.38 | 0.39 | 0.39 | 0.40 | 0.41 | 0.41 | 0.41 | 0.42 | 0.42 | 0.43 | 0.43 | 0.43 | 0.46 | 0.49 | 0.51 | 0.55 | 15 | 으 |
| 16 | 0.38 | 0.39 | 0.39 | 0.40 | 0.40 | 0.41 | 0.41 | 0.42 | 0.42 | 0.43 | 0.43 | 0.44 | 0.44 | 0.46 | 0.49 | 0.52 | 0.55 | 16 | freedom for the denominator |
| 17 | 0.38 | 0.39 | 0.40 | 0.40 | 0.41 | 0.41 | 0.42 | 0.42 | 0.43 | 0.43 | 0.44 | 0.44 | 0.44 | 0.47 | 0.50 | 0.52 | 0.56 | 17 | lo m |
| 18 | 0.39 | 0.39 | 0.40 | 0.41 | 0.41 | 0.42 | 0.42 | 0.43 | 0.43 | 0.44 | 0.44 | 0.44 | 0.45 | 0.47 | 0.50 | 0.53 | 0.57 | 18 | for t |
| 19 | 0.39 | 0.40 | 0.40 | 0.41 | 0.42 | 0.42 | 0.43 | 0.43 | 0.44 | 0.44 | 0.44 | 0.45 | 0.45 | 0.48 | 0.51 | 0.54 | 0.58 | 19 | heo |
| 20 | 0.39 | 0.40 | 0.41 | 0.41 | 0.42 | 0.42 | 0.43 | 0.43 | 0.44 | 0.44 | 0.45 | 0.45 | 0.46 | 0.48 | 0.51 | 0.54 | 0.59 | 20 | leno |
| 21 | 0.39 | 0.40 | 0.41 | 0.42 | 0.42 | 0.43 | 0.43 | 0.44 | 0.44 | 0.45 | 0.45 | 0.46 | 0.46 | 0.49 | 0.52 | 0.55 | 0.59 | 21 | min |
| 22 | 0.40 | 0.40 | 0.41 | 0.42 | 0.42 | 0.43 | 0.44 | 0.44 | 0.45 | 0.45 | 0.45 | 0.46 | 0.46 | 0.49 | 0.52 | 0.55 | 0.60 | 22 | ator |
| 23 | 0.40 | 0.41 | 0.41 | 0.42 | 0.43 | 0.43 | 0.44 | 0.44 | 0.45 | 0.45 | 0.46 | 0.46 | 0.47 | 0.49 | 0.53 | 0.56 | 0.60 | 23 | |
| 24 | 0.40 | 0.41 | 0.42 | 0.42 | 0.43 | 0.43 | 0.44 | 0.45 | 0.45 | 0.46 | 0.46 | 0.46 | 0.47 | 0.50 | 0.53 | 0.56 | 0.61 | 24 | |
| 25 | 0.40 | 0.41 | 0.42 | 0.42 | 0.43 | 0.44 | 0.44 | 0.45 | 0.45 | 0.46 | 0.46 | 0.47 | 0.47 | 0.50 | 0.54 | 0.56 | 0.62 | 25 | |
| 26 | 0.40 | 0.41 | 0.42 | 0.43 | 0.43 | 0.44 | 0.45 | 0.45 | 0.46 | 0.46 | 0.47 | 0.47 | 0.47 | 0.50 | 0.54 | 0.57 | 0.62 | 26 | |
| 27 | 0.40 | 0.41 | 0.42 | 0.43 | 0.44 | 0.44 | 0.45 | 0.45 | 0.46 | 0.46 | 0.47 | 0.47 | 0.48 | 0.51 | 0.54 | 0.57 | 0.63 | 27 | |
| 28 | 0.41 | 0.41 | 0.42 | 0.43 | 0.44 | 0.44 | 0.45 | 0.45 | 0.46 | 0.46 | 0.47 | 0.47 | 0.48 | 0.51 | 0.55 | 0.58 | 0.63 | 28 | |
| 29 | 0.41 | 0.42 | 0.42 | 0.43 | 0.44 | 0.44 | 0.45 | 0.46 | 0.46 | 0.47 | 0.47 | 0.48 | 0.48 | 0.51 | 0.55 | 0.58 | 0.63 | 29 | |
| 30 | 0.41 | 0.42 | 0.43 | 0.43 | 0.44 | 0.45 | 0.45 | 0.46 | 0.46 | 0.47 | 0.47 | 0.48 | 0.48 | 0.51 | 0.55 | 0.58 | 0.64 | 30 | |
| 40 | 0.42 | 0.43 | 0.44 | 0.45 | 0.45 | 0.46 | 0.47 | 0.47 | 0.48 | 0.48 | 0.49 | 0.49 | 0.50 | 0.53 | 0.57 | 0.61 | 0.67 | 40 | |
| 60 | 0.43 | 0.44 | 0.45 | 0.46 | 0.47 | 0.47 | 0.48 | 0.49 | 0.49 | 0.50 | 0.51 | 0.51 | 0.52 | 0.55 | 0.60 | 0.64 | 0.72 | 60 | |
| 100 | 0.44 | 0.45 | 0.46 | 0.47 | 0.48 | 0.49 | 0.49 | 0.50 | 0.51 | 0.51 | 0.52 | 0.53 | 0.53 | 0.57 | 0.63 | 0.67 | 0.77 | 100 | |
| 00 | 0.46 | 0.47 | 0.48 | 0.49 | 0.50 | 0.51 | 0.52 | 0.52 | 0.53 | 0.54 | 0.55 | 0.55 | 0.56 | 0.61 | 0.67 | 0.74 | 1.00 | ∞ | |

Appendix 18 *F*(0.95) Distribution Table

| | | | | | | | | 7 | <i>F</i> _{1-a} | ł | 1-α | | | | | | | | |
|----------------------------------------|-------------------|------|------|------|------|------|------|-------|-------------------------|------------------------|----------|--------|------|------|------|------|------|------|-------------------|
| | | | | | | | De | grees | of free | | or the r | numera | ator | | | | | | |
| | (v ₂) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | (v ₁) 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | (v ₂) |
| | 1 | 0.01 | 0.05 | 0.10 | 0.13 | 0.15 | 0.17 | 0.18 | 0.19 | 0.20 | 0.20 | 0.21 | 0.21 | 0.21 | 0.22 | 0.22 | 0.22 | 0.22 | 1 |
| | 2 | 0.01 | 0.05 | 0.10 | 0.14 | 0.17 | 0.19 | 0.21 | 0.22 | 0.23 | 0.24 | 0.25 | 0.26 | 0.26 | 0.27 | 0.27 | 0.28 | 0.28 | 2 |
| | 3 | 0.00 | 0.05 | 0.11 | 0.15 | 0.18 | 0.21 | 0.23 | 0.25 | 0.26 | 0.27 | 0.28 | 0.29 | 0.29 | 0.30 | 0.30 | 0.31 | 0.31 | 3 |
| | 4 | 0.00 | 0.05 | 0.11 | 0.16 | 0.19 | 0.22 | 0.24 | 0.26 | 0.28 | 0.29 | 0.30 | 0.31 | 0.31 | 0.32 | 0.33 | 0.33 | 0.34 | 4 |
| | 5 | 0.00 | 0.05 | 0.11 | 0.16 | 0.19 | 0.23 | 0.25 | 0.27 | 0.29 | 0.30 | 0.31 | 0.32 | 0.33 | 0.34 | 0.34 | 0.35 | 0.36 | 5 |
| | 6 | 0.00 | 0.05 | 0.11 | 0.16 | 0.20 | 0.23 | 0.26 | 0.28 | 0.30 | 0.31 | 0.32 | 0.33 | 0.34 | 0.35 | 0.36 | 0.36 | 0.37 | 6 |
| | 7 | 0.00 | 0.05 | 0.11 | 0.16 | 0.21 | 0.24 | 0.26 | 0.29 | 0.30 | 0.32 | 0.33 | 0.34 | 0.35 | 0.36 | 0.37 | 0.38 | 0.38 | 7 |
| | 8 | 0.00 | 0.05 | 0.11 | 0.17 | 0.21 | 0.24 | 0.27 | 0.29 | 0.31 | 0.33 | 0.34 | 0.35 | 0.36 | 0.37 | 0.38 | 0.39 | 0.39 | 8 |
| | 9 | 0.00 | 0.05 | 0.11 | 0.17 | 0.21 | 0.24 | 0.27 | 0.30 | 0.31 | 0.33 | 0.35 | 0.36 | 0.37 | 0.38 | 0.39 | 0.39 | 0.40 | 9 |
| | 10 | 0.00 | 0.05 | 0.11 | 0.17 | 0.21 | 0.25 | 0.27 | 0.30 | 0.32 | 0.34 | 0.35 | 0.36 | 0.37 | 0.38 | 0.39 | 0.40 | 0.41 | 10 |
| | 11 | 0.00 | 0.05 | 0.11 | 0.17 | 0.21 | 0.25 | 0.28 | 0.30 | 0.32 | 0.34 | 0.35 | 0.37 | 0.38 | 0.39 | 0.40 | 0.41 | 0.41 | 11 |
| | 12 | 0.00 | 0.05 | 0.11 | 0.17 | 0.21 | 0.25 | 0.28 | 0.30 | 0.33 | 0.34 | 0.36 | 0.37 | 0.38 | 0.39 | 0.40 | 0.41 | 0.42 | 12 |
| Degrees of freedom for the denominator | 13 | 0.00 | 0.05 | 0.11 | 0.17 | 0.21 | 0.25 | 0.28 | 0.31 | 0.33 | 0.35 | 0.36 | 0.38 | 0.39 | 0.40 | 0.41 | 0.42 | 0.42 | 13 |
| min | 14 | 0.00 | 0.05 | 0.11 | 0.17 | 0.22 | 0.25 | 0.28 | 0.31 | 0.33 | 0.35 | 0.37 | 0.38 | 0.39 | 0.40 | 0.41 | 0.42 | 0.43 | 14 |
| denc | 15 | 0.00 | 0.05 | 0.11 | 0.17 | 0.22 | 0.25 | 0.28 | 0.31 | 0.33 | 0.35 | 0.37 | 0.38 | 0.39 | 0.41 | 0.42 | 0.43 | 0.43 | 15 |
| the | 16 | 0.00 | 0.05 | 0.12 | 0.17 | 0.22 | 0.25 | 0.29 | 0.31 | 0.33 | 0.35 | 0.37 | 0.38 | 0.40 | 0.41 | 0.42 | 0.43 | 0.44 | 16 |
| for | 17 | 0.00 | 0.05 | 0.12 | 0.17 | 0.22 | 0.26 | 0.29 | 0.31 | 0.34 | 0.36 | 0.37 | 0.39 | 0.40 | 0.41 | 0.42 | 0.43 | 0.44 | 17 |
| - mop | 18 | 0.00 | 0.05 | 0.12 | 0.17 | 0.22 | 0.26 | 0.29 | 0.32 | 0.34 | 0.36 | 0.37 | 0.39 | 0.40 | 0.41 | 0.42 | 0.43 | 0.44 | 18 |
| free | 19 | 0.00 | 0.05 | 0.12 | 0.17 | 0.22 | 0.26 | 0.29 | 0.32 | 0.34 | 0.36 | 0.38 | 0.39 | 0.40 | 0.42 | 0.43 | 0.44 | 0.45 | 19 |
| s of | 20 | 0.00 | 0.05 | 0.12 | 0.17 | 0.22 | 0.26 | 0.29 | 0.32 | 0.34 | 0.36 | 0.38 | 0.39 | 0.41 | 0.42 | 0.43 | 0.44 | 0.45 | 20 |
| lree | 21 | 0.00 | 0.05 | 0.12 | 0.17 | 0.22 | 0.26 | 0.29 | 0.32 | 0.34 | 0.36 | 0.38 | 0.39 | 0.41 | 0.42 | 0.43 | 0.44 | 0.45 | 21 |
| Dec | 22 | 0.00 | 0.05 | 0.12 | 0.17 | 0.22 | 0.26 | 0.29 | 0.32 | 0.34 | 0.36 | 0.38 | 0.40 | 0.41 | 0.42 | 0.43 | 0.44 | 0.45 | 22 |
| | 23 | 0.00 | 0.05 | 0.12 | 0.17 | 0.22 | 0.26 | 0.29 | 0.32 | 0.34 | 0.36 | 0.38 | 0.40 | 0.41 | 0.42 | 0.44 | 0.45 | 0.45 | 23 |
| | 24 | 0.00 | 0.05 | 0.12 | 0.17 | 0.22 | 0.26 | 0.29 | 0.32 | 0.34 | 0.37 | 0.38 | 0.40 | 0.41 | 0.43 | 0.44 | 0.45 | 0.46 | 24 |
| | 25 | 0.00 | 0.05 | 0.12 | 0.17 | 0.22 | 0.26 | 0.29 | 0.32 | 0.35 | 0.37 | 0.38 | 0.40 | 0.41 | 0.43 | 0.44 | 0.45 | 0.46 | 25 |
| | 26 | 0.00 | 0.05 | 0.12 | 0.17 | 0.22 | 0.26 | 0.29 | 0.32 | 0.35 | 0.37 | 0.39 | 0.40 | 0.42 | 0.43 | 0.44 | 0.45 | 0.46 | 26 |
| | 27 | 0.00 | 0.05 | 0.12 | 0.17 | 0.22 | 0.26 | 0.29 | 0.32 | 0.35 | 0.37 | 0.39 | 0.40 | 0.42 | 0.43 | 0.44 | 0.45 | 0.46 | 27 |
| | 28 | 0.00 | 0.05 | 0.12 | 0.17 | 0.22 | 0.26 | 0.30 | 0.32 | 0.35 | 0.37 | 0.39 | 0.40 | 0.42 | 0.43 | 0.44 | 0.45 | 0.46 | 28 |
| | 29 | 0.00 | 0.05 | 0.12 | 0.17 | 0.22 | 0.26 | 0.30 | 0.32 | 0.35 | 0.37 | 0.39 | 0.40 | 0.42 | 0.43 | 0.44 | 0.45 | 0.46 | 29 |
| | 30 | 0.00 | 0.05 | 0.12 | 0.17 | 0.22 | 0.26 | 0.30 | 0.32 | 0.35 | 0.37 | 0.39 | 0.41 | 0.42 | 0.43 | 0.45 | 0.46 | 0.47 | 30 |
| | 40 | 0.00 | 0.05 | 0.12 | 0.17 | 0.22 | 0.26 | 0.30 | 0.33 | 0.35 | 0.38 | 0.40 | 0.41 | 0.43 | 0.44 | 0.45 | 0.46 | 0.48 | 40 |
| | 60 | 0.00 | 0.05 | 0.12 | 0.18 | 0.23 | 0.27 | 0.30 | 0.33 | 0.36 | 0.38 | 0.40 | 0.42 | 0.44 | 0.45 | 0.46 | 0.47 | 0.49 | 60 |
| | 100 | 0.00 | 0.05 | 0.12 | 0.18 | 0.23 | 0.27 | 0.31 | 0.34 | 0.36 | 0.39 | 0.41 | 0.43 | 0.44 | 0.46 | 0.47 | 0.48 | 0.49 | 100 |
| | ∞ | 0.00 | 0.05 | 0.12 | 0.18 | 0.23 | 0.27 | 0.31 | 0.34 | 0.37 | 0.39 | 0.42 | 0.44 | 0.45 | 0.47 | 0.48 | 0.50 | 0.51 | ∞ |



| | | | | | | | | F_{1-a} | v_1, v_2 | | | | | | | | | | |
|-----------------------------------------|------|------|------|------|------|------|---------|-----------|-------------------|----------|-------|------|------|------|------|------|------|-------------------|---------------------------------------|
| | | | | | | De | grees (| of free | dom fo | or the r | umera | tor | | | | | | | |
| | | | | | | | | | (v ₁) | | | | | | | | | | |
| (v ₂) | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 40 | 60 | 100 | 8 | (v ₂) | |
| 1 | 0.23 | 0.23 | 0.23 | 0.23 | 0.23 | 0.23 | 0.23 | 0.24 | 0.24 | 0.24 | 0.24 | 0.24 | 0.24 | 0.24 | 0.25 | 0.25 | 0.26 | 1 | |
| 2 | 0.28 | 0.28 | 0.29 | 0.29 | 0.29 | 0.29 | 0.29 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.31 | 0.32 | 0.32 | 0.33 | 2 | |
| 3 | 0.32 | 0.32 | 0.32 | 0.33 | 0.33 | 0.33 | 0.33 | 0.33 | 0.34 | 0.34 | 0.34 | 0.34 | 0.34 | 0.35 | 0.36 | 0.37 | 0.38 | 3 | |
| 4 | 0.34 | 0.35 | 0.35 | 0.35 | 0.36 | 0.36 | 0.36 | 0.36 | 0.36 | 0.37 | 0.37 | 0.37 | 0.37 | 0.38 | 0.40 | 0.41 | 0.42 | 4 | |
| 5 | 0.36 | 0.36 | 0.37 | 0.37 | 0.38 | 0.38 | 0.38 | 0.38 | 0.39 | 0.39 | 0.39 | 0.39 | 0.39 | 0.41 | 0.42 | 0.43 | 0.45 | 5 | |
| 6 | 0.38 | 0.38 | 0.38 | 0.39 | 0.39 | 0.40 | 0.40 | 0.40 | 0.40 | 0.41 | 0.41 | 0.41 | 0.41 | 0.43 | 0.44 | 0.46 | 0.48 | 6 | |
| 7 | 0.39 | 0.39 | 0.40 | 0.40 | 0.41 | 0.41 | 0.41 | 0.42 | 0.42 | 0.42 | 0.42 | 0.43 | 0.43 | 0.44 | 0.46 | 0.48 | 0.50 | 7 | |
| 8 | 0.40 | 0.40 | 0.41 | 0.41 | 0.42 | 0.42 | 0.42 | 0.43 | 0.43 | 0.43 | 0.44 | 0.44 | 0.44 | 0.46 | 0.48 | 0.49 | 0.52 | 8 | |
| 9 | 0.41 | 0.41 | 0.42 | 0.42 | 0.43 | 0.43 | 0.43 | 0.44 | 0.44 | 0.44 | 0.45 | 0.45 | 0.45 | 0.47 | 0.49 | 0.51 | 0.53 | 9 | |
| 10 | 0.41 | 0.42 | 0.43 | 0.43 | 0.44 | 0.44 | 0.44 | 0.45 | 0.45 | 0.45 | 0.46 | 0.46 | 0.46 | 0.48 | 0.50 | 0.52 | 0.55 | 10 | |
| 11 | 0.42 | 0.43 | 0.43 | 0.44 | 0.44 | 0.45 | 0.45 | 0.45 | 0.46 | 0.46 | 0.46 | 0.47 | 0.47 | 0.49 | 0.51 | 0.53 | 0.56 | 11 | |
| 12 | 0.43 | 0.43 | 0.44 | 0.44 | 0.45 | 0.45 | 0.46 | 0.46 | 0.47 | 0.47 | 0.47 | 0.48 | 0.48 | 0.50 | 0.52 | 0.54 | 0.57 | 12 | D |
| 13 | 0.43 | 0.44 | 0.44 | 0.45 | 0.46 | 0.46 | 0.46 | 0.47 | 0.47 | 0.48 | 0.48 | 0.48 | 0.48 | 0.51 | 0.53 | 0.55 | 0.58 | 13 | gre |
| 14 | 0.44 | 0.44 | 0.45 | 0.46 | 0.46 | 0.47 | 0.47 | 0.47 | 0.48 | 0.48 | 0.48 | 0.49 | 0.49 | 0.51 | 0.54 | 0.56 | 0.59 | 14 | o se |
| 15 | 0.44 | 0.45 | 0.45 | 0.46 | 0.46 | 0.47 | 0.47 | 0.48 | 0.48 | 0.49 | 0.49 | 0.49 | 0.50 | 0.52 | 0.54 | 0.57 | 0.60 | 15 | Degrees of freedom for the denominato |
| 16 | 0.44 | 0.45 | 0.46 | 0.46 | 0.47 | 0.47 | 0.48 | 0.48 | 0.49 | 0.49 | 0.49 | 0.50 | 0.50 | 0.53 | 0.55 | 0.57 | 0.61 | 16 | edor |
| 17 | 0.45 | 0.46 | 0.46 | 0.47 | 0.47 | 0.48 | 0.48 | 0.49 | 0.49 | 0.50 | 0.50 | 0.50 | 0.51 | 0.53 | 0.56 | 0.58 | 0.62 | 17 | n fo |
| 18 | 0.45 | 0.46 | 0.46 | 0.47 | 0.48 | 0.48 | 0.49 | 0.49 | 0.50 | 0.50 | 0.50 | 0.51 | 0.51 | 0.54 | 0.56 | 0.59 | 0.62 | 18 | r the |
| 19 | 0.45 | 0.46 | 0.47 | 0.47 | 0.48 | 0.49 | 0.49 | 0.49 | 0.50 | 0.50 | 0.51 | 0.51 | 0.51 | 0.54 | 0.57 | 0.59 | 0.63 | 19 | der |
| 20 | 0.46 | 0.46 | 0.47 | 0.48 | 0.48 | 0.49 | 0.49 | 0.50 | 0.50 | 0.51 | 0.51 | 0.51 | 0.52 | 0.54 | 0.57 | 0.60 | 0.64 | 20 | Iomi |
| 21 | 0.46 | 0.47 | 0.47 | 0.48 | 0.49 | 0.49 | 0.50 | 0.50 | 0.51 | 0.51 | 0.51 | 0.52 | 0.52 | 0.55 | 0.58 | 0.60 | 0.64 | 21 | nato |
| 22 | 0.46 | 0.47 | 0.48 | 0.48 | 0.49 | 0.49 | 0.50 | 0.50 | 0.51 | 0.51 | 0.52 | 0.52 | 0.52 | 0.55 | 0.58 | 0.61 | 0.65 | 22 | Ÿ |
| 23 | 0.46 | 0.47 | 0.48 | 0.48 | 0.49 | 0.50 | 0.50 | 0.51 | 0.51 | 0.52 | 0.52 | 0.52 | 0.53 | 0.55 | 0.58 | 0.61 | 0.65 | 23 | |
| 24 | 0.47 | 0.47 | 0.48 | 0.49 | 0.49 | 0.50 | 0.50 | 0.51 | 0.51 | 0.52 | 0.52 | 0.53 | 0.53 | 0.56 | 0.59 | 0.61 | 0.66 | 24 | |
| 25 | 0.47 | 0.47 | 0.48 | 0.49 | 0.50 | 0.50 | 0.51 | 0.51 | 0.52 | 0.52 | 0.52 | 0.53 | 0.53 | 0.56 | 0.59 | 0.62 | 0.66 | 25 | |
| 26 | 0.47 | 0.48 | 0.48 | 0.49 | 0.50 | 0.50 | 0.51 | 0.51 | 0.52 | 0.52 | 0.53 | 0.53 | 0.53 | 0.56 | 0.59 | 0.62 | 0.67 | 26 | |
| 27 | 0.47 | 0.48 | 0.49 | 0.49 | 0.50 | 0.50 | 0.51 | 0.52 | 0.52 | 0.52 | 0.53 | 0.53 | 0.54 | 0.57 | 0.60 | 0.63 | 0.67 | 27 | |
| 28 | 0.47 | 0.48 | 0.49 | 0.49 | 0.50 | 0.51 | 0.51 | 0.52 | 0.52 | 0.53 | 0.53 | 0.54 | 0.54 | 0.57 | 0.60 | 0.63 | 0.68 | 28 | |
| 29 | 0.47 | 0.48 | 0.49 | 0.50 | 0.50 | 0.51 | 0.51 | 0.52 | 0.52 | 0.53 | 0.53 | 0.54 | 0.54 | 0.57 | 0.60 | 0.63 | 0.68 | 29 | |
| 30 | 0.47 | 0.48 | 0.49 | 0.50 | 0.50 | 0.51 | 0.52 | 0.52 | 0.53 | 0.53 | 0.54 | 0.54 | 0.54 | 0.57 | 0.61 | 0.64 | 0.69 | 30 | |
| 40 | 0.48 | 0.49 | 0.50 | 0.51 | 0.52 | 0.52 | 0.53 | 0.53 | 0.54 | 0.54 | 0.55 | 0.55 | 0.56 | 0.59 | 0.63 | 0.66 | 0.72 | 40 | |
| 60 | 0.50 | 0.51 | 0.51 | 0.52 | 0.53 | 0.54 | 0.54 | 0.55 | 0.55 | 0.56 | 0.57 | 0.57 | 0.57 | 0.61 | 0.65 | 0.69 | 0.76 | 60 | |
| 100 | 0.51 | 0.52 | 0.52 | 0.53 | 0.54 | 0.55 | 0.56 | 0.56 | 0.57 | 0.57 | 0.58 | 0.58 | 0.59 | 0.63 | 0.68 | 0.72 | 0.80 | 100 | |
| ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~ | 0.52 | 0.53 | 0.54 | 0.55 | 0.56 | 0.57 | 0.58 | 0.58 | 0.59 | 0.60 | 0.60 | 0.61 | 0.62 | 0.66 | 0.72 | 0.78 | 1.00 | ∞ | |

Appendix 19 *F*(0.90) Distribution Table

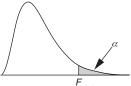
| | | | | | | | | 7 | <i>F</i> _{1-a} | ,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,, | 1-α | | | | | | | | |
|----------------------------------------|-------------------|------|------|------|------|------|------|-------|-------------------------|-----------------------------------------|----------|--------|------|------|------|------|------|------|-------------------|
| | | | | | | | De | grees | of free | | or the r | numera | ator | | | | | | |
| | (v ₂) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | (ν_1) 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | (v ₂) |
| | 1 | 0.03 | 0.12 | 0.18 | 0.22 | 0.25 | 0.26 | 0.28 | 0.29 | 0.30 | 0.30 | 0.31 | 0.31 | 0.32 | 0.32 | 0.33 | 0.33 | 0.33 | 1 |
| | 2 | 0.02 | 0.11 | 0.18 | 0.23 | 0.26 | 0.29 | 0.31 | 0.32 | 0.33 | 0.34 | 0.35 | 0.36 | 0.36 | 0.37 | 0.37 | 0.37 | 0.38 | 2 |
| | 3 | 0.02 | 0.11 | 0.19 | 0.24 | 0.28 | 0.30 | 0.33 | 0.34 | 0.36 | 0.37 | 0.38 | 0.38 | 0.39 | 0.40 | 0.40 | 0.41 | 0.41 | 3 |
| | 4 | 0.02 | 0.11 | 0.19 | 0.24 | 0.28 | 0.31 | 0.34 | 0.36 | 0.37 | 0.38 | 0.39 | 0.40 | 0.41 | 0.42 | 0.42 | 0.43 | 0.43 | 4 |
| | 5 | 0.02 | 0.11 | 0.19 | 0.25 | 0.29 | 0.32 | 0.35 | 0.37 | 0.38 | 0.40 | 0.41 | 0.42 | 0.43 | 0.43 | 0.44 | 0.45 | 0.45 | 5 |
| | 6 | 0.02 | 0.11 | 0.19 | 0.25 | 0.29 | 0.33 | 0.35 | 0.37 | 0.39 | 0.41 | 0.42 | 0.43 | 0.44 | 0.45 | 0.45 | 0.46 | 0.46 | 6 |
| | 7 | 0.02 | 0.11 | 0.19 | 0.25 | 0.30 | 0.33 | 0.36 | 0.38 | 0.40 | 0.41 | 0.43 | 0.44 | 0.45 | 0.46 | 0.46 | 0.47 | 0.48 | 7 |
| | 8 | 0.02 | 0.11 | 0.19 | 0.25 | 0.30 | 0.34 | 0.36 | 0.39 | 0.40 | 0.42 | 0.43 | 0.45 | 0.46 | 0.46 | 0.47 | 0.48 | 0.49 | 8 |
| | 9 | 0.02 | 0.11 | 0.19 | 0.25 | 0.30 | 0.34 | 0.37 | 0.39 | 0.41 | 0.43 | 0.44 | 0.45 | 0.46 | 0.47 | 0.48 | 0.49 | 0.49 | 9 |
| | 10 | 0.02 | 0.11 | 0.19 | 0.26 | 0.30 | 0.34 | 0.37 | 0.39 | 0.41 | 0.43 | 0.44 | 0.46 | 0.47 | 0.48 | 0.49 | 0.49 | 0.50 | 10 |
| | 11 | 0.02 | 0.11 | 0.19 | 0.26 | 0.30 | 0.34 | 0.37 | 0.40 | 0.42 | 0.43 | 0.45 | 0.46 | 0.47 | 0.48 | 0.49 | 0.50 | 0.51 | 11 |
| | 12 | 0.02 | 0.11 | 0.19 | 0.26 | 0.31 | 0.34 | 0.37 | 0.40 | 0.42 | 0.44 | 0.45 | 0.47 | 0.48 | 0.49 | 0.50 | 0.50 | 0.51 | 12 |
| ator | 13 | 0.02 | 0.11 | 0.19 | 0.26 | 0.31 | 0.35 | 0.38 | 0.40 | 0.42 | 0.44 | 0.46 | 0.47 | 0.48 | 0.49 | 0.50 | 0.51 | 0.52 | 13 |
| min | 14 | 0.02 | 0.11 | 0.19 | 0.26 | 0.31 | 0.35 | 0.38 | 0.40 | 0.43 | 0.44 | 0.46 | 0.47 | 0.48 | 0.49 | 0.50 | 0.51 | 0.52 | 14 |
| denc | 15 | 0.02 | 0.11 | 0.19 | 0.26 | 0.31 | 0.35 | 0.38 | 0.41 | 0.43 | 0.45 | 0.46 | 0.48 | 0.49 | 0.50 | 0.51 | 0.52 | 0.52 | 15 |
| Degrees of freedom for the denominator | 16 | 0.02 | 0.11 | 0.19 | 0.26 | 0.31 | 0.35 | 0.38 | 0.41 | 0.43 | 0.45 | 0.46 | 0.48 | 0.49 | 0.50 | 0.51 | 0.52 | 0.53 | 16 |
| for | 17 | 0.02 | 0.11 | 0.19 | 0.26 | 0.31 | 0.35 | 0.38 | 0.41 | 0.43 | 0.45 | 0.47 | 0.48 | 0.49 | 0.50 | 0.51 | 0.52 | 0.53 | 17 |
| dom | 18 | 0.02 | 0.11 | 0.19 | 0.26 | 0.31 | 0.35 | 0.38 | 0.41 | 0.43 | 0.45 | 0.47 | 0.48 | 0.49 | 0.51 | 0.52 | 0.52 | 0.53 | 18 |
| free | 19 | 0.02 | 0.11 | 0.19 | 0.26 | 0.31 | 0.35 | 0.38 | 0.41 | 0.43 | 0.45 | 0.47 | 0.48 | 0.50 | 0.51 | 0.52 | 0.53 | 0.53 | 19 |
| sof | 20 | 0.02 | 0.11 | 0.19 | 0.26 | 0.31 | 0.35 | 0.39 | 0.41 | 0.44 | 0.45 | 0.47 | 0.49 | 0.50 | 0.51 | 0.52 | 0.53 | 0.54 | 20 |
| lree | 21 | 0.02 | 0.11 | 0.19 | 0.26 | 0.31 | 0.35 | 0.39 | 0.41 | 0.44 | 0.46 | 0.47 | 0.49 | 0.50 | 0.51 | 0.52 | 0.53 | 0.54 | 21 |
| Dec | 22 | 0.02 | 0.11 | 0.19 | 0.26 | 0.31 | 0.35 | 0.39 | 0.41 | 0.44 | 0.46 | 0.47 | 0.49 | 0.50 | 0.51 | 0.52 | 0.53 | 0.54 | 22 |
| | 23 | 0.02 | 0.11 | 0.19 | 0.26 | 0.31 | 0.35 | 0.39 | 0.42 | 0.44 | 0.46 | 0.48 | 0.49 | 0.50 | 0.51 | 0.53 | 0.53 | 0.54 | 23 |
| | 24 | 0.02 | 0.11 | 0.19 | 0.26 | 0.31 | 0.35 | 0.39 | 0.42 | 0.44 | 0.46 | 0.48 | 0.49 | 0.50 | 0.52 | 0.53 | 0.54 | 0.54 | 24 |
| | 25 | 0.02 | 0.11 | 0.19 | 0.26 | 0.31 | 0.36 | 0.39 | 0.42 | 0.44 | 0.46 | 0.48 | 0.49 | 0.51 | 0.52 | 0.53 | 0.54 | 0.55 | 25 |
| | 26 | 0.02 | 0.11 | 0.19 | 0.26 | 0.31 | 0.36 | 0.39 | 0.42 | 0.44 | 0.46 | 0.48 | 0.49 | 0.51 | 0.52 | 0.53 | 0.54 | 0.55 | 26 |
| | 27 | 0.02 | 0.11 | 0.19 | 0.26 | 0.31 | 0.36 | 0.39 | 0.42 | 0.44 | 0.46 | 0.48 | 0.49 | 0.51 | 0.52 | 0.53 | 0.54 | 0.55 | 27 |
| | 28 | 0.02 | 0.11 | 0.19 | 0.26 | 0.31 | 0.36 | 0.39 | 0.42 | 0.44 | 0.46 | 0.48 | 0.50 | 0.51 | 0.52 | 0.53 | 0.54 | 0.55 | 28 |
| | 29 | 0.02 | 0.11 | 0.19 | 0.26 | 0.31 | 0.36 | 0.39 | 0.42 | 0.44 | 0.46 | 0.48 | 0.50 | 0.51 | 0.52 | 0.53 | 0.54 | 0.55 | 29 |
| | 30 | 0.02 | 0.11 | 0.19 | 0.26 | 0.32 | 0.36 | 0.39 | 0.42 | 0.44 | 0.46 | 0.48 | 0.50 | 0.51 | 0.52 | 0.53 | 0.54 | 0.55 | 30 |
| | 40 | 0.02 | 0.11 | 0.19 | 0.26 | 0.32 | 0.36 | 0.39 | 0.42 | 0.45 | 0.47 | 0.49 | 0.50 | 0.52 | 0.53 | 0.54 | 0.55 | 0.56 | 40 |
| | 60 | 0.02 | 0.11 | 0.19 | 0.26 | 0.32 | 0.36 | 0.40 | 0.43 | 0.45 | 0.47 | 0.49 | 0.51 | 0.53 | 0.54 | 0.55 | 0.56 | 0.57 | 60 |
| | 100 | 0.02 | 0.11 | 0.19 | 0.26 | 0.32 | 0.36 | 0.40 | 0.43 | 0.46 | 0.48 | 0.50 | 0.52 | 0.53 | 0.55 | 0.56 | 0.57 | 0.58 | 100 |
| | ∞ | 0.02 | 0.11 | 0.19 | 0.27 | 0.32 | 0.37 | 0.40 | 0.44 | 0.46 | 0.49 | 0.51 | 0.53 | 0.54 | 0.56 | 0.57 | 0.58 | 0.59 | ∞ |



| | | | | | | | | F_{1-a} | v_1, v_1, v_2 | | | | | | | | | | |
|-------------------|------|------|------|------|------|------|---------|-----------|-------------------|----------|-------|------|------|------|------|------|------|-------------------|----------------------------------------|
| | | | | | | Deg | grees (| of free | dom fo | or the r | umera | ator | | | | | | | |
| | | | | | | | | | (v ₁) | | | | | | | | | | |
| (v ₂) | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 40 | 60 | 100 | ∞ | (v ₂) | |
| 1 | 0.33 | 0.33 | 0.34 | 0.34 | 0.34 | 0.34 | 0.34 | 0.34 | 0.34 | 0.34 | 0.35 | 0.35 | 0.35 | 0.35 | 0.36 | 0.36 | 0.37 | 1 | |
| 2 | 0.38 | 0.38 | 0.39 | 0.39 | 0.39 | 0.39 | 0.39 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.41 | 0.42 | 0.42 | 0.43 | 2 | |
| 3 | 0.41 | 0.42 | 0.42 | 0.42 | 0.43 | 0.43 | 0.43 | 0.43 | 0.43 | 0.44 | 0.44 | 0.44 | 0.44 | 0.45 | 0.46 | 0.47 | 0.48 | 3 | |
| 4 | 0.44 | 0.44 | 0.44 | 0.45 | 0.45 | 0.45 | 0.46 | 0.46 | 0.46 | 0.46 | 0.46 | 0.47 | 0.47 | 0.48 | 0.49 | 0.50 | 0.51 | 4 | |
| 5 | 0.46 | 0.46 | 0.46 | 0.47 | 0.47 | 0.47 | 0.48 | 0.48 | 0.48 | 0.48 | 0.48 | 0.49 | 0.49 | 0.50 | 0.51 | 0.52 | 0.54 | 5 | |
| 6 | 0.47 | 0.47 | 0.48 | 0.48 | 0.49 | 0.49 | 0.49 | 0.49 | 0.50 | 0.50 | 0.50 | 0.50 | 0.50 | 0.52 | 0.53 | 0.55 | 0.56 | 6 | |
| 7 | 0.48 | 0.49 | 0.49 | 0.49 | 0.50 | 0.50 | 0.50 | 0.51 | 0.51 | 0.51 | 0.51 | 0.52 | 0.52 | 0.53 | 0.55 | 0.56 | 0.58 | 7 | |
| 8 | 0.49 | 0.50 | 0.50 | 0.50 | 0.51 | 0.51 | 0.52 | 0.52 | 0.52 | 0.52 | 0.53 | 0.53 | 0.53 | 0.55 | 0.56 | 0.58 | 0.60 | 8 | |
| 9 | 0.50 | 0.50 | 0.51 | 0.51 | 0.52 | 0.52 | 0.52 | 0.53 | 0.53 | 0.53 | 0.54 | 0.54 | 0.54 | 0.56 | 0.58 | 0.59 | 0.61 | 9 | |
| 10 | 0.51 | 0.51 | 0.52 | 0.52 | 0.53 | 0.53 | 0.53 | 0.54 | 0.54 | 0.54 | 0.54 | 0.55 | 0.55 | 0.57 | 0.59 | 0.60 | 0.63 | 10 | |
| 11 | 0.51 | 0.52 | 0.52 | 0.53 | 0.53 | 0.54 | 0.54 | 0.54 | 0.55 | 0.55 | 0.55 | 0.55 | 0.56 | 0.58 | 0.60 | 0.61 | 0.64 | 11 | |
| 12 | 0.52 | 0.52 | 0.53 | 0.53 | 0.54 | 0.54 | 0.55 | 0.55 | 0.55 | 0.56 | 0.56 | 0.56 | 0.56 | 0.58 | 0.60 | 0.62 | 0.65 | 12 | D |
| 13 | 0.52 | 0.53 | 0.53 | 0.54 | 0.54 | 0.55 | 0.55 | 0.56 | 0.56 | 0.56 | 0.56 | 0.57 | 0.57 | 0.59 | 0.61 | 0.63 | 0.66 | 13 | gre |
| 14 | 0.53 | 0.53 | 0.54 | 0.54 | 0.55 | 0.55 | 0.56 | 0.56 | 0.56 | 0.57 | 0.57 | 0.57 | 0.58 | 0.60 | 0.62 | 0.64 | 0.66 | 14 | es o |
| 15 | 0.53 | 0.54 | 0.54 | 0.55 | 0.55 | 0.56 | 0.56 | 0.56 | 0.57 | 0.57 | 0.57 | 0.58 | 0.58 | 0.60 | 0.62 | 0.64 | 0.67 | 15 | Degrees of freedom for the denominator |
| 16 | 0.53 | 0.54 | 0.55 | 0.55 | 0.56 | 0.56 | 0.56 | 0.57 | 0.57 | 0.58 | 0.58 | 0.58 | 0.59 | 0.61 | 0.63 | 0.65 | 0.68 | 16 | edo |
| 17 | 0.54 | 0.54 | 0.55 | 0.55 | 0.56 | 0.56 | 0.57 | 0.57 | 0.58 | 0.58 | 0.58 | 0.59 | 0.59 | 0.61 | 0.63 | 0.65 | 0.69 | 17 | m fo |
| 18 | 0.54 | 0.55 | 0.55 | 0.56 | 0.56 | 0.57 | 0.57 | 0.58 | 0.58 | 0.58 | 0.59 | 0.59 | 0.59 | 0.62 | 0.64 | 0.66 | 0.69 | 18 | r the |
| 19 | 0.54 | 0.55 | 0.55 | 0.56 | 0.57 | 0.57 | 0.58 | 0.58 | 0.58 | 0.59 | 0.59 | 0.59 | 0.60 | 0.62 | 0.64 | 0.66 | 0.70 | 19 | der |
| 20 | 0.54 | 0.55 | 0.56 | 0.56 | 0.57 | 0.57 | 0.58 | 0.58 | 0.59 | 0.59 | 0.59 | 0.60 | 0.60 | 0.62 | 0.65 | 0.67 | 0.70 | 20 | mor |
| 21 | 0.55 | 0.55 | 0.56 | 0.57 | 0.57 | 0.58 | 0.58 | 0.58 | 0.59 | 0.59 | 0.60 | 0.60 | 0.60 | 0.63 | 0.65 | 0.67 | 0.71 | 21 | inate |
| 22 | 0.55 | 0.56 | 0.56 | 0.57 | 0.57 | 0.58 | 0.58 | 0.59 | 0.59 | 0.60 | 0.60 | 0.60 | 0.61 | 0.63 | 0.66 | 0.68 | 0.71 | 22 | q |
| 23 | 0.55 | 0.56 | 0.56 | 0.57 | 0.58 | 0.58 | 0.59 | 0.59 | 0.59 | 0.60 | 0.60 | 0.60 | 0.61 | 0.63 | 0.66 | 0.68 | 0.72 | 23 | |
| 24 | 0.55 | 0.56 | 0.57 | 0.57 | 0.58 | 0.58 | 0.59 | 0.59 | 0.60 | 0.60 | 0.60 | 0.61 | 0.61 | 0.64 | 0.66 | 0.68 | 0.72 | 24 | |
| 25 | 0.55 | 0.56 | 0.57 | 0.57 | 0.58 | 0.58 | 0.59 | 0.59 | 0.60 | 0.60 | 0.61 | 0.61 | 0.61 | 0.64 | 0.66 | 0.69 | 0.73 | 25 | |
| 26 | 0.56 | 0.56 | 0.57 | 0.58 | 0.58 | 0.59 | 0.59 | 0.60 | 0.60 | 0.60 | 0.61 | 0.61 | 0.62 | 0.64 | 0.67 | 0.69 | 0.73 | 26 | |
| 27 | 0.56 | 0.56 | 0.57 | 0.58 | 0.58 | 0.59 | 0.59 | 0.60 | 0.60 | 0.61 | 0.61 | 0.61 | 0.62 | 0.64 | 0.67 | 0.69 | 0.73 | 27 | |
| 28 | 0.56 | 0.57 | 0.57 | 0.58 | 0.58 | 0.59 | 0.59 | 0.60 | 0.60 | 0.61 | 0.61 | 0.62 | 0.62 | 0.64 | 0.67 | 0.70 | 0.74 | 28 | |
| 29 | 0.56 | 0.57 | 0.57 | 0.58 | 0.59 | 0.59 | 0.60 | 0.60 | 0.61 | 0.61 | 0.61 | 0.62 | 0.62 | 0.65 | 0.68 | 0.70 | 0.74 | 29 | |
| 30 | 0.56 | 0.57 | 0.58 | 0.58 | 0.59 | 0.59 | 0.60 | 0.60 | 0.61 | 0.61 | 0.62 | 0.62 | 0.62 | 0.65 | 0.68 | 0.70 | 0.75 | 30 | |
| 40 | 0.57 | 0.58 | 0.59 | 0.59 | 0.60 | 0.60 | 0.61 | 0.61 | 0.62 | 0.62 | 0.63 | 0.63 | 0.64 | 0.66 | 0.70 | 0.72 | 0.77 | 40 | |
| 60 | 0.58 | 0.59 | 0.60 | 0.60 | 0.61 | 0.62 | 0.62 | 0.63 | 0.63 | 0.64 | 0.64 | 0.65 | 0.65 | 0.68 | 0.72 | 0.75 | 0.81 | 60 | |
| 100 | 0.59 | 0.60 | 0.61 | 0.61 | 0.62 | 0.63 | 0.63 | 0.64 | 0.64 | 0.65 | 0.65 | 0.66 | 0.66 | 0.70 | 0.74 | 0.77 | 0.84 | 100 | |
| 00 | 0.60 | 0.61 | 0.62 | 0.63 | 0.64 | 0.65 | 0.65 | 0.66 | 0.67 | 0.67 | 0.68 | 0.68 | 0.69 | 0.73 | 0.77 | 0.82 | 1.00 | ∞ | |

Appendix 20 *F*(0.10) Distribution Table

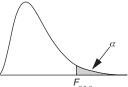
| | | | | | | | | | / | | | α | | | | | | | |
|----------------------------------------|-----------------------------------------|-------|-------|-------|-------|-------|-------|-------|---------|------------------------|----------------------------------|-------|-------|-------|-------|-------|-------|-------|-------------------|
| | | | | | | | | 1 | | | F | | | | | | | | |
| | | | | | | | | | | | <i>F</i> _{<i>α,ν</i>,,} | | | | | | | | |
| | | | | | | | De | grees | of free | $\frac{1}{(\nu_1)}$ | or the r | umera | itor | | | | | | |
| | (v ₂) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | (^µ 1) 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | (v ₂) |
| | 1 | 39.86 | 49.50 | 53.59 | 55.83 | 57.24 | 58.20 | 58.91 | 59.44 | 59.86 | 60.19 | 60.47 | 60.71 | 60.90 | 61.07 | 61.22 | 61.35 | 61.46 | 1 |
| | 2 | 8.53 | 9.00 | 9.16 | 9.24 | 9.29 | 9.33 | 9.35 | 9.37 | 9.38 | 9.39 | 9.40 | 9.41 | 9.41 | 9.42 | 9.42 | 9.43 | 9.43 | 2 |
| | 3 | 5.54 | 5.46 | 5.39 | 5.34 | 5.31 | 5.28 | 5.27 | 5.25 | 5.24 | 5.23 | 5.22 | 5.22 | 5.21 | 5.20 | 5.20 | 5.20 | 5.19 | 3 |
| | 4 | 4.54 | 4.32 | 4.19 | 4.11 | 4.05 | 4.01 | 3.98 | 3.95 | 3.94 | 3.92 | 3.91 | 3.90 | 3.89 | 3.88 | 3.87 | 3.86 | 3.86 | 4 |
| | 5 | 4.06 | 3.78 | 3.62 | 3.52 | 3.45 | 3.40 | 3.37 | 3.34 | 3.32 | 3.30 | 3.28 | 3.27 | 3.26 | 3.25 | 3.24 | 3.23 | 3.22 | 5 |
| | 6 | 3.78 | 3.46 | 3.29 | 3.18 | 3.11 | 3.05 | 3.01 | 2.98 | 2.96 | 2.94 | 2.92 | 2.90 | 2.89 | 2.88 | 2.87 | 2.86 | 2.85 | 6 |
| | 7 | 3.59 | 3.26 | 3.07 | 2.96 | 2.88 | 2.83 | 2.78 | 2.75 | 2.72 | 2.70 | 2.68 | 2.67 | 2.65 | 2.64 | 2.63 | 2.62 | 2.61 | 7 |
| | 8 | 3.46 | 3.11 | 2.92 | 2.81 | 2.73 | 2.67 | 2.62 | 2.59 | 2.56 | 2.54 | 2.52 | 2.50 | 2.49 | 2.48 | 2.46 | 2.45 | 2.45 | 8 |
| | 9 | 3.36 | 3.01 | 2.81 | 2.69 | 2.61 | 2.55 | 2.51 | 2.47 | 2.44 | 2.42 | 2.40 | 2.38 | 2.36 | 2.35 | 2.34 | 2.33 | 2.32 | 9 |
| | 10 | 3.29 | 2.92 | 2.73 | 2.61 | 2.52 | 2.46 | 2.41 | 2.38 | 2.35 | 2.32 | 2.30 | 2.28 | 2.27 | 2.26 | 2.24 | 2.23 | 2.22 | 10 |
| | 11 | 3.23 | 2.86 | 2.66 | 2.54 | 2.45 | 2.39 | 2.34 | 2.30 | 2.27 | 2.25 | 2.23 | 2.21 | 2.19 | 2.18 | 2.17 | 2.16 | 2.15 | 11 |
| | 12 | 3.18 | 2.81 | 2.61 | 2.48 | 2.39 | 2.33 | 2.28 | 2.24 | 2.21 | 2.19 | 2.17 | 2.15 | 2.13 | 2.12 | 2.10 | 2.09 | 2.08 | 12 |
| ator | 13 | 3.14 | 2.76 | 2.56 | 2.43 | 2.35 | 2.28 | 2.23 | 2.20 | 2.16 | 2.14 | 2.12 | 2.10 | 2.08 | 2.07 | 2.05 | 2.04 | 2.03 | 13 |
| min | 14 | 3.10 | 2.73 | 2.52 | 2.39 | 2.31 | 2.24 | 2.19 | 2.15 | 2.12 | 2.10 | 2.07 | 2.05 | 2.04 | 2.02 | 2.01 | 2.00 | 1.99 | 14 |
| leno | 15 | 3.07 | 2.70 | 2.49 | 2.36 | 2.27 | 2.21 | 2.16 | 2.12 | 2.09 | 2.06 | 2.04 | 2.02 | 2.00 | 1.99 | 1.97 | 1.96 | 1.95 | 15 |
| he c | 16 | 3.05 | 2.67 | 2.46 | 2.33 | 2.24 | 2.18 | 2.13 | 2.09 | 2.06 | 2.03 | 2.01 | 1.99 | 1.97 | 1.95 | 1.94 | 1.93 | 1.92 | 16 |
| for t | 17 | 3.03 | 2.64 | 2.44 | 2.31 | 2.22 | 2.15 | 2.10 | 2.06 | 2.03 | 2.00 | 1.98 | 1.96 | 1.94 | 1.93 | 1.91 | 1.90 | 1.89 | 17 |
| mo | 18 | 3.01 | 2.62 | 2.42 | 2.29 | 2.20 | 2.13 | 2.08 | 2.04 | 2.00 | 1.98 | 1.95 | 1.93 | 1.92 | 1.90 | 1.89 | 1.87 | 1.86 | 18 |
| eed | 19 | 2.99 | 2.61 | 2.40 | 2.27 | 2.18 | 2.11 | 2.06 | 2.02 | 1.98 | 1.96 | 1.93 | 1.91 | 1.89 | 1.88 | 1.86 | 1.85 | 1.84 | 19 |
| of fi | 20 | 2.97 | 2.59 | 2.38 | 2.25 | 2.16 | 2.09 | 2.04 | 2.00 | 1.96 | 1.94 | 1.91 | 1.89 | 1.87 | 1.86 | 1.84 | 1.83 | 1.82 | 20 |
| Degrees of freedom for the denominator | 21 | 2.96 | 2.57 | 2.36 | 2.23 | 2.14 | 2.08 | 2.02 | 1.98 | 1.95 | 1.92 | 1.90 | 1.87 | 1.86 | 1.84 | 1.83 | 1.81 | 1.80 | 21 |
| Jegr | 22 | 2.95 | 2.56 | 2.35 | 2.22 | 2.13 | 2.06 | 2.01 | 1.97 | 1.93 | 1.90 | 1.88 | 1.86 | 1.84 | 1.83 | 1.81 | 1.80 | 1.79 | 22 |
| | 23 | 2.94 | 2.55 | 2.34 | 2.21 | 2.11 | 2.05 | 1.99 | 1.95 | 1.92 | 1.89 | 1.87 | 1.84 | 1.83 | 1.81 | 1.80 | 1.78 | 1.77 | 23 |
| | 24 | 2.93 | 2.54 | 2.33 | 2.19 | 2.10 | 2.04 | 1.98 | 1.94 | 1.91 | 1.88 | 1.85 | 1.83 | 1.81 | 1.80 | 1.78 | 1.77 | 1.76 | 24 |
| | 25 | 2.92 | 2.53 | 2.32 | 2.18 | 2.09 | 2.02 | 1.97 | 1.93 | 1.89 | 1.87 | 1.84 | 1.82 | 1.80 | 1.79 | 1.77 | 1.76 | 1.75 | 25 |
| | 26 | 2.91 | 2.52 | 2.31 | 2.17 | 2.08 | 2.01 | 1.96 | 1.92 | 1.88 | 1.86 | 1.83 | 1.81 | 1.79 | 1.77 | 1.76 | 1.75 | 1.73 | 26 |
| | 27 | 2.90 | 2.51 | 2.30 | 2.17 | 2.07 | 2.00 | 1.95 | 1.91 | 1.87 | 1.85 | 1.82 | 1.80 | 1.78 | 1.76 | 1.75 | 1.74 | 1.72 | 27 |
| | 28 | 2.89 | 2.50 | 2.29 | 2.16 | 2.06 | 2.00 | 1.94 | 1.90 | 1.87 | 1.84 | 1.81 | 1.79 | 1.77 | 1.75 | 1.74 | 1.73 | 1.71 | 28 |
| | 29 | 2.89 | 2.50 | 2.28 | 2.15 | 2.06 | 1.99 | 1.93 | 1.89 | 1.86 | 1.83 | 1.80 | 1.78 | 1.76 | 1.75 | 1.73 | 1.72 | 1.71 | 29 |
| | 30 | 2.88 | 2.49 | 2.28 | 2.14 | 2.05 | 1.98 | 1.93 | 1.88 | 1.85 | 1.82 | 1.79 | 1.77 | 1.75 | 1.74 | 1.72 | 1.71 | 1.70 | 30 |
| | 40 | 2.84 | 2.44 | 2.23 | 2.09 | 2.00 | 1.93 | 1.87 | 1.83 | 1.79 | 1.76 | 1.74 | 1.71 | 1.70 | 1.68 | 1.66 | 1.65 | 1.64 | 40 |
| | 60 | 2.79 | 2.39 | 2.18 | 2.04 | 1.95 | 1.87 | 1.82 | 1.77 | 1.74 | 1.71 | 1.68 | 1.66 | 1.64 | 1.62 | 1.60 | 1.59 | 1.58 | 60 |
| | 100 | 2.76 | 2.36 | 2.14 | 2.00 | 1.91 | 1.83 | 1.78 | 1.73 | 1.69 | 1.66 | 1.64 | 1.61 | 1.59 | 1.57 | 1.56 | 1.54 | 1.53 | 100 |
| | ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~ | 2.71 | 2.30 | 2.08 | 1.94 | 1.85 | 1.77 | 1.72 | 1.67 | 1.63 | 1.60 | 1.57 | 1.55 | 1.52 | 1.50 | 1.49 | 1.47 | 1.46 | ∞ |



| | | | | | | | | | | $F_{\alpha,\nu_{1}}$ | <i>v</i> ₂ | | | | | | | | |
|-------------------|-------|-------|-------|-------|-------|-------|---------|---------|-----------|----------------------|-----------------------|-------|-------|-------|-------|-------|-------|-------------------|------------------------|
| | | | | | | De | grees (| of free | dom fo | or the r | numera | tor | | | | | | | |
| | | | | | | | | | (ν_1) | | | | | | | | | | |
| (v ₂) | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 40 | 60 | 100 | ∞ | (v ₂) | |
| 1 | 61.57 | 61.66 | 61.74 | 61.81 | 61.88 | 61.95 | 62.00 | 62.05 | 62.10 | 62.15 | 62.19 | 62.23 | 62.26 | 62.53 | 62.79 | 63.01 | 63.33 | 1 | |
| 2 | 9.44 | 9.44 | 9.44 | 9.44 | 9.45 | 9.45 | 9.45 | 9.45 | 9.45 | 9.45 | 9.46 | 9.46 | 9.46 | 9.47 | 9.47 | 9.48 | 9.49 | 2 | |
| 3 | 5.19 | 5.19 | 5.18 | 5.18 | 5.18 | 5.18 | 5.18 | 5.17 | 5.17 | 5.17 | 5.17 | 5.17 | 5.17 | 5.16 | 5.15 | 5.14 | 5.13 | 3 | |
| 4 | 3.85 | 3.85 | 3.84 | 3.84 | 3.84 | 3.83 | 3.83 | 3.83 | 3.83 | 3.82 | 3.82 | 3.82 | 3.82 | 3.80 | 3.79 | 3.78 | 3.76 | 4 | |
| 5 | 3.22 | 3.21 | 3.21 | 3.20 | 3.20 | 3.19 | 3.19 | 3.19 | 3.18 | 3.18 | 3.18 | 3.18 | 3.17 | 3.16 | 3.14 | 3.13 | 3.10 | 5 | |
| 6 | 2.85 | 2.84 | 2.84 | 2.83 | 2.83 | 2.82 | 2.82 | 2.81 | 2.81 | 2.81 | 2.81 | 2.80 | 2.80 | 2.78 | 2.76 | 2.75 | 2.72 | 6 | |
| 7 | 2.61 | 2.60 | 2.59 | 2.59 | 2.58 | 2.58 | 2.58 | 2.57 | 2.57 | 2.56 | 2.56 | 2.56 | 2.56 | 2.54 | 2.51 | 2.50 | 2.47 | 7 |] |
| 8 | 2.44 | 2.43 | 2.42 | 2.42 | 2.41 | 2.41 | 2.40 | 2.40 | 2.40 | 2.39 | 2.39 | 2.39 | 2.38 | 2.36 | 2.34 | 2.32 | 2.29 | 8 | |
| 9 | 2.31 | 2.30 | 2.30 | 2.29 | 2.29 | 2.28 | 2.28 | 2.27 | 2.27 | 2.26 | 2.26 | 2.26 | 2.25 | 2.23 | 2.21 | 2.19 | 2.16 | 9 | |
| 10 | 2.22 | 2.21 | 2.20 | 2.19 | 2.19 | 2.18 | 2.18 | 2.17 | 2.17 | 2.17 | 2.16 | 2.16 | 2.16 | 2.13 | 2.11 | 2.09 | 2.06 | 10 | |
| 11 | 2.14 | 2.13 | 2.12 | 2.12 | 2.11 | 2.11 | 2.10 | 2.10 | 2.09 | 2.09 | 2.08 | 2.08 | 2.08 | 2.05 | 2.03 | 2.01 | 1.97 | 11 |] |
| 12 | 2.08 | 2.07 | 2.06 | 2.05 | 2.05 | 2.04 | 2.04 | 2.03 | 2.03 | 2.02 | 2.02 | 2.01 | 2.01 | 1.99 | 1.96 | 1.94 | 1.90 | 12 | De |
| 13 | 2.02 | 2.01 | 2.01 | 2.00 | 1.99 | 1.99 | 1.98 | 1.98 | 1.97 | 1.97 | 1.96 | 1.96 | 1.96 | 1.93 | 1.90 | 1.88 | 1.85 | 13 | gree |
| 14 | 1.98 | 1.97 | 1.96 | 1.96 | 1.95 | 1.94 | 1.94 | 1.93 | 1.93 | 1.92 | 1.92 | 1.92 | 1.91 | 1.89 | 1.86 | 1.83 | 1.80 | 14 | s of |
| 15 | 1.94 | 1.93 | 1.92 | 1.92 | 1.91 | 1.90 | 1.90 | 1.89 | 1.89 | 1.88 | 1.88 | 1.88 | 1.87 | 1.85 | 1.82 | 1.79 | 1.76 | 15 | fre |
| 16 | 1.91 | 1.90 | 1.89 | 1.88 | 1.88 | 1.87 | 1.87 | 1.86 | 1.86 | 1.85 | 1.85 | 1.84 | 1.84 | 1.81 | 1.78 | 1.76 | 1.72 | 16 | Degrees of freedom for |
| 17 | 1.88 | 1.87 | 1.86 | 1.86 | 1.85 | 1.84 | 1.84 | 1.83 | 1.83 | 1.82 | 1.82 | 1.81 | 1.81 | 1.78 | 1.75 | 1.73 | 1.69 | 17 | n fo |
| 18 | 1.85 | 1.84 | 1.84 | 1.83 | 1.82 | 1.82 | 1.81 | 1.80 | 1.80 | 1.80 | 1.79 | 1.79 | 1.78 | 1.75 | 1.72 | 1.70 | 1.66 | 18 | r the |
| 19 | 1.83 | 1.82 | 1.81 | 1.81 | 1.80 | 1.79 | 1.79 | 1.78 | 1.78 | 1.77 | 1.77 | 1.76 | 1.76 | 1.73 | 1.70 | 1.67 | 1.63 | 19 | dei |
| 20 | 1.81 | 1.80 | 1.79 | 1.79 | 1.78 | 1.77 | 1.77 | 1.76 | 1.76 | 1.75 | 1.75 | 1.74 | 1.74 | 1.71 | 1.68 | 1.65 | 1.61 | 20 | denominator |
| 21 | 1.79 | 1.78 | 1.78 | 1.77 | 1.76 | 1.75 | 1.75 | 1.74 | 1.74 | 1.73 | 1.73 | 1.72 | 1.72 | 1.69 | 1.66 | 1.63 | 1.59 | 21 | inat |
| 22 | 1.78 | 1.77 | 1.76 | 1.75 | 1.74 | 1.74 | 1.73 | 1.73 | 1.72 | 1.72 | 1.71 | 1.71 | 1.70 | 1.67 | 1.64 | 1.61 | 1.57 | 22 | 9 |
| 23 | 1.76 | 1.75 | 1.74 | 1.74 | 1.73 | 1.72 | 1.72 | 1.71 | 1.70 | 1.70 | 1.69 | 1.69 | 1.69 | 1.66 | 1.62 | 1.59 | 1.55 | 23 | |
| 24 | 1.75 | 1.74 | 1.73 | 1.72 | 1.71 | 1.71 | 1.70 | 1.70 | 1.69 | 1.69 | 1.68 | 1.68 | 1.67 | 1.64 | 1.61 | 1.58 | 1.53 | 24 |] |
| 25 | 1.74 | 1.73 | 1.72 | 1.71 | 1.70 | 1.70 | 1.69 | 1.68 | 1.68 | 1.67 | 1.67 | 1.66 | 1.66 | 1.63 | 1.59 | 1.56 | 1.52 | 25 | |
| 26 | 1.72 | 1.71 | 1.71 | 1.70 | 1.69 | 1.68 | 1.68 | 1.67 | 1.67 | 1.66 | 1.66 | 1.65 | 1.65 | 1.61 | 1.58 | 1.55 | 1.50 | 26 | |
| 27 | 1.71 | 1.70 | 1.70 | 1.69 | 1.68 | 1.67 | 1.67 | 1.66 | 1.65 | 1.65 | 1.64 | 1.64 | 1.64 | 1.60 | 1.57 | 1.54 | 1.49 | 27 | |
| 28 | 1.70 | 1.69 | 1.69 | 1.68 | 1.67 | 1.66 | 1.66 | 1.65 | 1.64 | 1.64 | 1.63 | 1.63 | 1.63 | 1.59 | 1.56 | 1.53 | 1.48 | 28 | 1 |
| 29 | 1.69 | 1.68 | 1.68 | 1.67 | 1.66 | 1.65 | 1.65 | 1.64 | 1.63 | 1.63 | 1.62 | 1.62 | 1.62 | 1.58 | 1.55 | 1.52 | 1.47 | 29 | |
| 30 | 1.69 | 1.68 | 1.67 | 1.66 | 1.65 | 1.64 | 1.64 | 1.63 | 1.63 | 1.62 | 1.62 | 1.61 | 1.61 | 1.57 | 1.54 | 1.51 | 1.46 | 30 | |
| 40 | 1.62 | 1.61 | 1.61 | 1.60 | 1.59 | 1.58 | 1.57 | 1.57 | 1.56 | 1.56 | 1.55 | 1.55 | 1.54 | 1.51 | 1.47 | 1.43 | 1.38 | 40 | |
| 60 | 1.56 | 1.55 | 1.54 | 1.53 | 1.53 | 1.52 | 1.51 | 1.50 | 1.50 | 1.49 | 1.49 | 1.48 | 1.48 | 1.44 | 1.40 | 1.36 | 1.29 | 60 | |
| 100 | 1.52 | 1.50 | 1.49 | 1.48 | 1.48 | 1.47 | 1.46 | 1.45 | 1.45 | 1.44 | 1.43 | 1.43 | 1.42 | 1.38 | 1.34 | 1.29 | 1.21 | 100 |] |
| ∞ | 1.44 | 1.43 | 1.42 | 1.41 | 1.40 | 1.39 | 1.38 | 1.38 | 1.37 | 1.36 | 1.35 | 1.35 | 1.34 | 1.30 | 1.24 | 1.18 | 1.00 | 80 | |

Appendix 21 *F*(0.05) Distribution Table

| | | | | | | | | | | | F _{a,v,v} | <i>a</i> | _ | | | | | | |
|----------------------------------------|-------------------|--------|--------|--------|--------|--------|--------|--------|---------|-------------------|--------------------|----------|--------|--------|--------|--------|--------|--------|-------------------|
| | | | | | | | De | arees | of free | dom fo | r the n | umerat | or | | | | | | |
| | | | | | | | | grees | | (v ₁) | i ule li | unicia | .01 | | | | | | |
| | (v ₂) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | (v ₂) |
| | 1 | 161.45 | 199.50 | 215.71 | 224.58 | 230.16 | 233.99 | 236.77 | 238.88 | 240.54 | 241.88 | 242.98 | 243.91 | 244.69 | 245.36 | 245.95 | 246.46 | 246.92 | 1 |
| | 2 | 18.51 | 19.00 | 19.16 | 19.25 | 19.30 | 19.33 | 19.35 | 19.37 | 19.38 | 19.40 | 19.40 | 19.41 | 19.42 | 19.42 | 19.43 | 19.43 | 19.44 | 2 |
| | 3 | 10.13 | 9.55 | 9.28 | 9.12 | 9.01 | 8.94 | 8.89 | 8.85 | 8.81 | 8.79 | 8.76 | 8.74 | 8.73 | 8.71 | 8.70 | 8.69 | 8.68 | 3 |
| | 4 | 7.71 | 6.94 | 6.59 | 6.39 | 6.26 | 6.16 | 6.09 | 6.04 | 6.00 | 5.96 | 5.94 | 5.91 | 5.89 | 5.87 | 5.86 | 5.84 | 5.83 | 4 |
| | 5 | 6.61 | 5.79 | 5.41 | 5.19 | 5.05 | 4.95 | 4.88 | 4.82 | 4.77 | 4.74 | 4.70 | 4.68 | 4.66 | 4.64 | 4.62 | 4.60 | 4.59 | 5 |
| | 6 | 5.99 | 5.14 | 4.76 | 4.53 | 4.39 | 4.28 | 4.21 | 4.15 | 4.10 | 4.06 | 4.03 | 4.00 | 3.98 | 3.96 | 3.94 | 3.92 | 3.91 | 6 |
| | 7 | 5.59 | 4.74 | 4.35 | 4.12 | 3.97 | 3.87 | 3.79 | 3.73 | 3.68 | 3.64 | 3.60 | 3.57 | 3.55 | 3.53 | 3.51 | 3.49 | 3.48 | 7 |
| | 8 | 5.32 | 4.46 | 4.07 | 3.84 | 3.69 | 3.58 | 3.50 | 3.44 | 3.39 | 3.35 | 3.31 | 3.28 | 3.26 | 3.24 | 3.22 | 3.20 | 3.19 | 8 |
| | 9 | 5.12 | 4.26 | 3.86 | 3.63 | 3.48 | 3.37 | 3.29 | 3.23 | 3.18 | 3.14 | 3.10 | 3.07 | 3.05 | 3.03 | 3.01 | 2.99 | 2.97 | 9 |
| | 10 | 4.96 | 4.10 | 3.71 | 3.48 | 3.33 | 3.22 | 3.14 | 3.07 | 3.02 | 2.98 | 2.94 | 2.91 | 2.89 | 2.86 | 2.85 | 2.83 | 2.81 | 10 |
| | 11 | 4.84 | 3.98 | 3.59 | 3.36 | 3.20 | 3.09 | 3.01 | 2.95 | 2.90 | 2.85 | 2.82 | 2.79 | 2.76 | 2.74 | 2.72 | 2.70 | 2.69 | 11 |
| Ļ | 12 | 4.75 | 3.89 | 3.49 | 3.26 | 3.11 | 3.00 | 2.91 | 2.85 | 2.80 | 2.75 | 2.72 | 2.69 | 2.66 | 2.64 | 2.62 | 2.60 | 2.58 | 12 |
| nato | 13 | 4.67 | 3.81 | 3.41 | 3.18 | 3.03 | 2.92 | 2.83 | 2.77 | 2.71 | 2.67 | 2.63 | 2.60 | 2.58 | 2.55 | 2.53 | 2.51 | 2.50 | 13 |
| omir | 14 | 4.60 | 3.74 | 3.34 | 3.11 | 2.96 | 2.85 | 2.76 | 2.70 | 2.65 | 2.60 | 2.57 | 2.53 | 2.51 | 2.48 | 2.46 | 2.44 | 2.43 | 14 |
| denc | 15 | 4.54 | 3.68 | 3.29 | 3.06 | 2.90 | 2.79 | 2.71 | 2.64 | 2.59 | 2.54 | 2.51 | 2.48 | 2.45 | 2.42 | 2.40 | 2.38 | 2.37 | 15 |
| the c | 16 | 4.49 | 3.63 | 3.24 | 3.01 | 2.85 | 2.74 | 2.66 | 2.59 | 2.54 | 2.49 | 2.46 | 2.42 | 2.40 | 2.37 | 2.35 | 2.33 | 2.32 | 16 |
| for t | 17 | 4.45 | 3.59 | 3.20 | 2.96 | 2.81 | 2.70 | 2.61 | 2.55 | 2.49 | 2.45 | 2.41 | 2.38 | 2.35 | 2.33 | 2.31 | 2.29 | 2.27 | 17 |
| mo | 18 | 4.41 | 3.55 | 3.16 | 2.93 | 2.77 | 2.66 | 2.58 | 2.51 | 2.46 | 2.41 | 2.37 | 2.34 | 2.31 | 2.29 | 2.27 | 2.25 | 2.23 | 18 |
| eed | 19 | 4.38 | 3.52 | 3.13 | 2.90 | 2.74 | 2.63 | 2.54 | 2.48 | 2.42 | 2.38 | 2.34 | 2.31 | 2.28 | 2.26 | 2.23 | 2.21 | 2.20 | 19 |
| of fr | 20 | 4.35 | 3.49 | 3.10 | 2.87 | 2.71 | 2.60 | 2.51 | 2.45 | 2.39 | 2.35 | 2.31 | 2.28 | 2.25 | 2.22 | 2.20 | 2.18 | 2.17 | 20 |
| see | 21 | 4.32 | 3.47 | 3.07 | 2.84 | 2.68 | 2.57 | 2.49 | 2.42 | 2.37 | 2.32 | 2.28 | 2.25 | 2.22 | 2.20 | 2.18 | 2.16 | 2.14 | 21 |
| Degrees of freedom for the denominator | 22 | 4.30 | 3.44 | 3.05 | 2.82 | 2.66 | 2.55 | 2.46 | 2.40 | 2.34 | 2.30 | 2.26 | 2.23 | 2.20 | 2.17 | 2.15 | 2.13 | 2.11 | 22 |
| | 23 | 4.28 | 3.42 | 3.03 | 2.80 | 2.64 | 2.53 | 2.44 | 2.37 | 2.32 | 2.27 | 2.24 | 2.20 | 2.18 | 2.15 | 2.13 | 2.11 | 2.09 | 23 |
| | 24 | 4.26 | 3.40 | 3.01 | 2.78 | 2.62 | 2.51 | 2.42 | 2.36 | 2.30 | 2.25 | 2.22 | 2.18 | 2.15 | 2.13 | 2.11 | 2.09 | 2.07 | 24 |
| | 25 | 4.24 | 3.39 | 2.99 | 2.76 | 2.60 | 2.49 | 2.40 | 2.34 | 2.28 | 2.24 | 2.20 | 2.16 | 2.14 | 2.11 | 2.09 | 2.07 | 2.05 | 25 |
| | 26 | 4.23 | 3.37 | 2.98 | 2.74 | 2.59 | 2.47 | 2.39 | 2.32 | 2.27 | 2.22 | 2.18 | 2.15 | 2.12 | 2.09 | 2.07 | 2.05 | 2.03 | 26 |
| | 27 | 4.21 | 3.35 | 2.96 | 2.73 | 2.57 | 2.46 | 2.37 | 2.31 | 2.25 | 2.20 | 2.17 | 2.13 | 2.10 | 2.08 | 2.06 | 2.04 | 2.02 | 27 |
| | 28 | 4.20 | 3.34 | 2.95 | 2.71 | 2.56 | 2.45 | 2.36 | 2.29 | 2.24 | 2.19 | 2.15 | 2.12 | 2.09 | 2.06 | 2.04 | 2.02 | 2.00 | 28 |
| | 29 | 4.18 | 3.33 | 2.93 | 2.70 | 2.55 | 2.43 | 2.35 | 2.28 | 2.22 | 2.18 | 2.14 | 2.10 | 2.08 | 2.05 | 2.03 | 2.01 | 1.99 | 29 |
| | 30 | 4.17 | 3.32 | 2.92 | 2.69 | 2.53 | 2.42 | 2.33 | 2.27 | 2.21 | 2.16 | 2.13 | 2.09 | 2.06 | 2.04 | 2.01 | 1.99 | 1.98 | 30 |
| | 40 | 4.08 | 3.23 | 2.84 | 2.61 | 2.45 | 2.34 | 2.25 | 2.18 | 2.12 | 2.08 | 2.04 | 2.00 | 1.97 | 1.95 | 1.92 | 1.90 | 1.89 | 40 |
| | 60 | 4.00 | 3.15 | 2.76 | 2.53 | 2.37 | 2.25 | 2.17 | 2.10 | 2.04 | 1.99 | 1.95 | 1.92 | 1.89 | 1.86 | 1.84 | 1.82 | 1.80 | 60 |
| | 100 | 3.94 | 3.09 | 2.70 | 2.46 | 2.31 | 2.19 | 2.10 | 2.03 | 1.97 | 1.93 | 1.89 | 1.85 | 1.82 | 1.79 | 1.77 | 1.75 | 1.73 | 100 |
| | 00 | 3.84 | 3.00 | 2.60 | 2.37 | 2.21 | 2.10 | 2.01 | 1.94 | 1.88 | 1.83 | 1.79 | 1.75 | 1.72 | 1.69 | 1.67 | 1.64 | 1.62 | 00 |



| | | | | | | | | | | $F_{\alpha,\nu_{1},\nu}$ | 2 | | | | | | | | |
|------|--------|--------|--------|--------|--------|--------|--------|---------|-----------|--------------------------|--------|--------|--------|--------|--------|--------|--------|-------------------|-----------------|
| | | | | | | De | grees | of free | dom fo | r the n | umera | or | | | | | | | |
| | | | | | | | | | (ν_1) | | | | | | | | | | |
| (v2) | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 40 | 60 | 100 | ∞ | (v ₂) | |
| 1 | 247.32 | 247.69 | 248.01 | 248.31 | 248.58 | 248.83 | 249.05 | 249.26 | 249.45 | 249.63 | 249.80 | 249.95 | 250.10 | 251.14 | 252.20 | 253.04 | 254.31 | 1 | |
| 2 | 19.44 | 19.44 | 19.45 | 19.45 | 19.45 | 19.45 | 19.45 | 19.46 | 19.46 | 19.46 | 19.46 | 19.46 | 19.46 | 19.47 | 19.48 | 19.49 | 9.49 | 2 | |
| 3 | 8.67 | 8.67 | 8.66 | 8.65 | 8.65 | 8.64 | 8.64 | 8.63 | 8.63 | 8.63 | 8.62 | 8.62 | 8.62 | 8.59 | 8.57 | 8.55 | 5.13 | 3 | |
| 4 | 5.82 | 5.81 | 5.80 | 5.79 | 5.79 | 5.78 | 5.77 | 5.77 | 5.76 | 5.76 | 5.75 | 5.75 | 5.75 | 5.72 | 5.69 | 5.66 | 3.76 | 4 | |
| 5 | 4.58 | 4.57 | 4.56 | 4.55 | 4.54 | 4.53 | 4.53 | 4.52 | 4.52 | 4.51 | 4.50 | 4.50 | 4.50 | 4.46 | 4.43 | 4.41 | 3.10 | 5 | |
| 6 | 3.90 | 3.88 | 3.87 | 3.86 | 3.86 | 3.85 | 3.84 | 3.83 | 3.83 | 3.82 | 3.82 | 3.81 | 3.81 | 3.77 | 3.74 | 3.71 | 2.72 | 6 | |
| 7 | 3.47 | 3.46 | 3.44 | 3.43 | 3.43 | 3.42 | 3.41 | 3.40 | 3.40 | 3.39 | 3.39 | 3.38 | 3.38 | 3.34 | 3.30 | 3.27 | 2.47 | 7 | |
| 8 | 3.17 | 3.16 | 3.15 | 3.14 | 3.13 | 3.12 | 3.12 | 3.11 | 3.10 | 3.10 | 3.09 | 3.08 | 3.08 | 3.04 | 3.01 | 2.97 | 2.29 | 8 | |
| 9 | 2.96 | 2.95 | 2.94 | 2.93 | 2.92 | 2.91 | 2.90 | 2.89 | 2.89 | 2.88 | 2.87 | 2.87 | 2.86 | 2.83 | 2.79 | 2.76 | 2.16 | 9 | |
| 10 | 2.80 | 2.79 | 2.77 | 2.76 | 2.75 | 2.75 | 2.74 | 2.73 | 2.72 | 2.72 | 2.71 | 2.70 | 2.70 | 2.66 | 2.62 | 2.59 | 2.06 | 10 | |
| 11 | 2.67 | 2.66 | 2.65 | 2.64 | 2.63 | 2.62 | 2.61 | 2.60 | 2.59 | 2.59 | 2.58 | 2.58 | 2.57 | 2.53 | 2.49 | 2.46 | 1.97 | 11 | |
| 12 | 2.57 | 2.56 | 2.54 | 2.53 | 2.52 | 2.51 | 2.51 | 2.50 | 2.49 | 2.48 | 2.48 | 2.47 | 2.47 | 2.43 | 2.38 | 2.35 | 1.90 | 12 | Deg |
| 13 | 2.48 | 2.47 | 2.46 | 2.45 | 2.44 | 2.43 | 2.42 | 2.41 | 2.41 | 2.40 | 2.39 | 2.39 | 2.38 | 2.34 | 2.30 | 2.26 | 1.85 | 13 | Degrees |
| 14 | 2.41 | 2.40 | 2.39 | 2.38 | 2.37 | 2.36 | 2.35 | 2.34 | 2.33 | 2.33 | 2.32 | 2.31 | 2.31 | 2.27 | 2.22 | 2.19 | 1.80 | 14 | 오 |
| 15 | 2.35 | 2.34 | 2.33 | 2.32 | 2.31 | 2.30 | 2.29 | 2.28 | 2.27 | 2.27 | 2.26 | 2.25 | 2.25 | 2.20 | 2.16 | 2.12 | 1.76 | 15 | free |
| 16 | 2.30 | 2.29 | 2.28 | 2.26 | 2.25 | 2.24 | 2.24 | 2.23 | 2.22 | 2.21 | 2.21 | 2.20 | 2.19 | 2.15 | 2.11 | 2.07 | 1.72 | 16 | freedom for the |
| 17 | 2.26 | 2.24 | 2.23 | 2.22 | 2.21 | 2.20 | 2.19 | 2.18 | 2.17 | 2.17 | 2.16 | 2.15 | 2.15 | 2.10 | 2.06 | 2.02 | 1.69 | 17 | n foi |
| 18 | 2.22 | 2.20 | 2.19 | 2.18 | 2.17 | 2.16 | 2.15 | 2.14 | 2.13 | 2.13 | 2.12 | 2.11 | 2.11 | 2.06 | 2.02 | 1.98 | 1.66 | 18 | r the |
| 19 | 2.18 | 2.17 | 2.16 | 2.14 | 2.13 | 2.12 | 2.11 | 2.11 | 2.10 | 2.09 | 2.08 | 2.08 | 2.07 | 2.03 | 1.98 | 1.94 | 1.63 | 19 |) dei |
| 20 | 2.15 | 2.14 | 2.12 | 2.11 | 2.10 | 2.09 | 2.08 | 2.07 | 2.07 | 2.06 | 2.05 | 2.05 | 2.04 | 1.99 | 1.95 | 1.91 | 1.61 | 20 | denominator |
| 21 | 2.12 | 2.11 | 2.10 | 2.08 | 2.07 | 2.06 | 2.05 | 2.05 | 2.04 | 2.03 | 2.02 | 2.02 | 2.01 | 1.96 | 1.92 | 1.88 | 1.59 | 21 | inat |
| 22 | 2.10 | 2.08 | 2.07 | 2.06 | 2.05 | 2.04 | 2.03 | 2.02 | 2.01 | 2.00 | 2.00 | 1.99 | 1.98 | 1.94 | 1.89 | 1.85 | 1.57 | 22 | 9 |
| 23 | 2.08 | 2.06 | 2.05 | 2.04 | 2.02 | 2.01 | 2.01 | 2.00 | 1.99 | 1.98 | 1.97 | 1.97 | 1.96 | 1.91 | 1.86 | 1.82 | 1.55 | 23 | |
| 24 | 2.05 | 2.04 | 2.03 | 2.01 | 2.00 | 1.99 | 1.98 | 1.97 | 1.97 | 1.96 | 1.95 | 1.95 | 1.94 | 1.89 | 1.84 | 1.80 | 1.53 | 24 | |
| 25 | 2.04 | 2.02 | 2.01 | 2.00 | 1.98 | 1.97 | 1.96 | 1.96 | 1.95 | 1.94 | 1.93 | 1.93 | 1.92 | 1.87 | 1.82 | 1.78 | 1.52 | 25 | |
| 26 | 2.02 | 2.00 | 1.99 | 1.98 | 1.97 | 1.96 | 1.95 | 1.94 | 1.93 | 1.92 | 1.91 | 1.91 | 1.90 | 1.85 | 1.80 | 1.76 | 1.50 | 26 | |
| 27 | 2.00 | 1.99 | 1.97 | 1.96 | 1.95 | 1.94 | 1.93 | 1.92 | 1.91 | 1.90 | 1.90 | 1.89 | 1.88 | 1.84 | 1.79 | 1.74 | 1.49 | 27 | |
| 28 | 1.99 | 1.97 | 1.96 | 1.95 | 1.93 | 1.92 | 1.91 | 1.91 | 1.90 | 1.89 | 1.88 | 1.88 | 1.87 | 1.82 | 1.77 | 1.73 | 1.48 | 28 | |
| 29 | 1.97 | 1.96 | 1.94 | 1.93 | 1.92 | 1.91 | 1.90 | 1.89 | 1.88 | 1.88 | 1.87 | 1.86 | 1.85 | 1.81 | 1.75 | 1.71 | 1.47 | 29 | |
| 30 | 1.96 | 1.95 | 1.93 | 1.92 | 1.91 | 1.90 | 1.89 | 1.88 | 1.87 | 1.86 | 1.85 | 1.85 | 1.84 | 1.79 | 1.74 | 1.70 | 1.46 | 30 | |
| 40 | 1.87 | 1.85 | 1.84 | 1.83 | 1.81 | 1.80 | 1.79 | 1.78 | 1.77 | 1.77 | 1.76 | 1.75 | 1.74 | 1.69 | 1.64 | 1.59 | 1.38 | 40 | |
| 60 | 1.78 | 1.76 | 1.75 | 1.73 | 1.72 | 1.71 | 1.70 | 1.69 | 1.68 | 1.67 | 1.66 | 1.66 | 1.65 | 1.59 | 1.53 | 1.48 | 1.29 | 60 | |
| 100 | 1.71 | 1.69 | 1.68 | 1.66 | 1.65 | 1.64 | 1.63 | 1.62 | 1.61 | 1.60 | 1.59 | 1.58 | 1.57 | 1.52 | 1.45 | 1.39 | 1.21 | 100 | |
| ~ | 1.60 | 1.59 | 1.57 | 1.56 | 1.54 | 1.53 | 1.52 | 1.51 | 1.50 | 1.49 | 1.48 | 1.47 | 1.46 | 1.39 | 1.32 | 1.24 | 1.00 | 8 | |

Appendix 22 *F*(0.025) Distribution Table

| 542 | Part | VII: Appendices | |
|-----|------|-----------------|--|
|-----|------|-----------------|--|

100

00

5.18

5.02

3.83

3.69

3.25

3.12

2.92

2.79

2.70

2.57

2.54

2.41

2.42

2.29

2.32

2.19

2.24

2.11

2.18

2.05

2.12

1.99

2.08

1.94

2.04

1.90

2.00

1.87

1.97

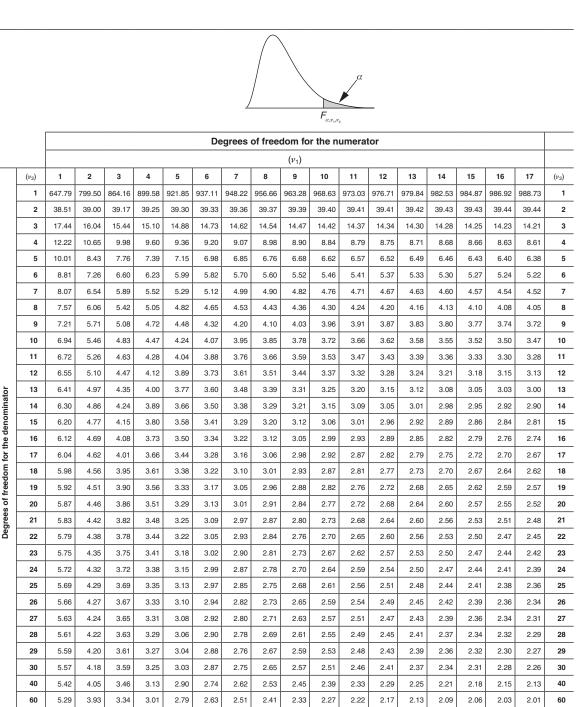
1.83

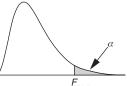
1.94

1.80

1.91 100

1.78 ∞

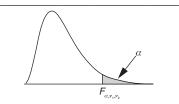




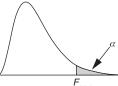
| | | | | | | | _ | | | F_{α,ν_1,ν_2} | 2 | _ | | | | | | | |
|-------------------|--------|--------|--------|--------|--------|--------|--------|---------|-------------------|--------------------------|---------|---------|---------|---------|---------|---------|---------|-------------------|--------------------|
| | | | | | | De | grees | of free | dom fo | r the n | umerat | or | | | | | | | |
| | | | | | | | | | (v ₁) | | | | | | | | | | |
| (v ₂) | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 40 | 60 | 100 | 8 | (v ₂) | |
| 1 | 990.35 | 991.80 | 993.10 | 994.29 | 995.36 | 996.35 | 997.25 | 998.08 | 998.85 | 999.56 | 1000.22 | 1000.84 | 1001.41 | 1005.60 | 1009.80 | 1013.17 | 1018.26 | 1 | |
| 2 | 39.44 | 39.45 | 39.45 | 39.45 | 39.45 | 39.45 | 39.46 | 39.46 | 39.46 | 39.46 | 39.46 | 39.46 | 39.46 | 39.47 | 39.48 | 39.49 | 39.50 | 2 | |
| 3 | 14.20 | 14.18 | 14.17 | 14.16 | 14.14 | 14.13 | 14.12 | 14.12 | 14.11 | 14.10 | 14.09 | 14.09 | 14.08 | 14.04 | 13.99 | 13.96 | 13.90 | 3 | |
| 4 | 8.59 | 8.58 | 8.56 | 8.55 | 8.53 | 8.52 | 8.51 | 8.50 | 8.49 | 8.48 | 8.48 | 8.47 | 8.46 | 8.41 | 8.36 | 8.32 | 8.26 | 4 | |
| 5 | 6.36 | 6.34 | 6.33 | 6.31 | 6.30 | 6.29 | 6.28 | 6.27 | 6.26 | 6.25 | 6.24 | 6.23 | 6.23 | 6.18 | 6.12 | 6.08 | 6.02 | 5 | |
| 6 | 5.20 | 5.18 | 5.17 | 5.15 | 5.14 | 5.13 | 5.12 | 5.11 | 5.10 | 5.09 | 5.08 | 5.07 | 5.07 | 5.01 | 4.96 | 4.92 | 4.85 | 6 | |
| 7 | 4.50 | 4.48 | 4.47 | 4.45 | 4.44 | 4.43 | 4.41 | 4.40 | 4.39 | 4.39 | 4.38 | 4.37 | 4.36 | 4.31 | 4.25 | 4.21 | 4.14 | 7 | |
| 8 | 4.03 | 4.02 | 4.00 | 3.98 | 3.97 | 3.96 | 3.95 | 3.94 | 3.93 | 3.92 | 3.91 | 3.90 | 3.89 | 3.84 | 3.78 | 3.74 | 3.67 | 8 | |
| 9 | 3.70 | 3.68 | 3.67 | 3.65 | 3.64 | 3.63 | 3.61 | 3.60 | 3.59 | 3.58 | 3.58 | 3.57 | 3.56 | 3.51 | 3.45 | 3.40 | 3.33 | 9 | |
| 10 | 3.45 | 3.44 | 3.42 | 3.40 | 3.39 | 3.38 | 3.37 | 3.35 | 3.34 | 3.34 | 3.33 | 3.32 | 3.31 | 3.26 | 3.20 | 3.15 | 3.08 | 10 | |
| 11 | 3.26 | 3.24 | 3.23 | 3.21 | 3.20 | 3.18 | 3.17 | 3.16 | 3.15 | 3.14 | 3.13 | 3.13 | 3.12 | 3.06 | 3.00 | 2.96 | 2.88 | 11 | |
| 12 | 3.11 | 3.09 | 3.07 | 3.06 | 3.04 | 3.03 | 3.02 | 3.01 | 3.00 | 2.99 | 2.98 | 2.97 | 2.96 | 2.91 | 2.85 | 2.80 | 2.72 | 12 | Degrees |
| 13 | 2.98 | 2.96 | 2.95 | 2.93 | 2.92 | 2.91 | 2.89 | 2.88 | 2.87 | 2.86 | 2.85 | 2.85 | 2.84 | 2.78 | 2.72 | 2.67 | 2.60 | 13 | rees |
| 14 | 2.88 | 2.86 | 2.84 | 2.83 | 2.81 | 2.80 | 2.79 | 2.78 | 2.77 | 2.76 | 2.75 | 2.74 | 2.73 | 2.67 | 2.61 | 2.56 | 2.49 | 14 | |
| 15 | 2.79 | 2.77 | 2.76 | 2.74 | 2.73 | 2.71 | 2.70 | 2.69 | 2.68 | 2.67 | 2.66 | 2.65 | 2.64 | 2.59 | 2.52 | 2.47 | 2.40 | 15 | free |
| 16 | 2.72 | 2.70 | 2.68 | 2.67 | 2.65 | 2.64 | 2.63 | 2.61 | 2.60 | 2.59 | 2.58 | 2.58 | 2.57 | 2.51 | 2.45 | 2.40 | 2.32 | 16 | of freedom for the |
| 17 | 2.65 | 2.63 | 2.62 | 2.60 | 2.59 | 2.57 | 2.56 | 2.55 | 2.54 | 2.53 | 2.52 | 2.51 | 2.50 | 2.44 | 2.38 | 2.33 | 2.25 | 17 | n fo |
| 18 | 2.60 | 2.58 | 2.56 | 2.54 | 2.53 | 2.52 | 2.50 | 2.49 | 2.48 | 2.47 | 2.46 | 2.45 | 2.44 | 2.38 | 2.32 | 2.27 | 2.19 | 18 | the |
| 19 | 2.55 | 2.53 | 2.51 | 2.49 | 2.48 | 2.46 | 2.45 | 2.44 | 2.43 | 2.42 | 2.41 | 2.40 | 2.39 | 2.33 | 2.27 | 2.22 | 2.13 | 19 | |
| 20 | 2.50 | 2.48 | 2.46 | 2.45 | 2.43 | 2.42 | 2.41 | 2.40 | 2.39 | 2.38 | 2.37 | 2.36 | 2.35 | 2.29 | 2.22 | 2.17 | 2.09 | 20 | denominato |
| 21 | 2.46 | 2.44 | 2.42 | 2.41 | 2.39 | 2.38 | 2.37 | 2.36 | 2.34 | 2.33 | 2.33 | 2.32 | 2.31 | 2.25 | 2.18 | 2.13 | 2.04 | 21 | iina |
| 22 | 2.43 | 2.41 | 2.39 | 2.37 | 2.36 | 2.34 | 2.33 | 2.32 | 2.31 | 2.30 | 2.29 | 2.28 | 2.27 | 2.21 | 2.14 | 2.09 | 2.00 | 22 | for |
| 23 | 2.39 | 2.37 | 2.36 | 2.34 | 2.33 | 2.31 | 2.30 | 2.29 | 2.28 | 2.27 | 2.26 | 2.25 | 2.24 | 2.18 | 2.11 | 2.06 | 1.97 | 23 | |
| 24 | 2.36 | 2.35 | 2.33 | 2.31 | 2.30 | 2.28 | 2.27 | 2.26 | 2.25 | 2.24 | 2.23 | 2.22 | 2.21 | 2.15 | 2.08 | 2.02 | 1.94 | 24 | |
| 25 | 2.34 | 2.32 | 2.30 | 2.28 | 2.27 | 2.26 | 2.24 | 2.23 | 2.22 | 2.21 | 2.20 | 2.19 | 2.18 | 2.12 | 2.05 | 2.00 | 1.91 | 25 | |
| 26 | 2.31 | 2.29 | 2.28 | 2.26 | 2.24 | 2.23 | 2.22 | 2.21 | 2.19 | 2.18 | 2.17 | 2.17 | 2.16 | 2.09 | 2.03 | 1.97 | 1.88 | 26 | |
| 27 | 2.29 | 2.27 | 2.25 | 2.24 | 2.22 | 2.21 | 2.19 | 2.18 | 2.17 | 2.16 | 2.15 | 2.14 | 2.13 | 2.07 | 2.00 | 1.94 | 1.85 | 27 | |
| 28 | 2.27 | 2.25 | 2.23 | 2.22 | 2.20 | 2.19 | 2.17 | 2.16 | 2.15 | 2.14 | 2.13 | 2.12 | 2.11 | 2.05 | 1.98 | 1.92 | 1.83 | 28 | |
| 29 | 2.25 | 2.23 | 2.21 | 2.20 | 2.18 | 2.17 | 2.15 | 2.14 | 2.13 | 2.12 | 2.11 | 2.10 | 2.09 | 2.03 | 1.96 | 1.90 | 1.81 | 29 | |
| 30 | 2.23 | 2.21 | 2.20 | 2.18 | 2.16 | 2.15 | 2.14 | 2.12 | 2.11 | 2.10 | 2.09 | 2.08 | 2.07 | 2.01 | 1.94 | 1.88 | 1.79 | 30 | |
| 40 | 2.11 | 2.09 | 2.07 | 2.05 | 2.03 | 2.02 | 2.01 | 1.99 | 1.98 | 1.97 | 1.96 | 1.95 | 1.94 | 1.88 | 1.80 | 1.74 | 1.64 | 40 | |
| 60 | 1.98 | 1.96 | 1.94 | 1.93 | 1.91 | 1.90 | 1.88 | 1.87 | 1.86 | 1.85 | 1.83 | 1.82 | 1.82 | 1.74 | 1.67 | 1.60 | 1.48 | 60 | |
| 100 | 1.89 | 1.87 | 1.85 | 1.83 | 1.81 | 1.80 | 1.78 | 1.77 | 1.76 | 1.75 | 1.74 | 1.72 | 1.71 | 1.64 | 1.56 | 1.48 | 1.35 | 100 | |
| 00 | 1.75 | 1.73 | 1.71 | 1.69 | 1.67 | 1.66 | 1.64 | 1.63 | 1.61 | 1.60 | 1.59 | 1.58 | 1.57 | 1.48 | 1.39 | 1.30 | 1.00 | ~ | |

Appendix 23 *F*(0.01) Distribution Table

| 546 | Part | VII: A | ppendices |
|-----|------|--------|-----------|
|-----|------|--------|-----------|



| | | α,y,y ₂ | | | | | | | | | | | | | | | | | |
|----------------------------------------|-------------------|--------------------------------------|---------|---------|---------|---------|---------|---------|---------|-------------------|---------|---------|---------|---------|---------|---------|---------|---------|-------------------|
| | | Degrees of freedom for the numerator | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | (v ₁) | | | | | | | | | |
| | (v ₂) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | (v ₂) |
| | 1 | 4052.18 | 4999.50 | 5403.35 | 5624.58 | 5763.65 | 5858.99 | 5928.36 | 5981.07 | 6022.47 | 6055.85 | 6083.32 | 6106.32 | 6125.86 | 6142.67 | 6157.28 | 6170.10 | 6181.43 | 1 |
| | 2 | 98.50 | 99.00 | 99.17 | 99.25 | 99.30 | 99.33 | 99.36 | 99.37 | 99.39 | 99.40 | 99.41 | 99.42 | 99.42 | 99.43 | 99.43 | 99.44 | 99.44 | 2 |
| | 3 | 34.12 | 30.82 | 29.46 | 28.71 | 28.24 | 27.91 | 27.67 | 27.49 | 27.35 | 27.23 | 27.13 | 27.05 | 26.98 | 26.92 | 26.87 | 26.83 | 26.79 | 3 |
| | 4 | 21.20 | 18.00 | 16.69 | 15.98 | 15.52 | 15.21 | 14.98 | 14.80 | 14.66 | 14.55 | 14.45 | 14.37 | 14.31 | 14.25 | 14.20 | 14.15 | 14.11 | 4 |
| | 5 | 16.26 | 13.27 | 12.06 | 11.39 | 10.97 | 10.67 | 10.46 | 10.29 | 10.16 | 10.05 | 9.96 | 9.89 | 9.82 | 9.77 | 9.72 | 9.68 | 9.64 | 5 |
| | 6 | 13.75 | 10.92 | 9.78 | 9.15 | 8.75 | 8.47 | 8.26 | 8.10 | 7.98 | 7.87 | 7.79 | 7.72 | 7.66 | 7.60 | 7.56 | 7.52 | 7.48 | 6 |
| | 7 | 12.25 | 9.55 | 8.45 | 7.85 | 7.46 | 7.19 | 6.99 | 6.84 | 6.72 | 6.62 | 6.54 | 6.47 | 6.41 | 6.36 | 6.31 | 6.28 | 6.24 | 7 |
| | 8 | 11.26 | 8.65 | 7.59 | 7.01 | 6.63 | 6.37 | 6.18 | 6.03 | 5.91 | 5.81 | 5.73 | 5.67 | 5.61 | 5.56 | 5.52 | 5.48 | 5.44 | 8 |
| | 9 | 10.56 | 8.02 | 6.99 | 6.42 | 6.06 | 5.80 | 5.61 | 5.47 | 5.35 | 5.26 | 5.18 | 5.11 | 5.05 | 5.01 | 4.96 | 4.92 | 4.89 | 9 |
| | 10 | 10.04 | 7.56 | 6.55 | 5.99 | 5.64 | 5.39 | 5.20 | 5.06 | 4.94 | 4.85 | 4.77 | 4.71 | 4.65 | 4.60 | 4.56 | 4.52 | 4.49 | 10 |
| | 11 | 9.65 | 7.21 | 6.22 | 5.67 | 5.32 | 5.07 | 4.89 | 4.74 | 4.63 | 4.54 | 4.46 | 4.40 | 4.34 | 4.29 | 4.25 | 4.21 | 4.18 | 11 |
| | 12 | 9.33 | 6.93 | 5.95 | 5.41 | 5.06 | 4.82 | 4.64 | 4.50 | 4.39 | 4.30 | 4.22 | 4.16 | 4.10 | 4.05 | 4.01 | 3.97 | 3.94 | 12 |
| ator | 13 | 9.07 | 6.70 | 5.74 | 5.21 | 4.86 | 4.62 | 4.44 | 4.30 | 4.19 | 4.10 | 4.02 | 3.96 | 3.91 | 3.86 | 3.82 | 3.78 | 3.75 | 13 |
| mina | 14 | 8.86 | 6.51 | 5.56 | 5.04 | 4.69 | 4.46 | 4.28 | 4.14 | 4.03 | 3.94 | 3.86 | 3.80 | 3.75 | 3.70 | 3.66 | 3.62 | 3.59 | 14 |
| leno | 15 | 8.68 | 6.36 | 5.42 | 4.89 | 4.56 | 4.32 | 4.14 | 4.00 | 3.89 | 3.80 | 3.73 | 3.67 | 3.61 | 3.56 | 3.52 | 3.49 | 3.45 | 15 |
| the c | 16 | 8.53 | 6.23 | 5.29 | 4.77 | 4.44 | 4.20 | 4.03 | 3.89 | 3.78 | 3.69 | 3.62 | 3.55 | 3.50 | 3.45 | 3.41 | 3.37 | 3.34 | 16 |
| for | 17 | 8.40 | 6.11 | 5.18 | 4.67 | 4.34 | 4.10 | 3.93 | 3.79 | 3.68 | 3.59 | 3.52 | 3.46 | 3.40 | 3.35 | 3.31 | 3.27 | 3.24 | 17 |
| dom | 18 | 8.29 | 6.01 | 5.09 | 4.58 | 4.25 | 4.01 | 3.84 | 3.71 | 3.60 | 3.51 | 3.43 | 3.37 | 3.32 | 3.27 | 3.23 | 3.19 | 3.16 | 18 |
| free | 19 | 8.18 | 5.93 | 5.01 | 4.50 | 4.17 | 3.94 | 3.77 | 3.63 | 3.52 | 3.43 | 3.36 | 3.30 | 3.24 | 3.19 | 3.15 | 3.12 | 3.08 | 19 |
| Degrees of freedom for the denominator | 20 | 8.10 | 5.85 | 4.94 | 4.43 | 4.10 | 3.87 | 3.70 | 3.56 | 3.46 | 3.37 | 3.29 | 3.23 | 3.18 | 3.13 | 3.09 | 3.05 | 3.02 | 20 |
| gree | 21 | 8.02 | 5.78 | 4.87 | 4.37 | 4.04 | 3.81 | 3.64 | 3.51 | 3.40 | 3.31 | 3.24 | 3.17 | 3.12 | 3.07 | 3.03 | 2.99 | 2.96 | 21 |
| ď | 22 | 7.95 | 5.72 | 4.82 | 4.31 | 3.99 | 3.76 | 3.59 | 3.45 | 3.35 | 3.26 | 3.18 | 3.12 | 3.07 | 3.02 | 2.98 | 2.94 | 2.91 | 22 |
| | 23 | 7.88 | 5.66 | 4.76 | 4.26 | 3.94 | 3.71 | 3.54 | 3.41 | 3.30 | 3.21 | 3.14 | 3.07 | 3.02 | 2.97 | 2.93 | 2.89 | 2.86 | 23 |
| | 24 | 7.82 | 5.61 | 4.72 | 4.22 | 3.90 | 3.67 | 3.50 | 3.36 | 3.26 | 3.17 | 3.09 | 3.03 | 2.98 | 2.93 | 2.89 | 2.85 | 2.82 | 24 |
| | 25 | 7.77 | 5.57 | 4.68 | 4.18 | 3.85 | 3.63 | 3.46 | 3.32 | 3.22 | 3.13 | 3.06 | 2.99 | 2.94 | 2.89 | 2.85 | 2.81 | 2.78 | 25 |
| | 26 | 7.72 | 5.53 | 4.64 | 4.14 | 3.82 | 3.59 | 3.42 | 3.29 | 3.18 | 3.09 | 3.02 | 2.96 | 2.90 | 2.86 | 2.81 | 2.78 | 2.75 | 26 |
| | 27 | 7.68 | 5.49 | 4.60 | 4.11 | 3.78 | 3.56 | 3.39 | 3.26 | 3.15 | 3.06 | 2.99 | 2.93 | 2.87 | 2.82 | 2.78 | 2.75 | 2.71 | 27 |
| | 28 | 7.64 | 5.45 | 4.57 | 4.07 | 3.75 | 3.53 | 3.36 | 3.23 | 3.12 | 3.03 | 2.96 | 2.90 | 2.84 | 2.79 | 2.75 | 2.72 | 2.68 | 28 |
| | 29 | 7.60 | 5.42 | 4.54 | 4.04 | 3.73 | 3.50 | 3.33 | 3.20 | 3.09 | 3.00 | 2.93 | 2.87 | 2.81 | 2.77 | 2.73 | 2.69 | 2.66 | 29 |
| | 30 | 7.56 | 5.39 | 4.51 | 4.02 | 3.70 | 3.47 | 3.30 | 3.17 | 3.07 | 2.98 | 2.91 | 2.84 | 2.79 | 2.74 | 2.70 | 2.66 | 2.63 | 30 |
| | 40 | 7.31 | 5.18 | 4.31 | 3.83 | 3.51 | 3.29 | 3.12 | 2.99 | 2.89 | 2.80 | 2.73 | 2.66 | 2.61 | 2.56 | 2.52 | 2.48 | 2.45 | 40 |
| | 60 | 7.08 | 4.98 | 4.13 | 3.65 | 3.34 | 3.12 | 2.95 | 2.82 | 2.72 | 2.63 | 2.56 | 2.50 | 2.44 | 2.39 | 2.35 | 2.31 | 2.28 | 60 |
| | 100 | 6.90 | 4.82 | 3.98 | 3.51 | 3.21 | 2.99 | 2.82 | 2.69 | 2.59 | 2.50 | 2.43 | 2.37 | 2.31 | 2.27 | 2.22 | 2.19 | 2.15 | 100 |
| | ∞ | 6.63 | 4.61 | 3.78 | 3.32 | 3.02 | 2.80 | 2.64 | 2.51 | 2.41 | 2.32 | 2.25 | 2.18 | 2.13 | 2.08 | 2.04 | 2.00 | 1.97 | ∞ |



| | | | | | | | | | | F_{α,ν_1,ν_2} | 2 | | | | | | | | |
|-----------------------------------------|---------|---------|---------|---------|---------|---------|---------|---------|-------------------|--------------------------|---------|---------|---------|---------|---------|---------|---------|-------------------|------------|
| | | | | | | De | grees | of free | dom fo | r the n | umerat | or | | | | | | | |
| | | | | | | | | | (v ₁) | | | | | | | | | | |
| (v ₂) | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 40 | 60 | 100 | 8 | (v ₂) | |
| 1 | 6191.53 | 6200.58 | 6208.73 | 6216.12 | 6222.84 | 6228.99 | 6234.63 | 6239.83 | 6244.62 | 6249.07 | 6253.20 | 6257.05 | 6260.65 | 6286.78 | 6313.03 | 6334.11 | 6365.86 | 1 | |
| 2 | 99.44 | 99.45 | 99.45 | 99.45 | 99.45 | 99.46 | 99.46 | 99.46 | 99.46 | 99.46 | 99.46 | 99.46 | 99.47 | 99.47 | 99.48 | 99.49 | 99.50 | 2 | J |
| 3 | 26.75 | 26.72 | 26.69 | 26.66 | 26.64 | 26.62 | 26.60 | 26.58 | 26.56 | 26.55 | 26.53 | 26.52 | 26.50 | 26.41 | 26.32 | 26.24 | 26.13 | 3 | |
| 4 | 14.08 | 14.05 | 14.02 | 13.99 | 13.97 | 13.95 | 13.93 | 13.91 | 13.89 | 13.88 | 13.86 | 13.85 | 13.84 | 13.75 | 13.65 | 13.58 | 13.46 | 4 | |
| 5 | 9.61 | 9.58 | 9.55 | 9.53 | 9.51 | 9.49 | 9.47 | 9.45 | 9.43 | 9.42 | 9.40 | 9.39 | 9.38 | 9.29 | 9.20 | 9.13 | 9.02 | 5 | |
| 6 | 7.45 | 7.42 | 7.40 | 7.37 | 7.35 | 7.33 | 7.31 | 7.30 | 7.28 | 7.27 | 7.25 | 7.24 | 7.23 | 7.14 | 7.06 | 6.99 | 6.88 | 6 | |
| 7 | 6.21 | 6.18 | 6.16 | 6.13 | 6.11 | 6.09 | 6.07 | 6.06 | 6.04 | 6.03 | 6.02 | 6.00 | 5.99 | 5.91 | 5.82 | 5.75 | 5.65 | 7 | |
| 8 | 5.41 | 5.38 | 5.36 | 5.34 | 5.32 | 5.30 | 5.28 | 5.26 | 5.25 | 5.23 | 5.22 | 5.21 | 5.20 | 5.12 | 5.03 | 4.96 | 4.86 | 8 | |
| 9 | 4.86 | 4.83 | 4.81 | 4.79 | 4.77 | 4.75 | 4.73 | 4.71 | 4.70 | 4.68 | 4.67 | 4.66 | 4.65 | 4.57 | 4.48 | 4.41 | 4.31 | 9 | |
| 10 | 4.46 | 4.43 | 4.41 | 4.38 | 4.36 | 4.34 | 4.33 | 4.31 | 4.30 | 4.28 | 4.27 | 4.26 | 4.25 | 4.17 | 4.08 | 4.01 | 3.91 | 10 | |
| 11 | 4.15 | 4.12 | 4.10 | 4.08 | 4.06 | 4.04 | 4.02 | 4.01 | 3.99 | 3.98 | 3.96 | 3.95 | 3.94 | 3.86 | 3.78 | 3.71 | 3.60 | 11 | |
| 12 | 3.91 | 3.88 | 3.86 | 3.84 | 3.82 | 3.80 | 3.78 | 3.76 | 3.75 | 3.74 | 3.72 | 3.71 | 3.70 | 3.62 | 3.54 | 3.47 | 3.36 | 12 | Degrees |
| 13 | 3.72 | 3.69 | 3.66 | 3.64 | 3.62 | 3.60 | 3.59 | 3.57 | 3.56 | 3.54 | 3.53 | 3.52 | 3.51 | 3.43 | 3.34 | 3.27 | 3.17 | 13 | rees |
| 14 | 3.56 | 3.53 | 3.51 | 3.48 | 3.46 | 3.44 | 3.43 | 3.41 | 3.40 | 3.38 | 3.37 | 3.36 | 3.35 | 3.27 | 3.18 | 3.11 | 3.00 | 14 | 으 |
| 15 | 3.42 | 3.40 | 3.37 | 3.35 | 3.33 | 3.31 | 3.29 | 3.28 | 3.26 | 3.25 | 3.24 | 3.23 | 3.21 | 3.13 | 3.05 | 2.98 | 2.87 | 15 | free |
| 16 | 3.31 | 3.28 | 3.26 | 3.24 | 3.22 | 3.20 | 3.18 | 3.16 | 3.15 | 3.14 | 3.12 | 3.11 | 3.10 | 3.02 | 2.93 | 2.86 | 2.75 | 16 | freedom |
| 17 | 3.21 | 3.19 | 3.16 | 3.14 | 3.12 | 3.10 | 3.08 | 3.07 | 3.05 | 3.04 | 3.03 | 3.01 | 3.00 | 2.92 | 2.83 | 2.76 | 2.65 | 17 | n for |
| 18 | 3.13 | 3.10 | 3.08 | 3.05 | 3.03 | 3.02 | 3.00 | 2.98 | 2.97 | 2.95 | 2.94 | 2.93 | 2.92 | 2.84 | 2.75 | 2.68 | 2.57 | 18 | r the |
| 19 | 3.05 | 3.03 | 3.00 | 2.98 | 2.96 | 2.94 | 2.92 | 2.91 | 2.89 | 2.88 | 2.87 | 2.86 | 2.84 | 2.76 | 2.67 | 2.60 | 2.49 | 19 | e de |
| 20 | 2.99 | 2.96 | 2.94 | 2.92 | 2.90 | 2.88 | 2.86 | 2.84 | 2.83 | 2.81 | 2.80 | 2.79 | 2.78 | 2.69 | 2.61 | 2.54 | 2.42 | 20 | denominato |
| 21 | 2.93 | 2.90 | 2.88 | 2.86 | 2.84 | 2.82 | 2.80 | 2.79 | 2.77 | 2.76 | 2.74 | 2.73 | 2.72 | 2.64 | 2.55 | 2.48 | 2.36 | 21 | nina |
| 22 | 2.88 | 2.85 | 2.83 | 2.81 | 2.78 | 2.77 | 2.75 | 2.73 | 2.72 | 2.70 | 2.69 | 2.68 | 2.67 | 2.58 | 2.50 | 2.42 | 2.31 | 22 | ę |
| 23 | 2.83 | 2.80 | 2.78 | 2.76 | 2.74 | 2.72 | 2.70 | 2.69 | 2.67 | 2.66 | 2.64 | 2.63 | 2.62 | 2.54 | 2.45 | 2.37 | 2.26 | 23 | |
| 24 | 2.79 | 2.76 | 2.74 | 2.72 | 2.70 | 2.68 | 2.66 | 2.64 | 2.63 | 2.61 | 2.60 | 2.59 | 2.58 | 2.49 | 2.40 | 2.33 | 2.21 | 24 | |
| 25 | 2.75 | 2.72 | 2.70 | 2.68 | 2.66 | 2.64 | 2.62 | 2.60 | 2.59 | 2.58 | 2.56 | 2.55 | 2.54 | 2.45 | 2.36 | 2.29 | 2.17 | 25 | |
| 26 | 2.72 | 2.69 | 2.66 | 2.64 | 2.62 | 2.60 | 2.58 | 2.57 | 2.55 | 2.54 | 2.53 | 2.51 | 2.50 | 2.42 | 2.33 | 2.25 | 2.13 | 26 | |
| 27 | 2.68 | 2.66 | 2.63 | 2.61 | 2.59 | 2.57 | 2.55 | 2.54 | 2.52 | 2.51 | 2.49 | 2.48 | 2.47 | 2.38 | 2.29 | 2.22 | 2.10 | 27 | |
| 28 | 2.65 | 2.63 | 2.60 | 2.58 | 2.56 | 2.54 | 2.52 | 2.51 | 2.49 | 2.48 | 2.46 | 2.45 | 2.44 | 2.35 | 2.26 | 2.19 | 2.06 | 28 | |
| 29 | 2.63 | 2.60 | 2.57 | 2.55 | 2.53 | 2.51 | 2.49 | 2.48 | 2.46 | 2.45 | 2.44 | 2.42 | 2.41 | 2.33 | 2.23 | 2.16 | 2.03 | 29 | |
| 30 | 2.60 | 2.57 | 2.55 | 2.53 | 2.51 | 2.49 | 2.47 | 2.45 | 2.44 | 2.42 | 2.41 | 2.40 | 2.39 | 2.30 | 2.21 | 2.13 | 2.01 | 30 | |
| 40 | 2.42 | 2.39 | 2.37 | 2.35 | 2.33 | 2.31 | 2.29 | 2.27 | 2.26 | 2.24 | 2.23 | 2.22 | 2.20 | 2.11 | 2.02 | 1.94 | 1.80 | 40 | 1 |
| 60 | 2.25 | 2.22 | 2.20 | 2.17 | 2.15 | 2.13 | 2.12 | 2.10 | 2.08 | 2.07 | 2.05 | 2.04 | 2.03 | 1.94 | 1.84 | 1.75 | 1.60 | 60 | |
| 100 | 2.12 | 2.09 | 2.07 | 2.04 | 2.02 | 2.00 | 1.98 | 1.97 | 1.95 | 1.93 | 1.92 | 1.91 | 1.89 | 1.80 | 1.69 | 1.60 | 1.43 | 100 | |
| ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~ | 1.93 | 1.90 | 1.88 | 1.85 | 1.83 | 1.81 | 1.79 | 1.77 | 1.76 | 1.74 | 1.72 | 1.71 | 1.70 | 1.59 | 1.47 | 1.36 | 1.00 | 8 | |

Appendix 24

Equivalent Sigma Levels, Percent Defective, and PPM Values

| | With no sigma | shift (centere | ed) | With 1.5 sigma shift | | | | | | | |
|----------------|--------------------------|-------------------|--------|----------------------|--------------------------|-------------------|--------|--|--|--|--|
| Sigma level | Percent in specification | Percent defective | РРМ | Sigma level | Percent in specification | Percent defective | РРМ | | | | |
| 0.10 | 7.9656 | 92.0344 | 920344 | 0.10 | 2.5957 | 97.40426 | 974043 | | | | |
| 0.20 | 15.8519 | 84.1481 | 841481 | 0.20 | 5.2235 | 94.77650 | 947765 | | | | |
| 0.30 | 23.5823 | 76.4177 | 764177 | 0.30 | 7.9139 | 92.08606 | 920861 | | | | |
| 0.40 | 31.0843 | 68.9157 | 689157 | 0.40 | 10.6950 | 89.30505 | 893050 | | | | |
| 0.50 | 38.2925 | 61.7075 | 617075 | 0.50 | 13.5905 | 86.40949 | 864095 | | | | |
| 0.60 | 45.1494 | 54.8506 | 548506 | 0.60 | 16.6196 | 83.38043 | 833804 | | | | |
| 0.70 | 51.6073 | 48.3927 | 483927 | 0.70 | 19.7952 | 80.20480 | 802048 | | | | |
| 0.80 | 57.6289 | 42.3711 | 423711 | 0.80 | 23.1240 | 76.87605 | 768760 | | | | |
| 0.90 | 63.1880 | 36.8120 | 368120 | 0.90 | 26.6056 | 73.39444 | 733944 | | | | |
| 1.00 | 68.2689 | 31.7311 | 317311 | 1.00 | 30.2328 | 69.76721 | 697672 | | | | |
| 1.10 | 72.8668 | 27.1332 | 271332 | 1.10 | 33.9917 | 66.00829 | 660083 | | | | |
| 1.20 | 76.9861 | 23.0139 | 230139 | 1.20 | 37.8622 | 62.13784 | 621378 | | | | |
| 1.30 | 80.6399 | 19.3601 | 193601 | 1.30 | 41.8185 | 58.18148 | 581815 | | | | |
| 1.40 | 83.8487 | 16.1513 | 161513 | 1.40 | 45.8306 | 54.16937 | 541694 | | | | |
| 1.50 | 86.6386 | 13.3614 | 133614 | 1.50 | 49.8650 | 50.13499 | 501350 | | | | |
| 1.60 | 89.0401 | 10.9599 | 109599 | 1.60 | 53.8860 | 46.11398 | 461140 | | | | |
| 1.70 | 91.0869 | 8.9131 | 89131 | 1.70 | 57.8573 | 42.14274 | 421427 | | | | |
| 1.80 | 92.8139 | 7.1861 | 71861 | 1.80 | 61.7428 | 38.25720 | 382572 | | | | |
| 1.90 | 94.2567 | 5.7433 | 57433 | 1.90 | 65.5085 | 34.49152 | 344915 | | | | |
| 2.00 | 95.4500 | 4.5500 | 45500 | 2.00 | 69.1230 | 30.87702 | 308770 | | | | |
| 2.10 | 96.4271 | 3.5729 | 35729 | 2.10 | 72.5588 | 27.44122 | 274412 | | | | |
| 2.20 | 97.2193 | 2.7807 | 27807 | 2.20 | 75.7929 | 24.20715 | 242071 | | | | |
| 2.30 | 97.8552 | 2.1448 | 21448 | 2.30 | 78.8072 | 21.19277 | 211928 | | | | |
| 2.40 | 98.3605 | 1.6395 | 16395 | 2.40 | 81.5892 | 18.41082 | 184108 | | | | |
| 2.50 | 98.7581 | 1.2419 | 12419 | 2.50 | 84.1313 | 15.86869 | 158687 | | | | |
| 2.60 | 99.0678 | 0.9322 | 9322 | 2.60 | 86.4313 | 13.56867 | 135687 | | | | |

| | With no sigma | shift (centere | ed) | With 1.5 sigma shift | | | | | | | |
|----------------|--------------------------|-------------------|-------|----------------------|--------------------------|-------------------|--------|--|--|--|--|
| Sigma level | Percent in specification | Percent defective | РРМ | Sigma level | Percent in specification | Percent defective | РРМ | | | | |
| 2.70 | 99.3066 | 0.6934 | 6934 | 2.70 | 88.4917 | 11.50830 | 115083 | | | | |
| 2.80 | 99.4890 | 0.5110 | 5110 | 2.80 | 90.3191 | 9.68090 | 96809 | | | | |
| 2.90 | 99.6268 | 0.3732 | 3732 | 2.90 | 91.9238 | 8.07621 | 80762 | | | | |
| 3.00 | 99.7300 | 0.2700 | 2700 | 3.00 | 93.3189 | 6.68106 | 66811 | | | | |
| 3.10 | 99.8065 | 0.1935 | 1935 | 3.10 | 94.5199 | 5.48014 | 54801 | | | | |
| 3.20 | 99.8626 | 0.1374 | 1374 | 3.20 | 95.5433 | 4.45668 | 44567 | | | | |
| 3.30 | 99.9033 | 0.0967 | 967 | 3.30 | 96.4069 | 3.59311 | 35931 | | | | |
| 3.40 | 99.9326 | 0.0674 | 674 | 3.40 | 97.1283 | 2.87170 | 28717 | | | | |
| 3.50 | 99.9535 | 0.0465 | 465 | 3.50 | 97.7250 | 2.27504 | 22750 | | | | |
| 3.60 | 99.9682 | 0.0318 | 318 | 3.60 | 98.2135 | 1.78646 | 17865 | | | | |
| 3.70 | 99.9784 | 0.0216 | 216 | 3.70 | 98.6096 | 1.39035 | 13904 | | | | |
| 3.80 | 99.9855 | 0.0145 | 145 | 3.80 | 98.9276 | 1.07242 | 10724 | | | | |
| 3.90 | 99.9904 | 0.0096 | 96.2 | 3.90 | 99.1802 | 0.81976 | 8198 | | | | |
| 4.00 | 99.9937 | 0.0063 | 63.3 | 4.00 | 99.3790 | 0.62097 | 6210 | | | | |
| 4.10 | 99.9959 | 0.0041 | 41.3 | 4.10 | 99.5339 | 0.46612 | 4661 | | | | |
| 4.20 | 99.9973 | 0.0027 | 26.7 | 4.20 | 99.6533 | 0.34670 | 3467 | | | | |
| 4.30 | 99.9983 | 0.0017 | 17.1 | 4.30 | 99.7445 | 0.25551 | 2555 | | | | |
| 4.40 | 99.9989 | 0.0011 | 10.8 | 4.40 | 99.8134 | 0.18658 | 1866 | | | | |
| 4.50 | 99.9993 | 0.0007 | 6.8 | 4.50 | 99.8650 | 0.13499 | 1350 | | | | |
| 4.60 | 99.9996 | 0.0004 | 4.2 | 4.60 | 99.9032 | 0.09676 | 968 | | | | |
| 4.70 | 99.9997 | 0.0003 | 2.6 | 4.70 | 99.9313 | 0.06871 | 687 | | | | |
| 4.80 | 99.9998 | 0.0002 | 1.6 | 4.80 | 99.9517 | 0.04834 | 483 | | | | |
| 4.90 | 99.99990 | 0.00010 | 1.0 | 4.90 | 99.9663 | 0.03369 | 337 | | | | |
| 5.00 | 99.99994 | 0.00006 | 0.6 | 5.00 | 99.9767 | 0.02326 | 233 | | | | |
| 5.10 | 99.99997 | 0.00003 | 0.3 | 5.10 | 99.9841 | 0.01591 | 159 | | | | |
| 5.20 | 99.99998 | 0.00002 | 0.2 | 5.20 | 99.9892 | 0.01078 | 108 | | | | |
| 5.30 | 99.999988 | 0.000012 | 0.12 | 5.30 | 99.9928 | 0.00723 | 72.3 | | | | |
| 5.40 | 99.999993 | 0.000007 | 0.07 | 5.40 | 99.9952 | 0.00481 | 48.1 | | | | |
| 5.50 | 99.999996 | 0.000004 | 0.04 | 5.50 | 99.9968 | 0.00317 | 31.7 | | | | |
| 5.60 | 99.999998 | 0.000002 | 0.02 | 5.60 | 99.9979 | 0.00207 | 20.7 | | | | |
| 5.70 | 99.9999988 | 0.0000012 | 0.012 | 5.70 | 99.9987 | 0.00133 | 13.3 | | | | |
| 5.80 | 99.9999993 | 0.0000007 | 0.007 | 5.80 | 99.9991 | 0.00085 | 8.5 | | | | |
| 5.90 | 99.9999996 | 0.0000004 | 0.004 | 5.90 | 99.9995 | 0.00054 | 5.4 | | | | |
| 6.00 | 99.9999998 | 0.0000002 | 0.002 | 6.00 | 99.9997 | 0.00034 | 3.4 | | | | |

Appendix 25

Glossary of Lean Six Sigma and Related Terms

A

- **acceptance control chart**—A control chart intended primarily to evaluate whether the plotted measure can be expected to satisfy specified tolerances.
- acceptance quality limit (AQL)—The quality level that is the worst tolerable product average when a continuing series of lots is submitted for acceptance sampling. Note the following: (1) This concept applies only when an acceptance sampling scheme with rules for switching and discontinuation is used. (2) Although individual lots with quality as bad as the AQL can be accepted with fairly high probability, the designation of an AQL does not suggest that this is a desirable quality level. (3) Acceptance sampling schemes found in standards with their rules for switching and discontinuation of sampling inspection are designed to encourage suppliers to have process averages consistently better than the AQL. If suppliers fail to do so, there is a high probability of being switched from normal inspection to tightened inspection, where lot acceptance becomes more difficult. Once on tightened inspection, unless corrective action is taken to improve product quality, it is very likely that the rule requiring discontinuation of sampling inspection pending such improvement will be invoked unless action is taken to improve the process. (4) The use of the abbreviation AQL to mean acceptable quality level is no longer recommended since modern thinking is that no fraction defective is really acceptable. Using "acceptance quality limit" rather than "acceptable quality level" indicates a technical value where acceptance occurs.
- **accuracy**—The closeness of agreement between a test result or measurement result and the true or reference value.
- action plan—See operational plan.
- activity—An action of some type that requires a time duration for accomplishment.
- **activity network diagram (AND)**—A tool used to illustrate a sequence of events or activities (nodes) and the interconnectivity of such nodes. It is used for scheduling and especially for determining the critical path through nodes. It is also known as an *arrow diagram*. The activity network diagram is one of the seven management and planning tools.

- activity-based costing (ABC)—A cost allocation method that allocates overhead expenses to activities based on the proportion of use, rather than proportion of costs.
- actual cost of work performed (ACWP)—See actual value.
- **actual value (AV)**—This is the actual cost incurred for work completed by a specific date. It is also known as the *actual cost of work performed* (ACWP).

ACWP—See actual value.

- adjourning—The fifth stage of team growth. See also stages of team growth.
- adjusted coefficient of determination (R^2_{adj}) —A statistic that takes into consideration the number of predictor variables in the model and is helpful when comparing models with a different number of predictor variables. It is computed as follows:

$$R_{adj}^{2} = 1 - \left(\frac{\sum_{i=1}^{n} (y_{i} - \hat{y}_{i})^{2}}{\left(\sum_{i=1}^{n} y_{i} - \overline{y}\right)^{2}}\right) \left(\frac{n-1}{n-p-1}\right)$$

where

n = number of observations

 $y_i = i$ th observed response value

 \overline{y}_i = mean observed response value

 $\hat{y}_i = i$ th fitted value

p = number of terms in the model including the constant term

Notice that when p = 2 (that is, simple linear regression), R^2 equals R^2_{adj} . Sometimes, R^2_{adj} is written as r^2_{adj} .

affinity diagram—A tool used to organize information and help achieve order out of the chaos that can develop in a brainstorming session. Large amounts of data, concepts, and ideas are grouped on the basis of their natural relationship to one another. It is more a creative process than a logical process. Also known as the *KJ method*, the affinity diagram is one of the seven management and planning tools.

agile approach—See Lean.

AIAG—Automotive Industry Action Group.

alias—An effect that is completely confounded with another effect due to the nature of the designed experiment. Aliases are the result of confounding, which may or may not be deliberate.

- **alpha** (α)—(1) The maximum probability or risk of making a Type I error when dealing with the significance level of a test. (2) The probability or risk of incorrectly deciding that a shift in the process mean has occurred when in fact the process has not changed (when referring to α in general or the *p*-value obtained in the test). (3) α is usually designated as *producer's risk*.
- **alpha** (*α*) **risk**—*See* Type I error.
- alternative hypothesis—A hypothesis formulated from new information.
- **analysis of means (ANOM)**—A statistical procedure for troubleshooting industrial processes and analyzing the results of experimental designs with factors at fixed levels. It provides a graphical display of data. Ellis R. Ott developed the procedure in 1967 because he observed that non-statisticians had difficulty understanding analysis of variance. Analysis of means is easier for quality practitioners to use because it is an extension of the control chart. In 1973, Edward G. Schilling further extended the concept, enabling analysis of means to be used with nonnormal distributions and attributes data where the normal approximation to the binomial distribution does not apply. This is referred to as *analysis of means for treatment effects*.
- **analysis of variance (ANOVA)**—A basic statistical technique for analyzing experimental data. It subdivides the total variation of a data set into meaningful component parts associated with specific sources of variation in order to test a hypothesis on the parameters of the model or to estimate variance components. There are three model types: fixed, random, and mixed.
- **analytical thinking**—Breaking down a problem or situation into discrete parts to understand how each part contributes to the whole.
- AND—See activity network diagram.
- **andon board**—A visual device (usually lights) displaying status alerts that can easily be seen by those who should respond.
- ANOM—See analysis of means.
- ANOVA—See analysis of variance.
- ANSI—American National Standards Institute.
- APC—See automated process control.
- **appraisal costs**—The costs associated with measuring, evaluating, or auditing products or services to ensure conformance to quality standards and performance requirements. These include costs such as incoming and source inspection/test of purchased material; in-process and final inspection/test; product, process, or service audit; calibration of measuring and test equipment; and the cost of associated supplies and materials.

AQL—See acceptance quality limit.

ARIMA—See autoregressive integrated moving average.

arrow diagram—See activity network diagram.

- **AS-9100**—A standard for the aeronautics industry embracing the ISO 9001 standard.
- ASQ—American Society for Quality.
- **assessment of training effectiveness**—According to the Manpower Services Commission (MSC) "Glossary of Training Terms": "A general term for the process of ascertaining whether training is efficient or effective in achieving prescribed objectives. It covers both evaluation and validation."
- **assignable cause**—A specifically identified factor that contributes to variation and is detectable. Eliminating assignable causes so that the points plotted on a control chart remain within the control limits helps achieve a state of statistical control. Note: Although *assignable cause* is sometimes considered synonymous with *special cause*, a special cause is assignable only when it is specifically identified. *See also* special cause.
- **assumption**—A condition that must be true in order for a statistical procedure to be valid.
- **attribute data**—Data that are categorized for analysis or evaluation. (Attribute data may involve measurements as long as they are used only to place given data in a category for further analysis or evaluation. Contrast with *variables data*.)
- attrition—Refers to loss of people to jobs outside the organization.
- **autocorrelation**—The internal correlation between members of series observations ordered in time.
- **automated process control (APC)**—The application of automation to effect process control by maintaining manipulated variables at set point levels such that controlled variables are maintained at specified levels. Generally, automation allows for a greater number of manipulated variables to be controlled with greater speed and accuracy than possible by human intervention.
- autonomation—See jidoka.
- **autoregressive integrated moving average (ARIMA)**—According to the SAS Institute, Inc., "an ARIMA model predicts a value in a response time series as a linear combination of its own past values, past errors (also called *shocks* or *innovations*), and current and past values of other time series."

AV—See actual value.

- avoidance, risk—Refers to the practice of eliminating the risk factor.
- **axiomatic design**—A theory by Dr. Nam Pyo Suh stressing that each functional requirement be designed to achieve robustness without affecting other functional requirements.

В

BAC—See budget at completion.

- **baka-yoke**—A term for a manufacturing technique for preventing mistakes by designing the manufacturing process, equipment, and tools so that an operation literally can not be performed incorrectly. In addition to preventing incorrect operation, the technique usually provides a warning signal of some sort for incorrect performance.
- **balanced design**—A design where all treatment combinations have the same number of observations. If replication in a design exists, it would be balanced only if the replication were consistent across all treatment combinations. In other words, the number of replicates of each treatment combination is the same.
- **balanced incomplete block design**—Incomplete block design in which each block contains the same number *k* of different levels from the *l* levels of the principal factor arranged so that every pair of levels occurs in the same number *l* of blocks from the *b* blocks. Note: This design implies that every level of the principal factor will appear the same number of times in the experiment.
- **balanced scorecard**—This translates an organization's mission and strategy into a comprehensive set of performance measures to provide a basis for strategic measurement and management, utilizing four balanced views: financial, customers, internal business processes, and learning and growth.
- **batch processing**—Running large batches of a single product through the process at one time, resulting in queues awaiting the next steps in the process.
- **bathtub curve**—A graphic representation of the relationship of the life of a product versus the probable failure rate. The curve contains three phases: early or infant failure (break-in), a stable rate during normal use, and wear-out. The bathtub curve is also known as the *life history curve*.
- BB—See Black Belt.
- BCWP—See earned value.
- BCWS—See planned value.
- **behavior**, **directive**—Establishes a clear path to a goal by specifying what is to be done, how it is to be done, and who is going to do it.
- **behavior**, **supportive**—Involves two-way communication and focuses on emotional and social support.
- **benchmark**—An organization, a part of an organization, or a measurement that serves as a reference point or point of comparison.
- **benchmarking**—An improvement process in which an organization measures its performance against that of best-in-class organizations (or others that are good performers), determines how those organizations achieved their performance levels, and uses the information to improve its own performance. Areas that can be benchmarked include strategies, operations, processes, and procedures.

- **benefit-cost analysis**—A collection of the dollar value of benefits derived from an initiative divided by the associated costs incurred. A benefit-cost analysis is also known as a *cost-benefit analysis*.
- benefit-cost ratio—The formula for computing a benefit-cost ratio is:

$$\overline{\sum}$$
NPV of all benefits anticipated $\overline{\sum}$ NPV of all costs anticipated

Occasionally, it is referred to as a cost-benefit ratio, which is the reciprocal.

- **best subsets**—A very useful tool to help select a regression model among many other viable models. It is based on the concept that for a model comprising m predictor variables, there are $2^m 1$ possible subsets of predictor variables possible. Each model can be compared using such measures as VIF, Mallows's C_{pr} PRESS, R_{adj}^2 , and squared error to aid in the model selection.
- **beta** (β)—(1) The maximum probability or risk of making a Type II error. (2) The probability or risk of incorrectly deciding that a shift in the process mean has not occurred when in fact the process has changed. (3) β is usually designated as *consumer's risk*.
- beta (β) risk—See Type II error.
- **bias**—A systematic difference between the mean of the test result or measurement result and a true or reference value. For example, if one measures the lengths of 10 pieces of rope that range from 1 foot to 10 feet and always concludes that the length of each piece is 2 inches shorter than the true length, then the individual is exhibiting a bias of 2 inches. Bias is a component of accuracy.
- **Black Belt (BB)**—A Lean Six Sigma role associated with an individual who is typically assigned full-time to train and mentor Green Belts as well as lead improvement projects using specified methodologies such as define, measure, analyze, improve, and control (DMAIC) and design for Six Sigma (DFSS).
- **block**—A collection of experimental units more homogeneous than the full set of experimental units. Blocks are usually selected to allow for special causes, in addition to those introduced as factors to be studied. These special causes may be avoidable within blocks, thus providing a more homogeneous experimental subspace.
- **block diagram**—A diagram that describes the operation, interrelationships, and interdependencies of components in a system. Boxes, or blocks (hence the name), represent the components; connecting lines between the blocks represent interfaces. There are two types of block diagrams: a *functional block diagram*, which shows a system's subsystems and lower-level products, their interrelationships, and interfaces with other systems, and a *reliability block diagram*, which is similar to the functional block diagram except that it is modified to emphasize those aspects influencing reliability.

- **block effect**—An effect resulting from a block in an experimental design. Existence of a block effect generally means that the method of blocking was appropriate and that an assignable cause has been found.
- **blocking**—Including blocks in an experiment in order to broaden the applicability of the conclusions or minimize the impact of selected assignable causes. The randomization of the experiment is restricted and occurs within blocks.
- BPM—See business process management.
- **brainstorming**—A problem-solving tool that teams use to generate as many ideas as possible that are related to a particular subject. Team members begin by offering all their ideas; the ideas are not discussed or reviewed until after the brainstorming session.
- **break-even point**—Conducted as an analysis, it is a useful approach when there are multiple alternatives, each of which may be economical under its own set of conditions. The break-even point is determined by altering one variable while holding all others constant. The variable generally represents some passage of time or number of units processed and can be calculated or determined graphically to identify where the alternatives are equal from an economic viewpoint. This point of equality is known as the break-even point.
- **budget at completion (BAC)**—This is the total planned value for a given task at the end of the project.
- budgeted cost of work performed (BCWP)—See earned value.
- budgeted cost of work scheduled (BCWS)—See planned value.
- **bullwhip effect**—When an organization changes the schedule or the demand for product frequently, these perturbations are rippled upstream throughout the value chain and amplified along the way, sending the supply chain into turmoil. This is known as the bullwhip effect. Some organizations have minimized the bullwhip effect by batching changes and releasing the changes at less frequent intervals. This introduces stability into the value chain.
- **business process management (BPM)**—A set of activities an organization implements to optimize its processes. Business process management encompasses both software tools and manual processes designed to assist organizations in achieving process optimization.

С

- **calibration**—The comparison of a measurement instrument or system of unverified accuracy to a measurement instrument or system of known accuracy to detect any variation from the true value.
- **capability**—The performance of a process demonstrated to be in a state of statistical control. *See also* process capability; process performance.
- capability index—See process capability index.

- **Capability Maturity Model (CMM)**—The capability maturity model was originally a tool developed by Carnegie Melon University to formally assess an organization's software development processes. However, it has evolved to include other organizational processes.
- **Capability Maturity Model Integration (CMMI)**—CMMI represents the evolution of the CMM to include a model for improving processes in the areas of: product and service development; service establishment, management, and delivery; and product and service acquisition.
- **causal factor**—A variable that when changed or manipulated in some manner serves to influence a given effect or result.
- **cause-and-effect analysis**—The process of identifying the likely causes of any outcome. Some cause-and-effect analyses produce an output like a fishbone diagram. Others produce a cause-and-effect tree with multiple sub-branches.
- **cause-and-effect diagram**—A diagram resembling a fish skeleton that is used to illustrate the main causes and sub-causes leading to an effect. One of the seven basic tools of quality, the cause-and-effect diagram is also known as the *Ishikawa diagram* or the *fishbone diagram*.
- **central limit theorem (CLT)**—A theorem stating that irrespective of the shape of the distribution of a population, the distribution of sample means is approximately normal when the sample size is large.
- **central tendency**—The propensity of data collected on a process to concentrate around a value situated approximately midway between the lowest and highest values. Three common measures of central tendency include (arithmetic) mean, median, and mode. *See also* mean; median; mode.
- **centralization versus decentralization**—In the context of organizational design, this refers to how much decision-making authority has been delegated to lower management levels.
- Certified Six Sigma Black Belt (CSSBB)—An ASQ certification.
- Certified Six Sigma Green Belt (CSSGB)—An ASQ certification.
- Certified Six Sigma Master Black Belt (CSSMBB)—An ASQ certification.
- **CFM**—*See* continuous flow manufacturing.
- **chaku-chaku**—A Japanese term meaning "load-load" in a cell layout where a part is taken from one machine and loaded into the next.
- **champion**—A Lean Six Sigma role of a senior executive who ensures that his or her projects are aligned with the organization's strategic goals and priorities, provides the Lean Six Sigma team with resources, removes organizational barriers for the team, participates in project tollgate reviews, and essentially serves as the team's backer. Although many organizations define the terms "champion" and "sponsor" differently, they are frequently used interchangeably. *See also* sponsor.

chance cause—See random cause.

- chance variation—Variation due to chance causes. See also random variation.
- **change agent**—According to Hutton (1994), a change agent is "an individual who plays a specific role in the planning and implementation of the change management process. They may be members of the organization or may be outsiders."
- **changeover time**—The time interval between the last good piece off the current run and the first good piece off the next run.
- **characteristic**—A property that helps differentiate between items of a given sample or population.
- **charter**—A documented statement officially initiating the formation of a committee, team, project, or other effort in which a clearly stated purpose and approval are conferred.
- **check sheet**—A simple data-recording device typically used to collect the frequency of occurrences of nominal data by category. Whenever the user observes the occurrence of one of the categories, he or she places a mark next to the category. When the observation process is complete, final counts by category can then be obtained. The user custom-designs the check sheet so that he or she can readily interpret the results. The check sheet is occasionally considered to be one of the seven basic tools of quality. Note: Check sheets are often confused with checklists. *See also* checklist.
- checklist—A quality tool that is used for processes where there is a large human element. Examples are audit checklists and an airplane pilot's preflight checklist. Checklists serve as reminders and, depending on design, evidence that important items have been observed or appropriate actions have been taken. A well-designed checklist helps gather information to support findings and observations and serves as a guide and a place to record information. Note: Checklists are often confused with check sheets. See also check sheet.
- churn—The movement of people to different job positions within the organization.
- **circle diagram**—A tool used to show linkages between various items. The circle diagram is constructed by evenly placing item descriptors around a circle. Such item descriptors might include products, services, organizations, individuals, and so forth. Arrows are drawn from each item to other items where a flow exists. Circle diagrams are highly useful in that they readily depict predecessor and successor relationships as well as potential bottlenecks. Too many inputs or outputs from any given descriptor around the circumference of the circle may indicate a limiting function. A circle diagram is also known as a *handoff map*.
- **closed-loop feedback system**—When a manipulated variable is adjusted by the final control element, and that variable, in turn, affects the next measurement, the system is said to be a closed-loop feedback system.

CLT—See central limit theorem.

CMM—See capability maturity model.

CMMI—See capability maturity model integration.

- **coaching**—A process by which a more experienced individual helps enhance the existing skills and capabilities that reside in a less experienced individual. Coaching is about listening, observing, and providing constructive, practical, and meaningful feedback. Typically, coaching is used on a one-to-one basis, or for a small group or team, and conducted at the job site or during the training process. During training, coaching helps the trainee translate the theoretical learning into applied learning while helping the trainee develop confidence in their newly developing knowledge and skills. Post training, coaches help projects stay on track and advance toward completion in a timely manner.
- **coefficient of determination** (R^2)—A statistic that indicates how much variation in the response variable is accounted for by the regression model $R^2 \le 1$. It is computed as follows:

$$R^{2} = 1 - \frac{\sum_{i=1}^{n} (y_{i} - \hat{y}_{i})^{2}}{\sum_{i=1}^{n} (y_{i} - \overline{y}_{i})^{2}}$$

where

n = Number of observations

 $y_i = i$ th observed response value

 \overline{y}_i = Mean observed response value

 $\hat{y}_i = i$ th fitted value

Sometimes, R^2 is written as r^2 . The coefficient of determination is known as the *coefficient of multiple determination* when used in multiple regression.

- **coefficient of multiple determination**—*See* coefficient of determination.
- **communications plan**—A document that defines what will be communicated, to whom, how often, and by what means.
- **complete block**—A block that accommodates a complete set of treatment combinations.
- **completely randomized design**—A design in which the treatments are randomly assigned to the full set of experimental units. No blocks are involved.
- **completely randomized factorial design**—A factorial design in which all the treatments are randomly assigned to the full set of experimental units. *See also* completely randomized design.

- **compliance**—An affirmative indication or judgment that the supplier of a product or service has met the requirements of the relevant specifications, contract, or regulation; also the state of meeting the requirements.
- **components of variation (COV)**—A statistical techniques that allows us to separate process variation by the inputs. Components of variation is useful in the situation where a problem can be formulated as a nested hierarchal structure.
- **concurrent engineering**—A way to reduce cost, improve quality, and shrink cycle time by simplifying a product's system of life cycle tasks during the early concept stages. Concurrent engineering is a process to get all departments from engineering, purchasing, marketing, manufacturing, and finance to work on a new design at once to speed development. The emphasis is on upstream prevention versus downstream correction. Concurrent engineering is also known as *simultaneous engineering*.
- **confidence coefficient** (1α) —*See* confidence level.
- **confidence interval**—An estimate of the interval between two statistics that includes the true value of the parameter with some probability. This probability is called the confidence level of the estimate. Confidence levels typically used are 90%, 95%, and 99%. Either the interval contains the parameter or it does not.
- **confidence level (1** α)—The probability that (1) the confidence interval described by a set of confidence limits actually includes the population parameter and (2) an interval about a sample statistic actually includes the population parameter. The confidence level is also known as the *confidence coefficient*.
- **confidence limits**—The endpoints of the interval about the sample statistic that is believed, with a specified confidence level, to include the population parameter. *See also* confidence interval.
- **conflict resolution**—A process for resolving disagreements in a manner acceptable to all parties.
- **confounding**—Indistinguishably combining an effect with other effects or blocks. When done deliberately, higher-order effects are systematically aliased so as to allow estimation of lower-order effects. Sometimes, confounding results from inadvertent changes to a design during the running of an experiment or from poor planning of the design. This can diminish or even invalidate the effectiveness of the experiment.
- **consensus**—Finding a proposal acceptable enough that all team members support the decision and no member opposes it.
- constraint—A bottleneck or limitation of the throughput of a process.
- **constraint management**—Pertains to identifying a constraint and working to remove or diminish it, while dealing with resistance to change.
- **consumer's risk** (β)—The probability of acceptance when the quality level has a value stated by the acceptance sampling plan as unsatisfactory. Note: (1) such acceptance is a Type II error; (2) consumer's risk is usually designated as beta

(β). Consumer's risk is also known as *beta* (β) *risk, beta* (β) *error, error of the second kind, Type 2 error,* and *Type II error.*

- **continuous flow manufacturing (CFM)**—A method in which items are produced and moved from one processing step to the next, one piece at a time. Each process makes only the one piece that the next process needs, and the transfer batch size is one. CFM is sometimes known as *one-piece flow* or *single-piece flow*.
- **continuous variable**—A variable whose possible values form an interval set of numbers such that between each two values in the set another member of the set occurs.
- **control chart**—A chart that plots a statistical measure of a series of samples in a particular order to steer the process regarding that measure and to control and reduce variation. The control chart comprises the plotted points, a set of upper and lower control limits, and a centerline. Specific rules are used to determine when the control chart goes out of control. Note: (1) the order is either time or sample number order based, and (2) the control chart operates most effectively when the measure is a process characteristic correlated with an ultimate product or service characteristic. The control chart is one of the seven basic tools of quality.
- control chart, acceptance—See acceptance control chart.
- **control limit**—A line on a control chart used for judging the stability of a process. Note: (1) control limits provide statistically determined boundaries for the deviations from the centerline of the statistic plotted on a Shewhart control chart due to random causes alone; (2) control limits (with the exception of the acceptance control chart) are based on actual process data, not on specification limits; (3) other than points outside the control limits, "out-of-control" criteria can include runs, trends, cycles, periodicity, and unusual patterns within the control limits; (4) the calculation of control limits depends on the type of control chart.
- **control plan**—A living document that identifies critical input or output variables and associated activities that must be performed to maintain control of the variation of processes, products, and services in order to minimize deviation from their preferred values.
- **controlled variable**—In the context of automated process control, the controlled variable is a variable that must be maintained at a set or specified level. The control variable is also known as the *process variable* or simply the *measurement*. In Lean Six Sigma terminology, it is the "Y" variable.
- **controller**—In the context of automated process control, the controller is considered the brain of the system since it compares the value of the signal to the set value (that is, nominal or desired value) and sends a signal to the final control element for action. Essentially, the controller makes a decision. The basic operation performed is "decide."

COPQ—*See* cost of poor quality.

COQ—*See* cost of quality.

- **corrective action**—Action taken to eliminate the root cause(s) and symptom(s) of an existing deviation or nonconformity to prevent recurrence.
- **correlation coefficient (***r***)**—A statistic that measures the degree of linear relationship between two sets of numbers and is computed as

$$r = \frac{s_{xy}}{\sqrt{s_{xx}s_{yy}}} = \frac{\sum_{i=1}^{n} (x_i - \overline{x})^2 (y_i - \overline{y})^2}{\sqrt{\sum_{i=1}^{n} (x_i - \overline{x})^2 \sum_{i=1}^{n} (y_i - \overline{y})^2}} = \frac{\sum_{i=1}^{n} x_i y_i - \frac{\left(\sum_{i=1}^{n} x_i\right) \left(\sum_{i=1}^{n} y_i\right)}{n}}{\sqrt{\left(\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}\right) \left(\sum_{i=1}^{n} y_i^2 - \frac{\left(\sum_{i=1}^{n} y_i\right)^2}{n}\right)}}$$

where $-1 \le r \le 1$. Also, *r* is used to estimate the population correlation coefficient ρ . Correlation values of -1 and +1 represent perfect linear agreement between two independent and dependent variables. An *r* = 0 means there is no linear relationship at all. If *r* is positive, the relationship between the variables is said to be positive. Hence, when *x* increases, *y* increases. If *r* is negative, the relationship is said to be negative. Hence, when *x* increases, *y* decreases. The correlation coefficient is also known as the *sample correlation coefficient* and is sometimes written as *R*.

- **cost avoidance (budget impacting)**—This type of cost avoidance eliminates or reduces items in the budget marked for future spending. For example, three engineers are budgeted for hire in the fourth quarter. The cost of the three engineers is not in the baseline, nor has any spending for these engineers occurred. Eliminating these planned expenses has an impact on the future budget and is thus cost avoidance.
- **cost avoidance (non-budget impacting)**—This type of cost avoidance results from productivity or efficiencies gained in a process (that is, reduction of non-value-added activities) without a head count reduction. For example, the process cycle time that involved two workers was reduced by ten percent. Assuming that the ten percent amounts to two hours per worker per week, the two workers save four hours per week. These four hours are allocated to other tasks.
- **cost avoidance savings**—These are, by exclusion, everything that is not hard dollar savings. Cost avoidance savings are also known as *soft dollar savings*. There are two types of cost avoidance savings: budget impacting, and non-budget impacting.
- **cost–benefit analysis**—*See* benefit–cost analysis.
- **cost of poor quality (COPQ)**—Costs associated with the production of nonconforming material. These costs include both external and internal failure costs. *See also* external failure costs; internal failure costs.

- **cost of quality (COQ)**—Costs specifically associated with the achievement or nonachievement of product or service quality, including all product or service requirements established by the organization and its contracts with customers and society. More specifically, quality costs are the total costs incurred by (1) investing in the prevention of nonconformances to requirements, (2) appraising a product or service for conformance to requirements, and (3) failing to meet requirements. These can then be categorized as prevention, appraisal, and failure costs. *See also* appraisal costs; failure costs; prevention costs.
- **cost performance index (CPI)**—This is a dimensionless index used to measure the project's cost efficiency. It is computed as

$$CPI = \frac{EV}{AC} = \frac{BCWP}{ACWP}$$

If the CPI is greater than 1, the project is under budget. If the CPI is equal to 1, the project is on budget. If the CPI is less than 1, the project is over budget, and corrective action must be taken.

- **cost savings**—For project savings to generate real cost savings, a prior baseline of spending must be established, the dollars must have been planned and in the budget, and the savings must affect the bottom line (that is, the profit and loss statement or balance sheet). Cost savings are also known as *hard dollar savings*.
- **cost variance (CV)**—The cost variance is the difference between the amount budgeted for the work performed and the actual cost of the work performed. It is computed as

$$CV = EV - AC$$

If the CV is positive, the project is under budget. If the CV is zero, the project is on budget. If the CV is negative, the project is over budget, and corrective action must be taken.

- COV—See components of variation.
- *C_p* (process capability index)—An index describing process capability in relation to specified tolerance of a characteristic divided by the natural process variation for a process in a state of statistical control:

$$C_p = \frac{USL - LSL}{6\sigma}$$

where USL = upper specification limit, LSL = lower specification limit, and 6σ = the natural process variation. *See also* P_p (process performance index), a similar term, except that the process may not be in a state of statistical control; process capability index.

CPI—*See* cost performance index.

 C_{pk} (minimum process capability index)—The smaller of the upper process capability index (C_{pk_i}) and the lower process capability index (C_{pk_i}). The C_{pk}

considers the mean of the process with respect to the upper and lower specifications. The C_{pk} can be converted to a sigma level using sigma level = $3C_{pk}$.

 C_{pk_L} (lower process capability index; CPL)—An index describing process capability in relation to the lower specification limit. The index is defined as

$$C_{pk_L} = \frac{\overline{x} - LSL}{3\sigma}$$

where \overline{x} = process average, *LSL* = lower specification limit, and 3σ = half of the process capability. Note: When the process average is obtained from a control chart, \overline{x} is replaced by $\overline{\overline{x}}$.

 $C_{pk_{u}}$ (upper process capability index; CPU)—An index describing process capability in relation to the upper specification limit. The index is defined as

$$C_{pk_{u}} = \frac{USL - \overline{x}}{3\sigma}$$

where \overline{x} = process average, USL = upper specification limit, and 3σ = half of the process capability. Note: When the process average is obtained from a control chart, \overline{x} is replaced by $\overline{\overline{x}}$.

- **CPL**—See C_{pk_l} .
- C_{pm} (process capability index of the mean)—An index that takes into account the location of the process average relative to a target value and is defined as

$$C_{pm} = \frac{(USL - LSL)/2}{3\sqrt{\sigma^2 + (\mu - T)^2}}$$

where USL = upper specification limit, LSL = lower specification limit, σ = standard deviation, μ = expected value, and T = target value. When the process average and the target value are the same, the $C_{pm} = C_{pk}$. When the process average drifts from the target value, the $C_{pm} < C_{pk}$.

CPM—See critical path method.

CPU—See $C_{pk_{ij}}$.

creativity—The mental capability to generate ideas.

- **critical path**—The sequence of tasks that takes the longest time and determines a project's completion date.
- **critical path method (CPM)**—An activity-oriented project management technique that uses arrow-diagramming methods to demonstrate both the time and the cost required to complete a project. It provides one time estimate: normal time.

critical success factor (CSF)—See key business driver.

criticality—An indication of the consequences expected to result from a failure.

- **critical-to-quality (CTQ)**—A characteristic of a product or service that is essential to ensure customer satisfaction.
- **cross-functional team**—A group consisting of members from more than one department that is organized to accomplish a project.
- CSF—See key business driver.
- CSSBB—See Certified Six Sigma Black Belt.
- CSSGB—See Certified Six Sigma Green Belt.
- CSSMBB—See Certified Six Sigma Master Black Belt.
- CTQ—See critical-to-quality.
- **culture**—Westcott (2006) refers to culture as "the collective beliefs, values, attitudes, manners, customs, behaviors, and artifacts unique to an organization."
- **cumulative sum (CUSUM) control chart**—A control chart on which the plotted value is the cumulative sum of deviations of successive samples from a target value. The ordinate of each plotted point represents the algebraic sum of the previous ordinate and the most recent deviations from the target.
- **customer loyalty**—A term used to describe the behavior of customers—in particular, customers who exhibit a high level of satisfaction, conduct repeat business, or provide referrals and testimonials. It is the result of an organization's processes, practices, and efforts designed to deliver its services or products in ways that drive such behavior.
- CUSUM—See cumulative sum (CUSUM) control chart.
- CV—See cost variance.
- **cycle time**—The time required to complete one cycle of an operation. If cycle time for every operation in a complete process can be reduced to equal takt time, products can be made in single-piece flow. *See also* takt time.
- **cycle time reduction**—A method for reducing the amount of time it takes to execute a process or build a specific product. Elimination of duplicate or unnecessary tasks also reduces the time that is necessary to execute a process. Improving handoffs, which tend to be points of substantial discontinuity, also eliminates rework or duplicate efforts.

D

- **decentralization versus centralization**—In the context of organizational design this refers to how much decision-making authority has been delegated to lower management levels.
- **defect**—The nonfulfillment of a requirement related to an intended or specified use for a product or service. In simpler terms, a defect is anything not done correctly the first time. Note: The distinction between the concepts *defect* and *nonconformity* is important because it has legal connotations, particularly

those associated with product liability issues. Consequently, the term "defect" should be used with extreme caution.

- **defective**—A unit of product that contains one or more defects with respect to the quality characteristic(s) under consideration.
- **defects per million opportunities (DPMO)**—A measure of capability for discrete (attribute) data found by dividing the number of defects by the opportunities for defects multiplied by a million. DPMO allows for comparison of different types of product. *See also* parts per million.
- **defects per unit (DPU)**—A measure of capability for discrete (attribute) data found by dividing the number of defects by the number of units.
- Deming cycle—See plan-do-study-act (PDSA) cycle.
- **Deming Prize**—Award given annually to organizations that, according to the award guidelines, have successfully applied organization-wide quality control based on statistical quality control and will keep up with it in the future. Although the award is named in honor of W. Edwards Deming, its criteria are not specifically related to Deming's teachings. There are three separate divisions for the award: the Deming Application Prize, the Deming Prize for Individuals, and the Deming Prize for Overseas Companies. The award process is overseen by the Deming Prize Committee of the Union of Japanese Scientists and Engineers in Tokyo.
- Deming wheel—See plan-do-study-act (PDSA) cycle.
- **dependability**—The degree to which a product is operable and capable of performing its required function at any randomly chosen time during its specified operating time, provided that the product is available at the start of that period. (Non-operation-related influences are not included.) Dependability can be expressed by the ratio:

time available time available+time required

- **dependent events**—Two events, A and B, are dependent if the probability of one event occurring is higher given the occurrence of the other event.
- **derating**—The practice of using components at lower stress levels than those stated by the component specifications.
- **descriptive statistics**—The collection of tools and techniques for displaying and summarizing data.
- **design failure mode and effects analysis (DFMEA)**—An analysis process used to identify and evaluate the relative risk associated with a particular design.
- **Design for Six Sigma (DFSS)**—A structured methodology that focuses on designing new products or services with the intent of achieving Six Sigma quality levels. *See also* DMADV; DMADOV; DMEDI; and IDOV.

- **Design for X (DFX)**—An umbrella term whereby *X* is a variable that can assume multiple descriptions such as maintainability, manufacturability, producibility, quality, testability, or whatever component of design, logistics, production, quality, and so forth, of the product needs to be emphasized. The various components of the product are collectively referred to as the "ilities." Design for X involves the principle of designing products so that they are cost-effective and achieve the objectives set forth for whatever "ilities" are of concern.
- **design of experiments (DOE; DOX)**—The arrangement in which an experimental program is to be conducted, including the selection of factor combinations and their levels. Note: The purpose of designing an experiment is to provide the most efficient and economical methods of reaching valid and relevant conclusions from the experiment. The selection of the design is a function of many considerations, such as the type of questions to be answered, the applicability of the conclusions, the homogeneity of experimental units, the randomization scheme, and the cost to run the experiment. A properly designed experiment will permit meaningful interpretation of valid results.
- **design review**—Documented, comprehensive, and systematic examination of a design to evaluate its capability to fulfill the requirements for quality.
- **design space**—The multidimensional region of possible treatment combinations formed by the selected factors and their levels.
- design-for-cost (DFC)—See design-to-cost.
- **design-to-cost (DTC)**—A set of tools, techniques, or methods used to set and achieve product cost goals through design trade-off analysis. Design-to-cost is also known as *design-for-cost*.
- **detection**—The likelihood of finding a failure once it has occurred. Detection may be evaluated based on a 10-point or similar scale. On the lowest end of the scale (1), it is assumed that a design control will detect a failure with certainty. On the highest end of the scale (10), it is assumed that a design control will not detect a failure when a failure occurs.
- DFC—An acronym for design-for-cost. See design-to-cost.
- DFM—An acronym for design for manufacturability. See Design for X.
- DFMaint—An acronym for design for maintainability. See Design for X.
- DFMEA—See design failure mode and effects analysis.
- **DFP**—An acronym for design for producibility. *See* Design for X.
- **DFQ**—An acronym for design for quality. *See* Design for X.
- DFSS—See Design for Six Sigma.
- **DFT**—An acronym for design for testability. *See* Design for X.
- **DFX**—*See* Design for X.

- **differences**—Difference between data values for a specified lag value k. For example, for a given set of ordered data, there is a lag k between values x_t and x_{t+k} . Differencing is used to introduce stationarity to a time series. Data transformations can also be used to stabilize the variance.
- **discrete scale**—A scale with only a set or sequence of distinct values. Examples include defects per unit, events in a given time period, types of defects, and number of orders on a truck.
- **discrete variable**—A variable whose possible values form a finite or, at most, countably infinite set.
- **discriminant analysis**—A statistical technique that is used "to classify observations into two or more groups if you have a sample with known groups. Discriminant analysis can also be used to investigate how variables contribute to group separation" (Minitab 16).
- **discrimination**—The capability of the measurement system to detect and indicate small changes in the characteristic measured. Discrimination is also known as *resolution*. For example, a tape measure with gradations of one inch would be unable to distinguish between lengths of objects that fall in between the inch marks. Hence, we would say the measurement system could not properly discriminate between the objects. If an object to be measured is 2.5 inches, the measurement system (that is, tape measure) would produce a value of 2 or 3 inches depending on how the individual decided to round. Therefore, to measure an object that is 2.5 inches, a tape measure with finer gradations would be required.
- **dispersion effect**—The influence of a single factor on the variance of the response variable.
- **dissatisfiers**—Those features or functions that the customer or employee has come to expect, and if they are no longer present, the customer will be dissatisfied.
- **disturbance**—In the context of automated process control, a disturbance causes the controlled variable to deviate from the set point. In Lean Six Sigma terminology, disturbances are noise variables. A disturbance is also known as an *upset*.
- **DMADOV**—A structured DFSS methodology. DMADOV is an acronym for: *define, measure, analyze, design, optimize,* and *verify.* Variations of DMADOV exist. *See also* Design for Six Sigma; DMADV; DMEDI; and IDOV.
- **DMADV**—A structured DFSS methodology. DMADV is an acronym for *define*, *measure*, *analyze*, *design*, and *verify*. Variations of DMADV exist. *See also* Design for Six Sigma; DMADOV; DMEDI; and IDOV.
- **DMAIC**—A structured methodology that focuses on improving existing processes with the intent of achieving Six Sigma quality levels. DMAIC is an acronym for *define, measure, analyze, improve,* and *control. See also* Design for Six Sigma; DMADV; DMADOV; DMEDI; and IDOV.

DMEDI—A structured DFSS methodology. DMEDI stands for: *define, measure, explore, develop,* and *implement*. Variations of DMEDI exist. *See also* Design for Six Sigma; DMADV; DMADOV; and IDOV.

DOE—See design of experiments.

DOX—See design of experiments.

DPMO—See defects per million opportunities.

DPU—See defects per unit.

driving forces—In the context of a force field analysis, driving forces are those forces that aid in achieving the objective.

DTC—See design-to-cost.

E

- **earned value (EV)**—This is the approved budget for the work actually completed by a specific date. It is also known as the *budgeted cost of work performed* (BCWP).
- education—Education focuses on broadening an individual's knowledge base and expands thinking processes. Further, it helps employees understand concepts and accept increased job responsibilities and prepares them for future jobs and leadership roles. Simply, education helps individuals learn how to think.
- **effect**—A relationship between factor(s) and response variable(s) based on cause. Note that there can be many factors and response variables. Specific types include main effect, dispersion effect, and interaction effect.
- efficiency—An unbiased estimator is considered to be more efficient than another unbiased estimator if it has a smaller variance.
- **eighth type of waste**—In addition to the traditional seven types of waste, an additional eighth has been added: people. The eighth category was added by Taiichi Ohno, as he saw the underutilization of employees (for example, brain-power, skills, experience, and talents) as a waste. *See also* seven types of waste.
- **eighty-twenty (80–20) rule**—A term referring to the Pareto principle, which was first defined by Joseph M. Juran in 1950. The principle suggests that most effects come from relatively few causes; that is, 80% of the effects come from 20% of the possible causes. *See also* Pareto principle.
- entity—An item that can be individually described and considered.
- **error**—(1) Error in measurement is the difference between the indicated value and the true value of a measured quantity. (2) A fault resulting from defective judgment, deficient knowledge, or carelessness. It is not to be confused with measurement error, which is the difference between a computed or measured value and the true or reference value.
- **error detection**—A hybrid form of error-proofing whereby a bad part can still be made but will be caught immediately, and corrective action will be taken to

prevent another bad part from being produced. An error detection device is used to spot the error and stop the process when a bad part is made. This is used when error-proofing is too expensive or not easily implemented.

error of the first kind—See Type I error.

- error of the second kind—See Type II error.
- **error-proofing**—The use of process or design features to prevent the acceptance or further processing of nonconforming products. *See also* foolproofing; mistake-proofing; and poka-yoke.
- estimate at completion (EAC)—This can be simply an estimate or it can be computed as:

1) Assuming cost performance for the remainder of the project will revert to what was originally planned,

EAC = Approved budget for the entire task – Cost variance for work performed to-date on task

= BAC + AC - EV

2) Assuming cost performance for the remainder of the project will continue as it has for the work performed to-date,

$$EAC = \frac{BAC}{Cumulative CPI}$$

estimate to complete (ETC)—This can be simply an estimate or it can be computed as

$$ETC = BAC - AC$$
 to-date.

ETC—See estimate to complete.

EV—See earned value.

- **evolutionary operations (EVOP)**—The procedure of adjusting variables in a process in small increments in search of a more optimum point on the response surface.
- EVOP—See evolutionary operations.
- expected value—The mean of a random variable.
- **experiential training**—The focus of this form of training is on having the learner experience the effects typically encountered in a real-life situation. The experiential approach employs many types of structured experiences (for example, games, simulations, role-plays) to facilitate learning. Typically, participants assume roles within a stated scenario designed to surface one or more learning points.
- **experimental design**—A formal plan that details the specifics for conducting an experiment, such as which responses, factors, levels, blocks, treatments, and tools are to be used. *See also* design of experiments.

- **experimental error**—Variation in the response variable beyond that accounted for by the factors, blocks, or other assignable sources in the conduct of the experiment.
- **experimental run**—A single performance of the experiment for a specific set of treatment combinations.
- **experimental unit**—The smallest entity receiving a particular treatment, subsequently yielding a value of the response variable.
- explanatory variable—See predictor variable.
- **external customer**—A person or organization that receives a product, service, or information but is not part of the organization supplying it. *See also* internal customer.
- **external failure costs**—The failure costs occurring after delivery or shipment of the product, or during or after furnishing of a service, to the customer. Examples include the costs of processing customer complaints, customer returns, warranty claims, and product recalls.

F

- **facilitator**—An individual responsible for creating favorable conditions that will enable a team to reach its purpose or achieve its goals by bringing together the necessary tools, information, and resources to get the job done.
- **factor**—An independent variable or assignable cause that may affect the responses and of which different levels are included in an experiment. Factors are also known as *explanatory*, *independent*, *regressor*, or *predictor variables*.
- **factor analysis**—A statistical technique used "to determine the underlying factors responsible for correlations in the data" (Minitab 15).
- **factor relationship diagram**—A visual tool that assists in the planning of experiment designs. Typically, a factor relationship diagram is composed of two structures: the design structure and the unit structure. The design structure contains those factors that are deliberately manipulated during the experimental design. The unit structure is determined by how we manage noise and other unmanipulated sources of variation during the experimental design. In addition to these two structures, we have the inference space. Hild (2009) states, "The inference space represents the range of conditions over which the experiment's results can expect to repeat."
- **failure**—The termination, due to one or more defects, of the ability of an item, product, or service to perform its required function when called on to do so. A failure may be partial, complete, or intermittent.
- **failure analysis**—The process of breaking down a failure to determine its cause and to put measures in place to prevent future problems.
- failure costs—The costs resulting from product or services not conforming to requirements or customer/user needs. Failure costs are further divided into

internal and external categories. *See also* external failure costs; internal failure costs.

- failure mode—The type of defect contributing to a failure.
- **failure mode and effects analysis (FMEA)**—A procedure in which each potential failure mode in every sub-item of an item is analyzed to determine its effect on other sub-items and on the required function of the item.
- failure rate—The number of failures per unit time (for equal time intervals).
- final control element—In the context of automated process control, the final control element, upon receiving direction from the controller, will take action to adjust the appropriate "manipulated variable." (that is, input variable). Examples of final control elements include control values, electric motors, conveyors, and variable-speed pumps. The basic operation of the final control element is "action."
- **first-pass yield (FPY)**—The percentage of units that complete a process and meet quality guidelines without being scrapped, rerun, retested, returned, or diverted into an off-line repair area. FPY is calculated by dividing the units entering the process minus the defective units by the total number of units entering the process. First-pass yield is also known as the *quality rate* or *first-time yield* (FTY).
- first-time yield (FTY)—See first-pass yield.
- fishbone diagram—See cause-and-effect diagram.
- **fitness for use**—A term used to indicate that a product or service fits the customer's or user's defined purpose for that product or service.
- 55's (five S's)—A term that derives its name from the five terms beginning with "s" that are used to create a workplace suited for visual control and lean production. Collectively, the 5S's are about how to create a workplace that is visibly organized, free of clutter, neatly arranged, and sparkling clean. The 5S's are defined as follows: *seiri* (*sort*; also *sifting*) means to separate needed tools, parts, and instructions from unneeded materials and to remove the latter; *seiton* (*set in order*) means to neatly arrange and identify parts and tools for ease of use; *seiso* (*shine*; also *sanitize* or *scrub*) means to conduct a cleanup campaign; *seiketsu* (*standardize*) means to conduct seiri, seiton, and seiso at frequent, indeed daily, intervals to maintain a workplace in perfect condition; and *shitsuke* (*sustain*; also *self-discipline*) means to form the habit of always following the first four S's. *See also* individual listings for each of the 5S's.
- **5W's and 1H**—Addressing the *who, what, where, when, why,* and *how* questions is a useful technique to help develop an objective and concise statement of a problem. *See also* 5 whys.
- **5 whys (five whys)**—A persistent questioning technique to probe deeper to reach the root cause of a problem. *See also* 5W's and 1H.

- **flowchart**—A basic quality tool that uses graphical representation for the steps in a process. Effective flowcharts include decisions, inputs, and outputs, as well as the sequence of process steps. The flowchart is occasionally considered to be one of the seven basic tools of quality.
- FMEA—See failure mode and effects analysis.
- **foolproofing**—A method of making a product or process immune to errors on the part of the user or operator. Foolproofing is synonymous with error-proofing. *See also* error-proofing; mistake-proofing; poka-yoke.
- **force field analysis**—A technique for analyzing the forces that aid or hinder an organization in reaching an objective.
- **forecasting**—A prediction of future values of a time series. For example, one might want to predict the next value, the next five values, or the next ten values in the series.
- forming—The first stage of team growth. See also stages of team growth.
- **Fourteen (14) Points for Management**—W. Edwards Deming's 14 management practices to help organizations increase their quality and productivity. They are as follows: (1) create constancy of purpose for improving products and services; (2) adopt a new philosophy; (3) cease dependence on inspection to achieve quality; (4) end the practice of awarding business on price alone; instead, minimize total cost by working with a single supplier; (5) improve constantly and forever every process for planning, production, and service; (6) institute training on the job; (7) adopt and institute leadership; (8) drive out fear; (9) break down barriers between staff areas; (10) eliminate slogans, exhortations, and targets for the workforce; (11) eliminate numerical quotas for the workforce and numerical goals for management; (12) remove barriers that rob people of pride of workmanship, and eliminate the annual rating or merit system; (13) institute a vigorous program of education and self-improvement for everyone; (14) put everybody in the company to work to accomplish the transformation.

FPY—*See* first-pass yield.

funnel experiment—An experiment that demonstrates the effects of tampering. Marbles are dropped through a funnel in an attempt to hit a mark on a flat surface target below. The experiment shows that adjusting a stable process to compensate for an undesirable result or an extraordinarily good result will produce output that is worse than if the process had been left alone.

G

gage repeatability and reproducibility (GR&R) study—A type of measurement system analysis done to evaluate the performance of a test method or measurement system. Such a study quantifies the capabilities and limitations of a measurement instrument, often estimating its repeatability and reproducibility. It typically involves multiple operators measuring a series of measurement items multiple times.

- **Gantt chart**—A type of bar chart used in process/project planning and control to display planned work and finished work in relation to time. Also called a *milestone chart*.
- **gap analysis**—A technique that compares an organization's existing state with its desired state (typically expressed by its long-term plans) to help determine what needs to be done to remove or minimize the gap.
- GB—See Green Belt.
- GD&T—See geometric dimensioning and tolerancing.
- **gemba visit**—The term "gemba" means "place of work" or "the place where the truth can be found." Still others may call it "the value proposition." A gemba visit is a method of obtaining voice of the customer information that requires the design team to visit and observe how the customer uses the product in his or her environment.
- **general linear model (GLM)**—A statistical tool that synthesizes many of the other common statistical techniques for analyzing both continuous and discrete data.
- **geometric dimensioning and tolerancing (GD&T)**—A method to minimize production costs by considering the functions or relationships of part features in order to define dimensions and tolerances.
- GLM—See general linear model.
- **goal**—A statement of general intent, aim, or desire. It is the point toward which the organization (or individual) directs its efforts; goals are often nonquantitative.
- **governance**—According to Bertin and Watson (2007), governance establishes the policy framework within which organization leaders will make strategic decisions to fulfill the organizational purpose, as well as the tactical actions that they take at the level of operational management to deploy and execute the organization's guiding policy and strategic direction. Governance in the context of Lean Six Sigma deployment includes all the processes, procedures, rules, roles, and responsibilities associated with the strategic, tactical, and operational deployment of Lean Six Sigma. It also includes the authoritative, decision-making body charged with the responsibility of providing the required governance.

GR&R study—See gage repeatability and reproducibility (GR&R) study.

Green Belt (GB)—A Lean Six Sigma role associated with an individual who retains his or her regular position within the firm but is trained in the tools, methods, and skills necessary to conduct Lean Six Sigma improvement projects either individually or as part of larger teams.

Н

handoff map—See circle diagram.

hard dollar savings—See cost savings.

- **heijunka**—The act of leveling the variety or volume of items produced at a process over a period of time. Heijunka is used to avoid excessive batching of product types and volume fluctuations, especially at a pacemaker process.
- **histogram**—A plot of a frequency distribution in the form of rectangles (cells) whose bases are equal to the class interval and whose areas are proportional to the frequencies. A histogram is a graphic summary of variation in a set of data. The graphical nature of the histogram permits easier visualization of patterns of variation that are difficult to detect in a simple table of numbers. The histogram is one of the seven basic tools of quality.
- hoshin kanri—A Japanese term referring to a top-down, bottom-up, systematic and structured strategic planning process that engages all levels of the organization while creating measurable and aligned goals that imbue the concept of continuous improvement through use of the plan–do–check–act cycle. (Note: Some sources will cite the PDSA cycle.) Hoshin kanri is also known as *hoshin planning, policy deployment, policy management, policy control,* and *management by policy.*
- **hoshin planning**—A term meaning "breakthrough planning." Hoshin planning is a strategic planning process in which an organization develops up to four vision statements that indicate where the company should be in the next five years. Company goals and work plans are developed on the basis of the vision statements. Periodic audits are then conducted to monitor progress. *See also* hoshin kanri.
- **house of quality**—A diagram named for its house-shaped appearance that clarifies the relationship between customer needs and product features. It helps correlate market or customer requirements and analysis of competitive products with higher-level technical and product characteristics, and makes it possible to bring several factors into a single figure. The house of quality is also known as *quality function deployment* (QFD).
- **hygiene factors**—A term used by Frederick Herzberg to label "dissatisfiers." *See also* dissatisfiers.

- ICC—See intra-class correlation coefficient.
- ID—See interrelationship digraph.
- ideation—According to Jonson (2005) this is "the creative process of generating, developing, and communicating new ideas, where an idea is understood as a basic element of thought that can be visual, concrete, or abstract."
- **IDOV**—A structured methodology as an alternative to Design for Six Sigma. IDOV is an acronym for *identify*, *design*, *optimize*, and *validate*. Variations of IDOV exist. *See also* Design for Six Sigma; DMADV; DMADOV; DMEDI.

- **imprecision**—A measure of precision computed as a standard deviation of the test or measurement results. Less precision is reflected by a larger standard deviation.
- **incomplete block**—A block that accommodates only a subset of treatment combinations.
- **incomplete block design**—A design in which the design space is subdivided into blocks in which there are insufficient experimental units available to run a complete replicate of the experiment.
- **in-control process**—A condition where the existence of special causes is no longer indicated by a Shewhart control chart. This does not indicate that only random causes remain, nor does it imply that the distribution of the remaining values is normal (Gaussian). It indicates (within limits) a predictable and stable process.
- **independent events**—Two events, A and B, are called independent if the probability that they both occur is the product of the probabilities of their individual occurrences. That is,

$$P(A \cap B) = P(A)P(B)$$

independent variable—See predictor variable.

- **inferential statistics**—Techniques for reaching conclusions about a population on the basis of analysis of data from a sample.
- **Information Technology Infrastructure Library (ITIL)**—The IT Infrastructure Library's "An Introductory Overview of ITIL® V3" Version 1.0 brochure describes the ITIL as "a public framework that describes Best Practice in IT service management. It provides a framework for the governance of IT, the 'service wrap,' and focuses on the continual measurement and improvement of the quality of IT service delivered, from both a business and a customer perspective."
- **inherent process variation**—The variation in a process when the process is operating in a state of statistical control.
- **innovation**—The ability to not only be creative, but to put ideas into practice.
- **inspection**—The process of measuring, examining, testing, gauging, or otherwise comparing a unit with the applicable requirements.
- **interaction effect**—The effect for which the apparent influence of one factor on the response variable depends on one or more other factors. Existence of an interaction effect means that the factors can not be changed independently of each other.
- **interaction plot**—A plot providing the average responses at the combinations of levels of two distinct factors.

- **internal customer**—The person or department receiving another person's or department's output (product, service, or information) within an organization. *See also* external customer.
- **internal failure costs**—The failure costs occurring prior to delivery or shipment of a product, or the furnishing of a service, to the customer. Examples include the costs of scrap, rework, reinspection, retesting, material review, and downgrading.
- **internal rate of return (IRR)**—The discount rate that causes the sum of NPV of the costs (negative cash flows) plus the sum of the NPV of the savings (positive cash flows) to equal zero.
- **interrelationship digraph (ID)**—A tool that displays the relationship between factors in a complex situation. It identifies meaningful categories from a mass of ideas and is useful when relationships are difficult to determine. Also known as a *relations diagram*, the interrelationship digraph is one of the seven management and planning tools.
- **interval scale**—A quantitative scale in which meaningful differences exist, but where multiplication and division are not allowed since there is no absolute zero. An example is temperature measured in °F because 20°F isn't twice as warm as 10°F.
- **intervention**—An action taken by a leader or a facilitator to support the effective functioning of a team or work group.
- **intra-class correlation coefficient (ICC)**—The intra-class correlation coefficient helps gauge the effectiveness of attribute (discrete) measurement systems and is used under the following conditions or assumptions: there is the need to classify things in order, rank, or scale; ranges are equally distributed, such as –2, –1, 0, 1, 2; there are consequences for misclassifying; units must be independent; raters make their classifications independently of other raters; and the categories are mutually exclusive and collectively exhaustive. There are six forms of the intra-class correlation coefficient.
- **IRR**—*See* internal rate of return.
- Ishikawa diagram—See cause-and-effect diagram.
- **ISO**—"Equal" (Greek). A prefix for a series of standards published by the International Organization for Standardization.
- **ISO 9000 series**—A set of individual, but related, international standards and guidelines on quality management and quality assurance developed to help organizations effectively document the quality system elements to be implemented to maintain an efficient quality system. The standards were developed by the International Organization for Standardization, a specialized international agency for standardization composed of the national standards bodies of nearly 100 countries.

ISO 14000 series—A set of individual, but related, international standards and guidelines relevant to developing and sustaining an environmental management system.

ITIL—See Information Technology Infrastructure Library.

- J
- **jidoka**—A method of autonomous control involving the adding of intelligent features to machines to start or stop operations as control parameters are reached, and to signal operators when necessary. Jidoka is also known as *autonomation*.
- JIT manufacturing—See just-in-time manufacturing.
- **job aids**—Virtually any type of media that can either substitute for formal training and/or provide reinforcement or reference after training.
- **job analysis**—A process for collecting information about duties, responsibilities, skills, and outcomes. Also, a description of the work environment may be included if it is unique or extraordinary in some manner. An example of such an environment might be a manufacturing clean room.
- **job description**—A job analysis, task analysis, and skills analysis are components of the job description. Once completed, these three aspects will be used to draft a job description and a job specification. Job descriptions, according to Rae (1997), "are statements of the outline of the whole job and show the duties and responsibilities involved in the job." A job description also "details the skills, knowledge, and attitudes which are required by the individual in order to carry out the duties involved in the job."
- **job specification**—A subset of the job description, the job specification, according to Rae (1997), defines "what the job holder should do and be capable of doing." Bee and Bee (1994) add that the job specification also includes carrying out the job to a prescribed standard. The greater the detail of the job specification, the easier it will be to determine performance gaps.
- Juran trilogy—See quality trilogy.
- **just-in-time manufacturing (JIT)**—A material requirement planning system for a manufacturing process in which there is little or no manufacturing material inventory on hand at the manufacturing site and little or no incoming inspection.

Κ

kaikaku—A term meaning a breakthrough improvement in eliminating waste.

kaizen—A term that means gradual, unending improvement by doing little things better and setting and achieving increasingly higher standards. The kaizen approach is usually implemented as a small, intensive event or project over a relatively short duration, such as a week.

- **kaizen blitz/event**—An intense team approach to employ the concepts and techniques of continuous improvement in a short time frame (for example, to reduce cycle time, increase throughput).
- kanban—A system that signals the need to replenish stock or materials or to produce more of an item. Kanban is also known as a *pull* approach. Kanban systems need not be elaborate, sophisticated, or even computerized to be effective. Taiichi Ohno of Toyota developed the concept of a kanban system after a visit to a U.S. supermarket.
- **Kano model**—A representation of the three levels of customer satisfaction, defined as dissatisfaction, neutrality, and delight.
- kappa (K)—Futrell (1995) defines kappa as "the proportion of agreement between raters after agreement by chance has been removed." Kappa techniques help gauge the effectiveness of attribute (discrete) measurement systems and are used under the following conditions or assumptions: there is the need to classify things in a nominal manner, some categories can be used more frequently than others, units must be independent, raters make their classifications independently of other raters, and the categories are mutually exclusive and collectively exhaustive. The formula for kappa is:

$$K = \frac{P_{Observed} - P_{Chance}}{1 - P_{Chance}}$$

When multiple raters are involved, we must be concerned with two types of kappa statistics: $K_{Overall}$ and $K_{Category}$. The overall kappa assesses the rater agreement across all of the categories, while the category kappa is used to compute individual kappa values for each category. This provides an indication regarding where each rater has trouble. These formulas are:

$$K_{Overall} = 1 - \frac{nm^2 - \sum_{i=1}^n \sum_{j=1}^k x_{ij}^2}{nm(m-1)\sum_{j=1}^k (\overline{p}_j)(1-\overline{p}_j)}$$
$$K_{Category} = 1 - \frac{\sum_{i=1}^n x_{ij}(m-x_{ij})}{mn(m-1)(\overline{p}_j)(1-\overline{p}_j)}$$

where

- m = Number of raters
- n = Number of units

$$k =$$
 Number of categories
 $\frac{1}{2}$ Ratings within category

KBD—See key business driver.

Kendall's coefficient of concordance—This coefficient measures the association between appraisers and the association within appraisers and is used with attributed (discrete) measurement systems. Kendall's coefficient of concordance ranges between -1 and 1. A positive value of -1 indicates positive association. Similarly, a negative value indicates negative association. A value of zero indicates no agreement or association. The formula for computing Kendall's coefficient of concordance is given by

$$W = \frac{12\sum_{i=1}^{N} R_i^2 - 3k^2 N(N+1)^2}{k^2 N(N+1) - k \sum_{j=1}^{k} T_j}$$

where

N = The number of units

$$\sum_{i=1}^{N} R_i$$
 = The sum of the squared sums of ranks for each of the ranked *N* units

- k = The number of appraisers
- T_j = Assigns the average of ratings to tied observation = $T_j = \sum_{i=1}^{s_j} (t_i^3 t_i)$

 t_i = The number of tied ranks in the *i*th grouping of ties

- g_i = The number of groups of ties in the *j*th set of ranks
- Kendall's correlation coefficient-This coefficient measures the association between all appraisers and the known standard, and each appraiser and the known standard, and is used with attribute (discrete) measurement systems. Kendall's correlation coefficient is also known as Kendall's rank-order correlation coefficient, Kendall's tau correlation coefficient, and Kendall's tau-b correlation coefficient. Kendall's correlation coefficient ranges between -1 and 1. A positive value of -1 indicates positive association. Similarly, a negative value indicates negative association. A value of zero indicates no agreement or association. The formula for computing Kendall's correlation coefficient is given by

$$\tau = \frac{C - D}{\sqrt{\left[N_{Total} \left(N_{Total} - 1\right)(0.5) - T_{X}\right]} \sqrt{\left[N_{Total} \left(N_{Total} - 1\right)(0.5) - T_{Y}\right]}}$$

where

$$C = \text{Number of concordant pairs} = \sum_{i < k} \sum_{j < i} n_{ij} n_{ki}$$
$$D = \text{Number of discordant pairs} = \sum_{i < k} \sum_{j > i} n_{ij} n_{ki}$$
$$n_{ij} = \text{Number of observations in the ith row}$$

 $n_{+i} = 1$ Number of observations in the *i*th row

 n_{+j} = Number of observations in the *j*th column

- n_{ii} = Number of observations in the cell in the *i*th row and *j*th column
- n_{ki} = Number of observations in the cell in the *k*th row and *i*th column

N = Total number of observations

$$T_X$$
 = Number of pairs tied on $X = 0.5 \sum n_{+i} (n_{+i} - 1)$

$$T_Y$$
 = Number of pairs tied on $Y = 0.5 \sum_j n_{+j} (n_{+j} - 1)$
 k = Number of appraisers

Kendall's rank-order correlation coefficient—See Kendall's correlation coefficient.

Kendall's tau correlation coefficient—See Kendall's correlation coefficient.

- Kendall's tau-b correlation coefficient—See Kendall's correlation coefficient.
- **key business drivers (KBDs)**—Those few things that must be done well for the organization to succeed. KBDs should reflect the organization's goals and objectives and are vital for its strategies to be successful. Key business drivers are also known as *critical success factors* (CSFs).
- **key performance indicators (KPIs)**—Quantifiable financial and nonfinancial measurements that reflect an organization's key business drivers (KBDs) and are usually long term.

KJ method—See affinity diagram.

KPI—See key performance indicator.

L

L—The average number of customers in the queueing system.

lag—The number of time units between differenced values.

- **latent defects**—Hidden defects in either material and/or workmanship that may cause a component to malfunction or fail, but are not discoverable through inspection.
- **law of unintended consequences**—Named by sociologist Robert K. Merton, this is meant to describe outcomes not intended by purposeful actions. "Unintended consequences" is sometimes known as *unanticipated consequences, unforeseen consequences,* or even *side effects.*
- LCI—See learner-controlled instruction.
- **Lean**—A comprehensive approach complemented by a collection of tools and techniques that focus on reducing cycle time, standardizing work, and reducing waste. Lean is also known as *lean approach* or *lean thinking*.

lean approach—See Lean.

- **lean manufacturing**—Applying the lean approach to improve manufacturing operations.
- Lean Six Sigma—A fact-based, data-driven philosophy of improvement that values defect prevention over defect detection. It drives customer satisfaction and bottom-line results by reducing variation, waste, and cycle time while promoting the use of work standardization and flow, thereby creating a competitive advantage. It applies anywhere variation and waste exist, and every employee should be involved. Note: In the first edition of *The Certified Six Sigma Black Belt Handbook*, this definition was attributed to Six Sigma. However, through experience and empirical evidence, it has become clear that Lean and Six Sigma are different sides of the same coin. Both concepts are required to effectively drive sustained breakthrough improvement. Subsequently, the definition has been changed to reflect the symbiotic relationship that must exist between Lean and Six Sigma to ensure lasting and positive change.

lean thinking—See Lean.

- learner-controlled instruction (LCI)—See self-directed learning.
- **learning**—According to Rothwell (2008), "learning gives individuals the potential to get results."
- **level**—The setting or assignment of a factor at a specific value.
- **linear programming**—A constrained optimization technique comprising both a linear objective function and linear constraints.
- **linear regression coefficients**—Numbers associated with each predictor variable in a linear regression equation that tell how the response variable changes with each unit increase in the predictor variable. This model also gives some sense of the degree of linearity present in the data. More specifically, when we talk about linear regression we mean we are linear in the coefficients or parameters.
- **linear regression equation**—A mathematical function or model that indicates the linear relationship between a set of predictor variables and a response variable.
- **linearity (general sense)**—The degree to which a pair of variables follows a straight-line relationship. Linearity can be measured by the correlation coefficient.
- **linearity (measurement system sense)**—The difference in bias through the range of measurement. A measurement system that has good linearity will have a constant bias no matter the magnitude of measurement. If one views the relation between the observed measurement result on the y axis and the true value on the x axis, an ideal measurement system would have a line of slope = 1. Linearity is a component of accuracy.

- **link function**—A function that transforms probabilities of a response variable from the closed interval, (0,1), to a continuous scale that is unbounded. Once the transformation is complete, the relationship between the predictor and response variable can be modeled with linear regression.
- **Ljung–Box Q (LBQ) statistic**—A statistic that tests the null hypothesis that autocorrelations up to lag *k* are equal to zero. The statistic is computed as

$$Q_k = n(n+2)\sum_{m=1}^k \frac{r_m^2}{n-m}$$

where

n = Number of observations

 r_m = Autocorrelation at lag m; m = 1,2,...,k

k = Lag j; k = 1, 2, ...

Q is distributed as $\chi^2_{\alpha,k}$ and tests the hypothesis of model adequacy. If the value of *Q* exceeds the chi-square value, the hypothesis is rejected.

- **logistic regression**—A type of regression that is used when the response variable is discrete. Although there are many types of logistic regression, three common types include binary, nominal, and ordinal.
- L_q —The average number of customers in the queue. Note: In queueing theory, the queue length includes the customer being served.

Μ

main effect—The influence of a single factor on the mean of the response variable.

- **main effects plot**—The plot giving the average responses at the various levels of individual factors.
- **maintainability**—The measure of the ability of an item to be retained or restored to a specified condition when maintenance is performed by personnel having specified skill levels, using prescribed procedures and resources, at each prescribed level of maintenance and repair.
- Malcolm Baldrige National Quality Award (MBNQA)—An award established by Congress in 1987 to raise awareness of quality management and to recognize U.S. organizations that have implemented successful quality management systems. A Criteria for Performance Excellence is published each year. Three awards may be given annually in each of five categories: manufacturing businesses, service businesses, small businesses, education institutions, and healthcare organizations. The award is named after the late secretary of commerce Malcolm Baldrige, a proponent of quality management. The U.S. Commerce Department's National Institute of Standards and Technology manages the award, and ASQ administers it. The major emphasis in determining success is achieving results driven by effective processes.

Mallows's *C*_{*p*}—A statistic used to measure the goodness of a prediction. It is computed by using

Mallows's
$$C_p = \frac{SSE_p}{MSE_m} - (n-2p)$$

where

 SSE_{v} = Sum of the squared error for the model under consideration

 MSE_m = Mean squared error for the model with all predictors included

n = Number of observations

- p = Number of terms in the model, including the constant term
- **manipulated variable**—In the context of automated process control, the manipulated variable is the variable used to maintain the controlled variable at the set point. In Lean Six Sigma terminology, it is the "*X*" variable. There can be many manipulated variables. In the event there are, the APC system becomes more complex. Examples of such variables include concentration, temperature, pressure, and flow rate.
- Manpower Services Commission (MSC)— A United Kingdom non-departmental public body of the Department of Employment Group created in 1973. The MSC was created to coordinate employment and training services in the UK through an industry, union, and education-based commission. The MSC was replaced in the late '80s or early '90s with a network of 72 training and education councils known as TECs.
- margin—Refers to the difference between income and cost.
- **market share**—An organization's market share of a particular product or service is that percentage of the dollar value that is sold relative to the total dollar value sold by all organizations in a given market.
- **Master Black Belt (MBB)**—A Lean Six Sigma role associated with an individual typically assigned full-time to train and mentor Black Belts as well as lead the strategy to ensure that improvement projects chartered are the right strategic projects for the organization. Master Black Belts are usually the authorizing body to certify Green Belts and Black Belts within an organization.
- **materials review board (MRB)**—A quality control committee or team usually employed in manufacturing or other materials-processing installations that has the responsibility and authority to deal with items or materials that do not conform to fitness-for-use specifications.
- matrix chart—See matrix diagram.
- **matrix diagram**—A tool that shows the relationships between various groups of data; it yields information about the relationships and the importance of tasks and method elements of the subjects. For each required task (listed along the left side of the diagram), the matrix diagram shows which departments were in charge, which had some involvement, and which had no involvement

(listed across the top of the diagram). At each intersecting point, the relationship is either present or absent. This is frequently used to determine who or what department is responsible for the different elements of an implementation plan. The matrix diagram is one of the seven management and planning tools.

- MBB—See Master Black Belt.
- MBNQA—See Malcolm Baldrige National Quality Award.
- **mean**—A measure of central tendency; the arithmetic average of all measurements in a data set is the most common form of mean.
- **mean absolute deviation (MAD)**—A measure of accuracy of the fitted time series values, computed as

$$MAD = \sum_{t=1}^{n} \left| \left(\hat{Y}_t - \hat{Y}_t \right) \right|$$

where

n = Number of observations

 Y_t = Actual value at time t

 \hat{Y}_t = Fitted value at time *t*

mean absolute percentage error (MAPE)—A measure of accuracy of the fitted time series values, computed as

$$MAPE = \frac{\left(\sum_{t=1}^{n} \frac{\left|\left(Y_{t} - \hat{Y}_{t}\right)\right|}{Y_{t}}\right)}{n} (100)$$

where

n = Number of observations

 Y_t = Actual value at time t

 \hat{Y}_t = Fitted value at time *t*

mean squared deviation (MSD)—A measure of accuracy of the fitted time series values, computed as

$$MSD = \frac{\sum_{t=1}^{n} \left| Y_t - \hat{Y}_t \right|^2}{n}$$

where

n = Number of forecasted values

 Y_t = Forecast value at time t

 \hat{Y}_t = Fitted value at time *t*

- **mean time between failures (MTBF)**—A basic measure of reliability for repairable items. The mean number of life units during which all parts of an item perform within their specified limits, during a particular measurement interval, under stated conditions.
- **mean time to failure (MTTF)**—A basic measure of system reliability for nonrepairable items. The total number of life units for an item divided by the total number of failures within that population, during a particular measurement interval, under stated conditions.
- **mean time to repair (MTTR)**—A basic measure of maintainability. The sum of corrective maintenance times at any specific level of repair, divided by the total number of failures within an item repaired at that level, during a particular interval, under stated conditions.
- **measurement error**—The difference between the actual value and the measured value of a quality characteristic.
- **median**—The middle number or center value of a set of data when all the data are arranged in increasing order.
- **megaprojects**—Projects that are so large they can't be managed effectively. Consequently, they are decomposed into smaller, more manageable projects that can be completed in shorter times with fewer resources. Megaprojects provide a focused way of advancing an organization's strategies. However, they require additional coordination between projects and functional groups to be successful.
- **mentoring**—While coaching focuses on the individual as they relate to Lean Six Sigma, mentoring focuses on the individual from the career perspective. Mentors are usually experienced individuals (not necessarily in Lean Six Sigma) who have in-depth knowledge about the organization as well as the individual (that is, mentee). Usually, they come from within the organization, though not necessarily from the same department as their mentee. Their role is to help provide guidance, wisdom, and a possible road map to career advancement.
- **method of least squares**—A technique for estimating a parameter that minimizes the sum of the squared differences between the observed and the predicted values derived from the model.
- metrology—The science and practice of measurements.
- **mind mapping**—A technique for creating a visual representation of a multitude of issues or concerns by forming a map of the interrelated ideas.
- **mistake-proofing**—The use of process or design features to prevent manufacture of nonconforming product. It is typically engineering techniques that make the process or product sufficiently robust so as to avoid failing. *See also* error-proofing; foolproofing; and poka-yoke.
- mitigation—Minimizing the impact of a risk.

- **mode**—The value that occurs most frequently in a data set.
- monitoring—Continuing to observe low risks where the cost of mitigation or avoidance is too high.
- **motivation, extrinsic**—The satisfaction of either material or psychological needs that is applied by others or the organization through pre-action (incentive) or post-action (reward).
- **motivation, intrinsic**—The qualities of work itself or of relationships, events, or situations that satisfy basic psychological needs (for example, achievement, power, affiliation, autonomy, responsibility, creativity, and self-actualization) in a self-rewarding process.
- MRB—See materials review board.
- MSC—See Manpower Services Commission.
- **MTBF**—*See* mean time between failures.
- MTTF—See mean time to failure.
- **MTTR**—*See* mean time to repair.
- **muda**—An activity that consumes resources but creates no value; the seven categories are correction, processing, inventory, waiting, overproduction, internal transport, and motion. *See also* seven types of waste.
- **multicollinearity**—Multicollinearity is said to occur when two or more predictor variables in a multiple regression model are correlated. Multicollinearity may cause the coefficient estimates and significance tests for each predictor variable to be underestimated and difficult to interpret.
- **multiphase planning**—This is the process of dissecting the planning time frame into smaller time elements or phases to recognize or even celebrate significant events or accomplishments and to demonstrate progress.
- **multiple analysis of variance (MANOVA)**—A statistical technique that can be used to analyze both balanced and unbalanced experimental designs. MANOVA is used to perform multivariate analysis of variance for balanced designs when there is more than one dependent variable. The advantage of MANOVA over multiple one-way ANOVAs is that MANOVA controls the family error rate. By contrast, multiple one-way ANOVAs work to increase the alpha error rate.
- **multiple linear regression**—a linear regression equation in which multiple predictor variables are used.
- multi-vari analysis—A graphical technique for viewing multiple sources of process variation. Different sources of variation are categorized into families of related causes and quantified to reveal the largest causes.
- **multivariate control chart**—A variables control chart that allows plotting of more than one variable. These charts make use of the T^2 statistic to combine information from the dispersion and mean of several variables.

multi-voting—A decision-making tool that enables a group to sort through a long list of ideas to identify priorities.

Ν

n—*See* sample size.

- National Institute of Standards and Technology (NIST)—According to the National Institute of Standards and Technology website, "Founded in 1901, NIST is a non-regulatory federal agency within the U.S. Department of Commerce. NIST's mission is to promote U.S. innovation and industrial competitiveness by advancing measurement science, standards, and technology in ways that enhance economic security and improve our quality of life." In carrying out its mission, NIST also provides oversight to Malcolm Baldrige National Quality Award program.
- natural process variation—See voice of the process.
- natural team—A work group responsible for a particular process.
- needs analysis—See training needs analysis.
- **net present value**—A discounted cash flow technique for finding the present value of each future year's cash flow.
- next operation as customer—The concept that the organization is composed of service/product providers and service/product receivers or "internal customers." See also internal customer.

NGT—See nominal group technique.

- NIST—See National Institute of Standards and Technology.
- **noise factor**—In robust parameter design, a noise factor is a predictor variable that is hard to control or is not desirable to control as part of the standard experimental conditions. In general, it is not desirable to make inference on noise factors, but they are included in an experiment to broaden the conclusions regarding control factors.
- **nominal group technique (NGT)**—A technique similar to brainstorming that is used by teams to generate ideas on a particular subject. Team members are asked to silently come up with as many ideas as possible and write them down. Each member then shares one idea, which is recorded. After all the ideas are recorded, they are discussed and prioritized by the group.
- **nominal scale**—A scale with unordered, labeled categories, or a scale ordered by convention. Examples include type of defect, breed of dog, and complaint category. Note: It is possible to count by category but not by order or measure.

nonconformance—See defective.

nonconformity—See defect.

- **nonlinear regression**—In nonlinear regression, as with linear regression, we are talking about the form of the coefficients or parameters. In the case of non-linear regression, the coefficients are nonlinear. With linear regression the parameters have a closed-form solution. However, with nonlinear regression they do not. Instead, the parameters must be solved for using iterative optimization techniques that may or may not converge.
- **non-value-added (NVA)**—Tasks or activities that can be eliminated with no deterioration in product or service functionality, performance, or quality in the eyes of the customer. Generally, customers are unwilling to pay for non-value-added activities.
- **norming**—The third stage of team growth. *See also* stages of team growth.
- **norms**—Behavioral expectations, mutually agreed-on rules of conduct, protocols to be followed, or social practices.
- **NPV**—See net present value.
- **null hypothesis**—A hypothesis that is formulated without considering any new information, or formulated based on the belief that what already exists is true.
- NVA—See non-value-added.

Ο

- **objective**—A quantitative statement of future expectations and an indication of when the expectations should be achieved; it flows from goal(s) and clarifies what people must accomplish.
- **observation**—The process of determining the presence or absence of attributes or measuring a variable. Also, the result of the process of determining the presence or absence of attributes or measuring a variable.
- **observational study**—Analysis of data collected from a process without imposing changes on the process.
- **observed value**—The particular value of a response variable determined as a result of a test or measurement.
- **occurrence**—The likelihood of a failure occurring. Occurrence is commonly evaluated using a 10-point or similar scale. On the lowest end of the scale (1) it is assumed the probability of a failure is unlikely. On the highest end of the scale (10) it is assumed the probability of a failure is nearly inevitable.
- 1 α —*See* confidence level.
- 1β —See power.
- **operational plan**—A set of short-term plans that support the achievement of an organization's tactical plans. The planning horizon for operational plans is often one year or less, but may vary depending on the nature of the organization. Operational plans are sometimes called *action plans*.

- order—There are two types of order used by Minitab when creating experimental designs: (1) standard order (StdOrder) shows what the order of the runs in the experiment would be if the experiment were done in standard order (also called Yates's order), and (2) run order (RunOrder) shows what the order of the runs in the experiment would be if the experiment were run in random order.
- **ordinal scale**—A scale with ordered, labeled categories. Note: (1) The borderline between ordinal and discrete scales is sometimes blurred. When subjective opinion ratings such as excellent, very good, neutral, poor, and very poor are coded (as numbers 1–5), the apparent effect is conversion from an ordinal to a discrete scale. However, such numbers should not be treated as ordinary numbers because the distance between 1 and 2 may not be the same as that between 2 and 3, 3 and 4, and so forth. On the other hand, some categories that are ordered objectively according to magnitude—such as the Richter scale, which ranges from 0 to 8 according to the amount of energy released—could equally well be related to a discrete scale. (2) Sometimes, nominal scales are ordered by convention. An example is the blood groups A, B, and O, which are always stated in this order. It's the same case if different categories are denoted by single letters; they are then ordered, by convention, according to the alphabet.
- **organizational dynamics**—A field of study that deals with the behavioral nature of organizations, specifically with regard to the following elements: organizational culture, organizational structure and alignment, theories of motivation, leadership styles, group or team dynamics, conflict management and resolution, and change management. More importantly, organizational dynamics deals with the interaction of the above elements when they are all brought together.
- **out of control**—A process is described as operating out of control when special causes are present.

Ρ

- **parameter**—A constant or coefficient that describes some characteristic of a population.
- **Pareto chart**—A graphical tool based on the Pareto principle for ranking causes from most significant to least significant. It utilizes a vertical bar graph in which the bar height reflects the frequency or impact of causes. The graph is distinguished by the inclusion of a cumulative percentage line that identifies the vital few opportunities for improvement. It is also known as the *Pareto diagram*. The Pareto chart is one of the seven basic tools of quality.

Pareto diagram—See Pareto chart.

Pareto principle—An empirical rule named after the nineteenth-century economist Vilfredo Pareto that suggests that most effects come from relatively few causes; that is, about 80% of the effects come from about 20% of the possible causes. The Pareto principle is also known as the *eighty-twenty* (80–20) rule.

- **parts per billion (ppb)**—The number of times an occurrence happens in one billion chances. In a typical quality setting it usually indicates the number of times a defective part will happen in a billion parts produced; the calculation is projected into the future on the basis of past performance. Parts per billion allows for comparison of different types of product. Two ppb corresponds to a Six Sigma level of quality, assuming there is not a 1.5-sigma shift of the mean.
- **parts per million (ppm)**—The number of times an occurrence happens in one million chances. In a typical quality setting it usually indicates the number of times a defective part will happen in a million parts produced; the calculation is often projected into the future on the basis of past performance. Parts per million allows for comparison of different types of product. A ppm of 3.4 corresponds to a Six Sigma level of quality, assuming a 1.5-sigma shift of the mean.
- payback period—The number of units it will take the results of a project or capital investment to recover the investment from net cash flows. (Units may refer to the amount of product, passage of time, or other meaningful component of measure.)
- payout period—See payback period.
- PDCA cycle—See plan-do-check-act (PDCA) cycle.
- **PDPC**—See process decision program chart.
- **PDSA cycle**—*See* plan–do–study–act (PDSA) cycle.
- **percent agreement**—The percentage of time in an attribute GR&R study that appraisers agree with (1) themselves (that is, repeatability), (2) other appraisers (that is, reproducibility), or (3) a known standard (that is, bias) when classifying or rating items using nominal or ordinal scales, respectively.
- **percent gross margin**—This metric measures the efficiency of producing sales. It is computed as follows:

Percent gross margin =
$$\left(\frac{\text{Gross profit}}{\text{Net sales}}\right)(100)$$

percent net margin—This metric measures the effectiveness of an organization at generating net profit from sales. It is computed as:

Percent net margin =
$$\left(\frac{\text{Net profit}}{\text{Net sales}}\right)(100)$$

percent operating margin—This metric measures how effectively an organization controls costs not related to the production and sales of a product or service. It is computed as:

Percent operating margin =
$$\left(\frac{\text{Operating profit}}{\text{Net sales}}\right)$$
(100)

=

percent profit margin—This metric provides an overall sense of how efficiently an organization converts sales into profits. It is computed as follows:

$$= \left(\frac{\text{Sales of a product or service} - \text{Cost of a product or service}}{\text{Sales of a product or service}}\right) (100)$$
$$= \left(\frac{\text{Net profit}}{\text{Net sales}}\right) (100)$$

- performance-According to Rothwell (2008), "performance is the actual realization of that potential" gained by learning.
- **performing**—The fourth stage of team growth. *See also* stages of team growth.
- **PERT**—See program evaluation and review technique.
- **PEST analysis**—An analysis similar to SWOT, the PEST analysis brings together four environmental scanning perspectives that serve as useful input into an organization's strategic planning process. PEST stands for political, economic, social, and technological.
- **PFMEA**—See process failure mode and effects analysis.
- pipeline—A term applied to where projects "reside" that have been selected and/ or prioritized and are waiting to be assigned.
- **PIT**—See process improvement team.
- plan-do-check-act (PDCA) cycle—A four-step process for quality improvement. In the first step (plan), a plan to effect improvement is developed. In the second step (do), the plan is carried out, preferably on a small scale. In the third step (check), the effects of the plan are observed. In the last step (act), the results are studied to determine what was learned and what can be predicted. The plan-do-check-act cycle is sometimes referred to as the *Shewhart cycle*, after Walter A. Shewhart.
- plan-do-study-act (PDSA) cycle-A variation on the plan-do-check-act (PDCA) cycle, with the variation indicating that additional study is required after a change is made. The PDSA cycle is attributed to W. Edwards Deming.
- planned value (PV)—This is the approved budget for work scheduled for completion by a specific date. It is also known as the budgeted cost of work scheduled (BCWS).
- P_n —The probability that there are *n* customers in the system.
- point estimate—The single value used to estimate a population parameter. Point estimates are commonly referred to as the points at which the interval estimates are centered; these estimates give information about how much uncertainty is associated with the estimate.

- poka-yoke—A term that means to mistake-proof a process by building safeguards into the system that avoid or immediately find errors. A poka-yoke device prevents incorrect parts from being made or assembled and easily identifies a flaw or error. The term comes from the Japanese terms *poka*, which means "error," and *yokeru*, which means "to avoid." *See also* baka-yoke; errorproofing; foolproofing; mistake-proofing.
- **policy**—A high-level overall statement embracing the general goals and acceptable practices of a group.

policy control—See hoshin kanri.

policy deployment—See hoshin kanri.

policy management—See hoshin kanri.

PONC—See price of nonconformance.

population—The totality of items or units of material under consideration.

- **portfolio management**—The process of managing both active (assigned) and inactive (pipeline) projects.
- potential process capability—See P_{pk}.
- **power** (1β) —The probability of rejecting the null hypothesis when the alternative is true. We write this as *P*(rejecting $H_0|H_0$ is not true) = 1β . The stronger the power of a hypothesis test, the more sensitive the test is to detecting small differences.
- *P_p* (process performance index)—An index describing process performance in relation to specified tolerance:

$$P_p = \frac{USL - LSL}{6s}$$

s is used for standard deviation instead of σ since both random and special causes may be present. Note: A state of statistical control is not required. Also note the similarity to the formula for C_p .

ppb—See parts per billion.

- P_{pk} (minimum process performance index)—The smaller of the upper process performance index (P_{pk_u}) and the lower process performance index (P_{pk_L}). The P_{pk} similar to the C_{pk} considers the mean of the process with respect to the upper and lower specifications. The P_{pk} is known as the *potential process capability* and is used in the validation stage of a new product launch. A state of statistical control is not required.
- P_{pk_L} (process performance index, lower; PPL)—An index describing process performance in relation to the lower specification limit. The index is defined as:

$$P_{pk_L} = \frac{\overline{x} - LSL}{3s}$$

where \overline{x} = process average, *LSL* = lower specification limit, and 3*s* = half of the process capability. Note: When the process average is obtained from a control chart, \overline{x} is replaced by $\overline{\overline{x}}$.

 P_{pk_u} (process performance index, upper; PPU)—An index describing process performance in relation to the upper specification limit. The index is defined as

$$P_{pk_{u}} = \frac{USL - \overline{x}}{3s}$$

where \overline{x} = process average, USL = upper specification limit, and 3s = half of the process capability. Note: When the process average is obtained from a control chart, \overline{x} is replaced by $\overline{\overline{x}}$.

- **PPL**—See P_{vk_t} .
- P_{pm} (process performance index of the mean)—An index that takes into account the location of the process average relative to a target value and is defined as

$$P_{cm} = \frac{(USL - LSL)/2}{3\sqrt{s^2 + (\overline{x} - T)^2}}$$

where USL = upper specification limit, LSL = lower specification limit, σ = standard deviation, \overline{x} = process average, and T = target value. If the process average is obtained from a control chart, \overline{x} is replaced by $\overline{\overline{x}}$. When the process average and the target value are the same, the $P_{pm} = P_{pk}$. Also, should the process average drift from the target value, the $P_{pm} < P_{pk}$. Note: A state of statistical control is not required. Also note the similarity to the formula for P_{cm} .

ppm—See parts per million.

PPU—See $P_{pk_{ij}}$.

- **precision**—The closeness of agreement between randomly selected individual measurements or test results. It is this aspect of measure that addresses repeatability or consistency when an identical item is measured several times.
- **precision-to-tolerance ratio (PTR)**—A measure of the capability of the measurement system. It can be computed as

$$PTR = \frac{5.15\hat{\sigma}_{ms}}{USL - LSL}$$

where $\hat{\sigma}_{ms}$ is the estimated standard deviation of the total measurement system variability. In general, reducing the PTR will yield an improved measurement system.

prediction interval—Similar to a confidence interval, this is based on the predicted value that is likely to contain the values of future observations. It will be wider than the confidence interval because it contains bounds on individual observations rather than a bound on the mean of a group of observations. The $100(1 - \alpha)$ % prediction interval for a single future observation from a normal distribution is given by

$$\overline{X} - t_{\alpha/2}s\sqrt{1 + \frac{1}{n}} \le X_{n+1} \le \overline{X} + t_{\alpha/2}s\sqrt{1 + \frac{1}{n}}$$

- **predictor variable**—A variable that can contribute to the explanation of the outcome of an experiment. A predictor variable is also known as an *independent variable, explanatory variable,* or *regressor variable*.
- **PRESS**—A statistic that is used to assess the fit of a regression model for a given set of observations that were not used to estimate the model's parameters. The smaller the PRESS value the better. The PRESS statistic is computed as

$$PRESS = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

where

n = Number of observations

yi = ith observed value

- \hat{y}_i = *i*th predicted value based on a model fitted to the remaining n-1 points.
- **prevention costs**—The cost of all activities specifically designed to prevent poor quality in products or services. Examples include the cost of new product review, quality planning, supplier capability surveys, process capability evaluations, quality improvement team meetings, quality improvement projects, and quality education and training.
- **prevention versus detection**—A phrase used to contrast two types of quality activities. Prevention refers to those activities designed to prevent nonconformances in products and services. Detection refers to those activities designed to find nonconformances already in products and services. Another phrase used to describe this distinction is "designing-in quality versus inspecting-in quality."
- **price of nonconformance (PONC)**—The cost of not doing things right the first time.
- **principal components**—Used "to form a smaller number of uncorrelated variables from a large set of data. The goal of principal components analysis is to explain the maximum amount of variance with the fewest number of principal components" (Minitab 16). Principal components have wide applicability in the social sciences and market research.
- **principles**—Covey (1989) defines principles as "guidelines for human conduct that are proven to have enduring, permanent value. They're fundamental. They're essentially unarguable because they are self-evident. One way to

quickly grasp the self-evident nature of principles is to simply consider the absurdity of attempting to live an effective life based on their opposites. I doubt that anyone would seriously consider unfairness, deceit, baseness, use-lessness, mediocrity, or degeneration to be a solid foundation for lasting happiness and success."

- **prioritization matrix**—A tool used to choose between several options that have many useful benefits, but where not all of them are of equal value. The choices are prioritized according to known weighted criteria and then narrowed down to the most desirable or effective one(s) to accomplish the task or problem at hand. The prioritization matrix is one of the seven management and planning tools.
- **process**—A series of interrelated steps consisting of resources and activities that transform inputs into outputs and work together toward a common end. A process can be graphically represented using a flowchart. A process may or may not add value.
- **process capability**—The calculated inherent variability of a characteristic of a product. It represents the best performance of the process over a period of stable operations. Process capability is expressed as $6\hat{\sigma}$, where $\hat{\sigma}$ is the sample standard deviation (short-term component of variation) of the process under a state of statistical control. Note: $\hat{\sigma}$ is often shown as σ in most process capability formulas. A process is said to be "capable" when the output of the process always conforms to the process specifications.
- **process capability index**—A single-number assessment of ability to meet specification limits on a quality characteristic of interest. This index compares the variability of the characteristic to the specification limits. Three basic process capability indices are C_p , C_{pk} , and C_{pm} . See also P_p ; P_{pk} ; and P_{pm} .
- **process decision program chart (PDPC)**—A tool that identifies all events that can go wrong and the appropriate countermeasures for these events. It graphically represents all sequences that lead to a desirable effect. It is one of the seven management and planning tools.
- **process failure mode and effects analysis (PFMEA)**—An analysis process used to identify and evaluate the relative risks associated with a particular process.
- **process improvement team (PIT)**—A natural work group or cross-functional team whose responsibility is to achieve needed improvements in existing processes. The life span of the team is based on the completion of the team purpose and specific tasks.
- process management—The set of techniques and tools applied to a process to implement and improve process effectiveness, hold the gains, and ensure process integrity in fulfilling customer requirements.
- **process mapping**—The flowcharting of a process in detail that includes the identification of inputs and outputs.
- **process owner**—A Lean Six Sigma role associated with an individual who coordinates the various functions and work activities at all levels of a process, has

the authority or ability to make changes in the process as required, and manages the entire process cycle so as to ensure performance effectiveness.

- **process performance**—A statistical measure of the outcome of a characteristic from a process that may not have been demonstrated to be in a state of statistical control. Note: Use this measure cautiously since it may contain a component of variability from special causes of unpredictable value. It differs from process capability because a state of statistical control is not required.
- **process performance index**—A single-number assessment of ability to meet specification limits on a quality characteristic of interest. The index compares the variability of the characteristic with the specification limits. Three basic process performance indices are P_{pr} , P_{pkr} , and P_{pm} . The P_{pm} is analogous to the C_{pm} . *See also* P_{pr} ; P_{pkr} , and P_{pm} .
- **producer's risk (** α **)**—The probability of nonacceptance when the quality level has a value stated by the acceptance sampling plan as acceptable. Note: (1) Such nonacceptance is a Type I error; (2) producer's risk is usually designated as alpha (α); (3) quality level could relate to fraction nonconforming and acceptable to AQL; (4) interpretation of producer's risk requires knowledge of the stated quality level. Producer's risk is also known as *alpha* (α) *risk, alpha* (α) *error, error of the first kind, Type 1 error,* and *Type I error*.
- **product identification**—A means of marking parts with a label, etching, engraving, ink, or other means so that part numbers and other key attributes can be identified.
- **program evaluation and review technique (PERT)**—An event-oriented project management planning and measurement technique that utilizes an arrow diagram or road map to identify all major project events and demonstrates the amount of time (critical path) needed to complete a project. It provides three time estimates: optimistic, most likely, and pessimistic.
- **program management**—Occasionally, project management is used interchangeably with *program management*. However, program management is actually a higher-level activity often involving the management of multiple projects with a common goal.
- **project**—A project exists for producing results. Three components must be present for an activity to be defined as a project: scope, schedule, and resources.
- **project, active**—Projects that have been assigned and are under way. Note: Projects that have been assigned and under way, but have been placed on hold are considered to be active.
- **project**, **inactive**—Projects that currently reside in the pipeline. These include both selected and prioritized projects.
- project assignment—The process by which a project is assigned for execution out of the project pipeline. A prioritized project and a resource (that is, Black Belt or Black Belt candidate) are available to be matched. If the matching is successful, the resource is assigned to the project and the project gets under way.

- **project closure**—The process by which a project is closed and ready to be removed from active status. A project has been terminated or closed for one of two reasons: successful completion or it is no longer viable.
- **project identification**—The process by which a project is identified as a possible improvement opportunity. The delineation of clear and specific improvement opportunities that are required in the organization.
- **project life cycle**—A typical project life cycle consists of five sequential phases in project management: concept, planning, design, implementation, and evaluation.
- **project management**—The discipline of planning, organizing, and managing resources to bring about the successful completion of specific project goals and objectives, categorized into five component processes (sometimes called life cycle phases): initiating, planning, executing, controlling, and closing. The Project Management Institute notes that project management is the "application of knowledge, skills, tools, and techniques" to activities to meet the requirements of a specific project.
- **project plan**—Documents that contain the details of why the project is to be initiated, what the project is to accomplish, when and where it is to be implemented, who will be responsible, how implementation will be carried out, how much it will cost, what resources are required, and how the project's progress and results will be measured.
- **project portfolio**—The totality of the projects that have been selected and prioritized and waiting in the pipeline, plus those that have been assigned and are currently under way.
- **project prioritization**—The process by which a project is prioritized once it has been selected and waiting in the project pipeline. A selected project will be independently assessed against specific criteria in order to establish a rank score. Rank scores of other selected projects will be compared, and a final ranking of the projects will be established. This final ranking will set the precedence and order for project assignment.
- **project qualification**—The process by which a project is qualified for consideration as a Lean Six Sigma project. The project qualification process translates an identified improvement opportunity into Lean Six Sigma charter format for recognizing its completeness and the ability to assess its potential as a viable project.
- **project reserve budget**—In instances where doubt exists, it may be possible to establish a project reserve budget. Such budgets may be set as a percentage (for example, 10%, 15%, 20%) of the actual dollar budget. These budgets are typically set aside as contingency dollars to help mitigate project risk.
- project selection—The process by which a project is selected to enter the project pipeline. To select a project means that it has met the criteria of being a Lean Six Sigma project (that is, it has been qualified) and now must be judged to determine whether it is sufficiently worthy such that the organization is willing to invest time and resources to achieve the expected benefits of the project.

propagation of errors—Also known as *transmission of error*, this has application in two major areas of Lean Six Sigma: (1) response model analysis where the analysis will address how random noise in the x_i values affects the predicted value of y and (2) tolerance stack-up analysis where the analysis will address tolerance stack-up problems.

PTR—See precision-to-tolerance ratio.

- **PV**—See planned value.
- *p***-value**—The smallest level of significance leading to the rejection of the null hypothesis.

 P_w —The probability that an arriving customer will have to wait for service.

 P_0 —The probability that there are no customers in the system.

Q

QFD—See quality function deployment.

- **quality assurance**—The planned or systematic actions necessary to provide adequate confidence that a product or service will satisfy given needs.
- **quality control**—The operational techniques and activities that sustain a quality of product or service that will satisfy given needs; also, the use of such techniques and activities.
- **quality council**—A group within an organization that drives the quality improvement effort and usually has oversight responsibility for the implementation and maintenance of the quality management system; it operates in parallel with the normal operation of the business. A quality council is sometimes referred to as a *quality steering committee*.
- **quality function deployment (QFD)**—A method used to translate voice of the customer information into product requirements/CTQs and to continue deployment (for example, cascading) of requirements to parts and process requirements. Quality function deployment is also known as the *house of quality. See also* house of quality.
- **quality improvement**—The actions taken throughout an organization to increase the effectiveness and efficiency of activities and processes in order to provide added benefits to both the organization and its customers.
- **quality loss function**—A parabolic approximation of the quality loss that occurs when a quality characteristic deviates from its target value. The quality loss function is expressed in monetary units. The cost of deviating from the target increases as a quadratic function the farther the quality characteristic moves from the target. The formula used to compute the quality loss function depends on the type of quality characteristic used. Genichi Taguchi first introduced the quality loss function in this form.
- **quality management**—The totality of functions involved in organizing and leading the effort to determine and achieve quality.

- **quality manual**—A document stating the quality policy and describing the quality system of an organization.
- **quality planning**—The activity of establishing quality objectives and quality requirements.
- **quality policy**—Top management's formally stated intentions and direction for the organization pertaining to quality.
- **quality system**—The organizational structure, procedures, processes, and resources needed to implement quality management.
- **quality trilogy**—A three-pronged approach identified by Joseph M. Juran for managing for quality. The three legs are quality planning (developing the products and processes required to meet customer needs), quality control (meeting product and process goals), and quality improvement (achieving unprecedented levels of performance).

queue—A waiting line.

queueing theory—The mathematical study of queues. Alternate spelling of "queueing" is "queuing." ("Queueing" is thought to be the only word in the English language that contains five consecutive vowels.)

R

r—*See* correlation coefficient.

- *R*—*See* correlation coefficient.
- **random cause**—A source of process variation that is inherent in a process over time. Note: In a process subject only to random cause variation, the variation is predictable within statistically established limits. Random cause is also known as *chance cause* and *common cause*.
- **random sampling**—The process of selecting units for a sample in such a manner that all combinations of units under consideration have an equal or ascertainable chance of being selected for the sample.
- random variable—A variable whose value depends on chance.
- random variation—Variation due to random cause.
- **randomization**—The process used to assign treatments to experimental units so that each experimental unit has an equal chance of being assigned a particular treatment.
- **randomized block design**—An experimental design consisting of *b* blocks with *t* treatments assigned via randomization to the experimental units within each block. This is a method for controlling the variability of experimental units. For the completely randomized design, no stratification of the experimental units is made. In the randomized block design, the treatments are randomly allotted within each block; that is, the randomization is restricted.

- **randomized block factorial design**—A factorial design run in a randomized block design where each block includes a complete set of factorial combinations.
- **ratio scale**—A scale where meaningful differences are shown, where absolute zero exists, and where multiplication and division are permitted. One example of a ratio scale is length in inches, because zero length is defined as having no length, and 20 inches is twice as long as 10 inches.
- **rational subgroup**—A subgroup that is expected to be as free as possible from assignable causes (usually consecutive items). In a rational subgroup, variation is presumed to be only from random cause.
- **recognition**—In the context of team growth, recognition is the sixth or final stage. *See also* stages of team growth.
- red bead experiment—An experiment developed by W. Edwards Deming to illustrate that it is impossible to put employees in rank order of performance for the coming year based on their performance during the past year because performance differences must be attributed to the system, not to the employees. Four thousand red and white beads in a jar (of which 20% are red) and six people are needed for the experiment. The participants' goal is to produce white beads, as the customer will not accept red beads. One person stirs the beads and then, blindfolded, selects a sample of 50 beads. That person hands the jar to the next person, who repeats the process, and so on. When all participants have their samples, the number of red beads for each is counted. The limits of variation between employees that can be attributed to the system are calculated. Everyone will fall within the calculated limits of variation that could arise from the system. The calculations will show that there is no evidence that one person will be a better performer than another in the future. The experiment shows that it would be a waste of management's time to try to find out why, say, John produced 4 red beads and Jane produced 15; instead, management should improve the system, making it possible for everyone to produce more white beads.
- reengineering—The process of completely redesigning or restructuring a whole organization, an organizational component, or a complete process. It's a "start all over again from the beginning" approach, sometimes called a *breakthrough*. In terms of improvement approaches, reengineering is contrasted with incremental improvement (kaizen).
- **regression analysis**—A technique that typically uses continuous predictor variable(s) (that is, regressor variables) to predict the variation in a continuous response variable. Regression analysis uses the method of least squares to determine the values of the linear regression coefficients and the corresponding model.
- regressor variable—See predictor variable.
- **reliability**—The probability that an item can perform its intended function for a specified interval under stated conditions. In the context of training, reliability

refers to the consistency and stability of the measurement process over time. For example, if an individual's skills have not been enhanced between two designated time periods, a reliable measurement process would yield the same results for the same participant.

reliability growth—The improvement in product reliability over a period of time.

- **repeatability**—The precision under conditions where independent measurement results are obtained with the same method on identical measurement items by the same appraiser (that is, operator) using the same equipment within a short period of time. Although misleading, repeatability is often referred to as *equipment variation* (EV). It is also referred to as *within-system variation* when the conditions of measurement are fixed and defined (that is, equipment, appraiser, method, and environment). Repeatability is a component of precision.
- **repeated measures**—The measurement of a response variable more than once under similar conditions. Repeated measures allow one to determine the inherent variability in the measurement system. Repeated measures are also known as *duplication* or *repetition*.
- **replication**—The performance of an experiment more than once for a given set of predictor variables. Each repetition of the experiment is called a *replicate*. Replication differs from repeated measures in that it is a repeat of the entire experiment for a given set of predictor variables, not just a repeat of measurements on the same experiment. Note: Replication increases the precision of the estimates of the effects in an experiment. It is more effective when all elements contributing to the experimental error are included.
- **reproducibility**—The precision under conditions where independent measurement results are obtained with the same method on identical measurement items with different operators using different equipment. Although misleading, reproducibility is often referred to as *appraiser variation* (AV). The term "appraiser variation" is used because it is common practice to have different operators with identical measuring systems. Reproducibility, however, can refer to any changes in the measurement system. For example, assume the same appraiser uses the same material, equipment, and environment, but uses two different measurement methods; the reproducibility calculation will show the variation due to change in methods. It is also known as the *average variation between systems* or *between-conditions variation of measurement*. Reproducibility is a component of precision.
- **resolution**—(1) The smallest measurement increment that can be detected by the measurement system. (2) In the context of experimental design, resolution refers to the level of confounding in a fractional factorial design. For example, in a resolution III design, the main effects are confounded with the two-way interaction effects.
- **response variable**—A variable that shows the observed results of an experimental treatment. It is sometimes known as the *dependent variable* or *y variable*. There may be multiple response variables in an experimental design.

- **restraining forces**—In the context of a force field analysis, restraining forces are those forces that hinder or oppose the objective.
- **RETAD**—See single-minute exchange of die.
- **return on equity (ROE)**—The net profit after taxes divided by last year's tangible stockholders' equity and then multiplied by 100 to provide a percentage.
- **return on investment (ROI)**—An umbrella term for a variety of ratios measuring an organization's business performance, the ROI metric measures the effectiveness of an organization's ability to use its resources to generate income. It is computed as

$$ROI = \frac{Income}{Cost}(100\%).$$

In its most basic form, ROI indicates what remains from all money taken in after all expenses are paid.

- **revenue growth**—In the context of Lean Six Sigma, the projected increase in income that will result from a project. This is calculated as the increase in gross income minus the cost. Revenue growth may be stated in dollars per year or as a percentage per year.
- *ρ* (rho)—In the context of measuring the relationship between two sets of numbers, it is the population correlation coefficient. *See also* correlation coefficient. In the context of queueing theory, it is the utilization factor for each server and is the fraction of time that each server is busy.
- **risk**—The probability of not achieving a project goal or schedule, budget, or resource target because something did or didn't occur. Risk may be negative, in the form of threats, or positive, in the form of opportunities.
- **risk, consumer's** (β)—See consumer's risk.
- **risk**, **producer's** (*α*)—*See* producer's risk.
- **risk assessment/management**—The process of determining what risks are present in a situation and what actions might be taken to eliminate or mediate them.
- **robust designs**—Products or processes that continue to perform as intended in spite of manufacturing variation and extreme environmental conditions during use.
- **robustness**—The condition of a product or process design that remains relatively stable with a minimum of variation even though factors that influence operations or usage, such as environment and wear, are constantly changing.
- **ROE**—*See* return on equity.
- ROI—See return on investment.
- rolled throughput yield (RTY)—The probability of a unit of product passing through an entire process defect-free. The rolled throughput yield is

determined by multiplying first-pass yield from each subprocess of the total process.

- **root cause**—A factor (that is, original cause) that, through a chain of cause and effect, causes a defect or nonconformance to occur. Root causes should be permanently eliminated through process improvement. Note: Several root causes may be present and may work either together or independently to cause a defect.
- **root cause analysis**—A structured approach or process of identifying root (that is, original) causes. Many techniques and statistical tools are available for analyzing data to ultimately determine the root cause.
- r^2 —See coefficient of determination.
- *R*²—*See* coefficient of determination.
- r_{adj}^2 —See adjusted coefficient of determination.
- R_{adj}^2 —See adjusted coefficient of determination.
- RTY—See rolled throughput yield.
- **run chart**—A chart showing a line connecting consecutive data points collected from a process running over a period of time. The run chart is occasionally considered to be one of the seven basic tools of quality. Note: A trend is indicated when the series of collected data points heads up or down.
- **run rates**—A projection estimate of future performance based on the assumption that the remainder of the project will continue as it has for the work performed to date. These estimates may be 12-month, year-end, or project-end projections.

S

- **sample**—A group of units, portions of material, or observations taken from a larger collection of units, quantity of material, or observations that serves to provide information that may be used as a basis for making a decision concerning the larger quantity.
- **sample autocorrelation function (ACF)**—The sample autocorrelation function is computed as

$$r_k = \frac{\sum_{t=1}^n (x_{t-k} - \overline{x})(x_t - \overline{x})}{\sum_{t=1}^n (x_t - \overline{x})^2}$$

where

n = Number of observations

 x_t = Value of x at time t

k = Lag j; k = 1, 2, ...

- **sample size** (*n*)—The number of units in a sample. Note: In a multistage sample, the sample size is the total number of units at the conclusion of the final stage of sampling.
- **Sarbanes–Oxley Act of 2002 (SOX)**—A legislative act enacted after a series of financial scandals that mandates strict requirements for financial accounting and for top management's responsibility and accountability in disclosing the financial status of their organization.
- scatter diagram—A plot of two variables, one on the *y* axis and the other on the *x* axis. The resulting graph allows visual examination for patterns to determine if the variables show any relationship or if there is just random "scatter." This pattern, or lack thereof, aids in choosing the appropriate type of model for estimation. Note: Evidence of a pattern does not imply that a causal relationship exists between the variables. The scatter diagram is one of the seven tools of quality.

scatter plot—See scatter diagram.

schedule performance index (SPI)—This is a dimensionless index used to measure the project's schedule efficiency. It is computed as

$$SPI = \frac{EV}{PV} = \frac{BCWP}{BCWS}$$

If the SPI is greater than 1, the project is ahead of schedule. If the SPI is equal to 1, the project is on schedule. If the SPI is less than 1, the project is behind schedule, and corrective action must be taken.

schedule variance (SV)—The schedule variance is the difference between the amount budgeted for the work performed and the planned cost of the work performed. It is computed as

$$SV = EV - PC$$

If the SV is positive, the project is ahead of schedule. If the SV is zero, the project is on schedule. If the SV is negative, the project is behind schedule, and corrective action must be taken.

- scribe—The member of a team assigned to record minutes of meetings.
- SDL—See self-directed learning.

SDWT-See self-directed work team.

SEI—See Software Engineering Institute.

seiban—The name of a management practice taken from the Japanese words *sei*, which means "manufacturing," and *ban*, which means "number." A seiban number is assigned to all parts, materials, and purchase orders associated with a particular customer, job, or project. This enables a manufacturer to track everything related to a particular product, project, or customer and facilitates setting aside inventory for specific projects or priorities. Seiban is an effective practice for project and build-to-order manufacturing.

- **seiketsu**—One of the 5S's that means to conduct seiri, seiton, and seiso at frequent, indeed daily, intervals to maintain a workplace in perfect condition.
- **seiri**—One of the 5S's that means to separate needed tools, parts, and instructions from unneeded materials and to remove the latter.
- seiso—One of the 5S's that means to conduct a cleanup campaign.
- **seiton**—One of the 5S's that means to neatly arrange and identify parts and tools for ease of use.
- **self-directed learning (SDL)** A learning method by which students learn without an instructor and at their own pace. This permits adult learners to build toward achieving the desired competency level in the needed knowledge or skill.
- self-directed work team (SDWT)—A team that requires little supervision and manages itself and the day-to-day work it does; self-directed teams are responsible for whole work processes and schedules, with each individual performing multiple tasks.
- **self-sustaining department**—A department that must promote itself to the remainder of the organization with the hope that the organization will fund its services.
- sensor/transmitter—In the context of automated process control, these components are often combined. They capture a measurement, convert it to a signal, and transmit it to the controller for interpretation. These are also called the *primary* and *secondary elements*. The basic operation performed is "measurement."
- **set point**—In the context of automated process control, the set point is the specified value of the controlled variable. This variable is equivalent to the nominal value in a specification.
- **seven basic tools of quality**—A set of both qualitative and quantitative tools that help organizations understand and improve. Unfortunately, there is no longer a unified set of seven tools that are universally agreed on, but most authors agree that five of the seven basic tools are the cause-and-effect diagram, control chart, histogram, Pareto chart, and the scatter diagram. Additional tools suggested as part of the seven basic tools of quality include the check sheet, flowchart, graph, run chart, trend chart, and stratification. Note: (1) Graphs are sometimes included with control charts and are distinct at other times; (2) run charts are sometimes included with control charts and are distinct at other times; (3) run charts are sometimes synonymous with trend charts and are distinct at other times; (4) run charts are sometimes included with graphs and are distinct at other times. Some authors are known to have posited eight basic tools of quality. *See also* individual entries.
- 7M's—The 6M's with the addition of management. See also 6M's.
- seven management and planning tools—The seven commonly recognized management and planning tools are the affinity diagram, tree diagram, process

decision program chart (PDPC), matrix diagram, interrelationship diagram, prioritization matrices, and the activity network diagram. *See also* individual entries.

- seven tools of quality—See seven basic tools of quality.
- seven types of waste—Taiichi Ohno proposed value as the opposite of waste and identified seven categories: (1) defects, (2) overproduction (ahead of demand), (3) overprocessing (beyond customer requirements), (4) waiting, (5) unnecessary motion, (6) transportation, and (7) inventory (in excess of the minimum).
- **severity**—An indicator of the degree of a failure should a failure occur. Severity can be evaluated based on a 10-point or similar scale. On the lowest end of the scale (1), it is assumed a failure will have no noticeable effect. On the highest end of the scale (10), it is assumed a failure will impact safe operation or violate compliance with a regulatory mandate.
- Shewhart cycle—See plan-do-check-act (PDCA) cycle.
- **shitsuke**—One of the 5S's that means to form the habit of always following the first four S's.
- side effects—See law of unintended consequences.
- **simple linear regression**—A linear regression equation in which one predictor variable is used.
- simulation—The act of modeling the characteristics or behaviors of a physical or abstract system.
- simultaneous engineering—See concurrent engineering.
- **single-minute exchange of die (SMED)**—A series of techniques pioneered by Shigeo Shingo for changeovers of production machinery in less than 10 minutes. The long-term objective is always zero setup, in which changeovers are instantaneous and do not interfere in any way with continuous flow. Setup in a single minute is not required but used as a reference. SMED is also known as *rapid exchange of tooling and dies* (RETAD).
- SIPOC—A macro-level analysis of suppliers, inputs, processes, outputs, and customers.
- **6M's**—Typically, the primary categories of the cause-and-effect diagram: machines, manpower, materials, measurements, methods, and Mother Nature. Using these categories as a structured approach provides some assurance that few causes will be overlooked. *See also* 7M's.
- **Six Sigma**—"Six Sigma" can take on various definitions across a broad spectrum depending on the level of focus and implementation. (1) Philosophy—the philosophical perspective views all work as processes that can be defined, measured, analyzed, improved, and controlled (DMAIC). Processes require inputs and produce outputs. If you control the inputs, you will control the outputs. This is generally expressed as the y = f(x) concept. (2) Set of tools—Six Sigma

as a set of tools includes all the qualitative and quantitative techniques used by the Six Sigma practitioner to drive process improvement through defect reduction and the minimization of variation. A few such tools include statistical process control (SPC), control charts, failure mode and effects analysis (FMEA), and process mapping. There is probably little agreement among Six Sigma professionals as to what constitutes the tool set. (3) Methodology—this view of Six Sigma recognizes the underlying and rigorous approach known as DMAIC. DMAIC defines the steps a Six Sigma practitioner is expected to follow, starting with identifying the problem and ending with implementing long-lasting solutions. While DMAIC is not the only Six Sigma methodology in use, it is certainly the most widely adopted and recognized. (4) Metrics in simple terms, Six Sigma quality performance means 3.4 defects per million opportunities (accounting for a 1.5-sigma shift in the mean).

- **skills analysis**—The purpose of the skills analysis is to identify the specific skills required to perform the specific tasks identified by a task analysis. In many instances, it is likely multiple skills will be required to perform a specific task. Examples of skills associated with the task example might be "ability to listen effectively" and "ability to write clearly."
- **slack time**—The time an activity can be delayed without delaying the entire project; it is determined by calculating the difference between the latest allowable date and the earliest expected date.
- **SMART**—SMART stands for *specific, measurable, achievable, relevant,* and *timely,* and is used generally in regard to the creation of goal statements.
- SMED—See single-minute exchange of die.
- soft dollar savings—See cost avoidance savings.
- **Software Engineering Institute (SEI)**—According to the Carnegie Mellon University website, "The Carnegie Mellon Software Engineering Institute (SEI) works closely with defense and government organizations, industry, and academia to continually improve software-intensive systems. Our core purpose is to help organizations such as yours to improve their software engineering capabilities and to develop or acquire the right software, defect free, within budget and on time, every time. To accomplish this, the SEI: 1) performs research to explore promising solutions to software engineering problems; 2) identifies and codifies technological and methodological solutions; 3) tests and refines the solutions through pilot programs that help industry and government solve their problems; and 4) widely disseminates proven solutions through training, licensing, and publication of best practices."
- SOW—See statement of work.
- **SOX**—*See* Sarbanes–Oxley Act of 2002.
- **spaghetti chart**—A before-improvement chart of existing steps in a process and the many back and forth interrelationships (can resemble a bowl of spaghetti); used to see the redundancies and other wasted movements of people and material.

- **span of control**—The number of subordinates a manager can effectively and efficiently manage. From a practical standpoint, many organizations do not actively consider the "effectively and efficiently" aspect of managing subordinates. Consequently, spans of control often exceed the number of subordinates who can be managed effectively and efficiently. Moreover, indirect reports are frequently overlooked.
- SPC—See statistical process control.
- **special cause**—A source of process variation other than inherent process variation. Note: (1) Sometimes special cause is considered synonymous with assignable cause, but a special cause is assignable only when it is specifically identified; (2) a special cause arises because of specific circumstances that are not always present. Therefore, in a process subject to special causes, the magnitude of the variation over time is unpredictable. *See also* assignable cause.
- **specification**—An engineering requirement used for judging the acceptability of a particular product/service based on product characteristics such as appearance, performance, and size. In statistical analysis, specifications refer to the document that prescribes the requirements with which the product or service has to perform.
- **specification limits**—Limiting value(s) stated for a characteristic. *See also* tolerance.
- **SPI**—*See* schedule performance index.
- sponsor—A member of senior management who oversees, supports, and implements the efforts of a team or initiative. Although many organizations define the terms "champion" and "sponsor" differently, they are frequently used interchangeably. See also champion.
- **SQC**—*See* statistical quality control.
- **stability (of a measurement system)**—The change in bias of a measurement system over time and usage when that system is used to measure a master part or standard. Thus, a stable measurement system is one in which the variation is in statistical control, which is typically demonstrated through the use of control charts. Stability is a component of accuracy.
- **stable process**—A process that is predictable within limits; a process that is subject only to random causes. (This is also known as a state of statistical control.) Note: (1) A stable process will generally behave as though the results are simple random samples from the same population; (2) this state does not imply that the random variation is large or small, within or outside specification limits, but rather that the variation is predictable using statistical techniques; (3) the process capability of a stable process is usually improved by fundamental changes that reduce or remove some of the random causes present and/or adjusting the mean toward the target value.
- stages of team growth—The six development stages through which groups typically progress: forming, storming, norming, performing, adjourning, and

recognition. Knowledge of the stages helps team members accept the normal problems that occur on the path from forming a group to becoming a team.

- **stakeholders**—The people, departments, and groups that have an investment or interest in the success or actions taken by the organization.
- **standard**—A statement, specification, or quantity of material against which measured outputs from a process may be judged as acceptable or unacceptable.
- standard error of the ACF-The standard error of the ACF is computed as

$$SE(r_k) = \sqrt{\frac{\left(1 + 2\sum_{m=1}^{k-1} r_m^2\right)}{n}}; SE(r_1) = \sqrt{\frac{1}{n}}$$

where

n = Number of observations

 r_m = Autocorrelation at lag m; m = 1,2,...,k

k = Lag j; k = 1, 2, ...

- **standard work**—A concept whereby each work activity is organized around human motion to minimize waste. Each work activity is precisely described and includes specifying the cycle time, takt time, task sequence, and the minimum inventory of parts on hand required to conduct the activity. Standard work is also known as *standardized work*.
- statement of work (SOW)—A description of the actual work to be accomplished. It is derived from the work breakdown structure and, when combined with the project specifications, becomes the basis for the contractual agreement on the project. The statement of work is also known as *scope of work*.
- stationarity—The statistical characteristics of the mean, variance, and autocorrelation structure over time. Time-series data are assumed stationary when the mean, variance, and autocorrelation structure are constant over the entire range of the data.
- **statistic**—A quantity calculated from a sample of observations, most often to form an estimate of some population parameter.
- **statistical control**—A process is considered to be in a state of statistical control if variations among the observed sampling results from it can be attributed to a constant system of chance causes.
- statistical process control (SPC)—The use of statistical techniques such as control charts to reduce variation, increase knowledge about the process, and steer the process in the desired way. Note: (1) SPC operates most efficiently by controlling variation of process or in-process characteristics that correlate with a final product characteristic, and/or by increasing the robustness of the process against this variation; (2) a supplier's final product characteristic can be a process characteristic of the next downstream supplier's process.

- **statistical quality control (SQC)**—The application of statistical techniques to control quality. The term "statistical process control" is often used interchangeably with "statistical quality control," although statistical quality control includes acceptance sampling as well as statistical process control.
- **statistical tolerance interval**—An interval estimator determined from a random sample so as to provide a specified level of confidence that the interval covers at least a specified proportion of the sampled population.
- storming—The second stage of team growth. See also stages of team growth.
- **strategic plan**—A set of long-term plans generally focused on a three- to fiveyear planning horizon and designed to achieve an organization's goals and objectives.
- **strategic planning**—A process to set an organization's long-range goals and identify the actions needed to reach those goals.
- **stratification**—The layering of objects or data; also, the process of classifying data into subgroups based on characteristics or categories. Stratification is occasionally considered to be one of the seven tools of quality.
- **subsystem**—A combination of sets, groups, and so on, that performs an operational function within a system and its major subdivision of the system.
- **supply chain**—The series of processes and/or organizations that are involved in producing and delivering a product or service to the customer or user.
- SV—See schedule variance.
- **SWOT analysis**—An assessment of an organization's key *strengths, weaknesses, opportunities,* and *threats.* It considers factors such as the organization's industry, competitive position, functional areas, and management.
- **system**—A composite of equipment, skills, and techniques capable of performing or supporting an operational role, or both. A complete system includes all equipment, related facilities, material, software, services, and personnel required for its operation and support to the degree that it can be considered self-sufficient in its intended operating environment.
- **systems thinking**—A problem-solving approach that expands the view of something under analysis to determine how it interacts with other elements of the system of which it is a part.

Т

- **tactical plan**—A set of mid-term plans that support the achievement of an organization's strategic plans. The planning horizon for tactical plans is often in the one- to three-year range, but may vary depending on the nature of the organization.
- **Taguchi methods**—The American Supplier Institute's trademarked term for the quality engineering methodology developed by Genichi Taguchi. In this engineering approach to quality control, Taguchi calls for off-line quality control,

online quality control, and a system of experimental designs to improve quality and reduce costs.

- **takt time**—A term derived from the German word *taktzeit*, meaning "clock cycle." Takt time is the available production time divided by the rate of customer demand. Operating under takt time sets the production pace to customer demand.
- **task analysis**—The purpose of the task analysis is to identify the specific tasks required to perform a particular job. Often, this is accomplished by direct observation of the individual performing the job. It is important that all tasks be identified regardless of whether or not they are observable at the time the task analysis is being performed. An example of a task might be "prepare minutes for all management meetings."
- **team**—Two or more people who are equally accountable for the accomplishment of a purpose and specific performance goals; it is also defined as a small number of people with complementary skills who are committed to a common purpose.
- **team building**—The process of transforming a group of people into a team and developing the team to achieve its purpose.
- **testing**—A means of determining the capability of an item to meet specified requirements by subjecting the item to a set of physical, chemical, environmental, and operating actions and conditions.
- **theory of constraints (TOC)**—Eliyahu Goldratt's theory dealing with techniques and tools for identifying and eliminating the constraints (bottlenecks) in a process.
- **theory of knowledge**—A belief that management is about prediction, and that people learn not only from experience but also from theory. When people study a process and develop a theory, they can compare their predictions with their observations; profound learning results.
- **theory X and theory Y**—A theory developed by Douglas McGregor maintaining that there are two contrasting assumptions about people, each of which is based on the manager's view of human nature. Theory X managers take a negative view and assume that most employees do not like work and try to avoid it. Theory Y managers take a positive view and believe that employees want to work, will seek and accept responsibility, and can offer creative solutions to organizational problems.
- **theory Z**—A term coined by William G. Ouchi, theory Z refers to a Japanese style of management that is characterized by long-term employment, slow promotions, considerable job rotation, consensus-style decision making, and concern for the employee as a whole.
- **three-sixty-degree (360°) feedback process**—An evaluation method that provides feedback from the perspectives of self, peers, direct reports, superiors, customers, and suppliers.

- **throughput time**—The total time required (processing + queue time) from concept to launch, from order received to delivery, or from raw materials received to delivery to customer.
- **time series**—A sequence of measurements of some quantity taken at different times, often at equally spaced intervals. Time-series models often have the following components:

 T_i = Long-term trend

 C_i = Cyclical effect (due to business or economic upturns or downturns)

 S_i = Seasonal effect

 R_i = Residual or error effect

 t_k —This is the *t* statistic at lag *k*. The formula is

$$t_k = \frac{r_k}{SE(r_k)}$$

where $SE(r_k)$ = standard error of the autocorrelation at lag *k*.

TOC—*See* theory of constraints.

tolerance—The difference between upper and lower specification limits.

- **tolerance design (Taguchi)**—A rational grade limit for components of a system that determines which parts and processes need to be modified and to what degree it is necessary to increase their control capacity; a method for rationally determining tolerances.
- **tollgate review**—A formal review process conducted by a champion who asks a series of focused questions aimed at ensuring that the team has performed diligently during the current phase. The end result of a tollgate is a "go" or "no-go" decision. The "go" decision allows the team to move forward to the next phase. If it is in the last phase, the "go" decision brings about project closure. If the decision is "no-go," the team must remain in the phase or retreat to an earlier phase, or perhaps the project is terminated or suspended.
- **total productive maintenance (TPM)**—A methodology pioneered by Nippondenso (a member of the Toyota group) that works to ensure that every machine in a production process is always able to perform its required tasks such that production is never interrupted. TPM maximizes equipment effectiveness by using a preventive maintenance program throughout the life of the equipment.
- **TPM**—*See* total productive maintenance.
- **training**—A subset of education that focuses on increasing proficiency in a skill. Therefore, training typically refers to skill-based instruction and addresses the specific skills employees need to perform a current job or task.

- **training need**—Essentially, a performance gap between what people know or are able to do and what they should know or be able to do to perform their work competently. For example, consider the situation where a new piece of equipment is to be installed and an employee needs to know how to operate it.
- **training needs analysis**—A diagnostic method to identify the gap between current performance and desired performance. A needs analysis is also known as a *needs assessment* or *training requirements analysis*.
- **training plan**—A road map for meeting critical training requirements. All training plans, though, should be a result of a strategic planning process and should provide a mechanism for ensuring that training and education build the organization's capabilities as well as enable employees to a make positive contribution to the organization's success.
- training requirements analysis—See training needs analysis.
- training want—A training want may surface when an employee's proficiency is low in certain job tasks or behaviors. However, those tasks and behaviors may not necessarily be important to work performance outcomes. For example, consider a customer service employee who wants to learn more about an engineering topic because of a personal interest.
- **transfer, risk**—This refers to moving the risk to another party or individual and allowing them to assume the risk responsibility.
- transmission of error-See propagation of errors.
- transmitter—See sensor/transmitter.
- **treatment**—The specific setting or combination of factor levels for an experimental unit.
- **tree diagram**—A tool that depicts the hierarchy of tasks and subtasks needed to complete an objective. The finished diagram resembles a tree. The tree diagram is one of the seven management and planning tools.
- **trend chart**—A control chart in which the deviation of the subgroup average, \overline{X} , from an expected trend in the process level is used to evaluate the stability of a process. The trend chart is also known as the *trend control chart*. The trend chart is occasionally considered to be one of the seven basic tools of quality.
- trend control chart—See trend chart.
- **TRIZ**—A Russian acronym for "theory of inventive problem solving"; a systematic means of inventing and solving design conflicts. TRIZ involves three items to solve technical problems: (1) various tricks, (2) methods based on utilizing physical effects and phenomena, and (3) complex methods.
- Type 1 error—See Type I error.
- **Type I error**—An error that occurs when the null hypothesis is rejected when it is true. We refer to the *P*(Type I error) = *P*(rejecting H_0 when H_0 is true) = α . A Type I error is also known as an *alpha* (α) *error* and *error of the first kind*. The

P(Type I error) is also known as α – value, producer's risk, level of significance, and significance level.

Type 2 error—See Type II error.

Type II error—An error that occurs when the alternative hypothesis is accepted when it is false. We refer to the *P*(Type II error) = *P*(not rejecting H_0 when H_0 is false) = β . A Type II error is also known as a *beta* (β) *error* and *error of the second kind*. The *P*(Type II error) is also known as β –*value* and *consumer's risk*.

U

unanticipated consequences—See law of unintended consequences.

underwritten department—A department that receives a budget and is expected to meet its objectives within the constraints of that budget.

unforeseen consequences—See law of unintended consequences.

upset—See disturbance.

V

VA—See value-added.

- valence—The value placed on achievement; it reveals the "why" beneath the "what" that is being asked of someone.
- **validation**—This refers to the effectiveness of the design process itself and is intended to ensure that the design process is capable of meeting the requirements of the final product or process.
- validation, external—According to the Manpower Services Commission (MSC) "Glossary of Training Terms," "A series of tests and assessments designed to ascertain whether the behavioural objectives of an internally valid training programme were realistically based on an accurate initial identification of training needs in relation to the criteria of effectiveness adopted by the organization."
- validation, internal—According to the Manpower Services Commission (MSC) "Glossary of Training Terms," "A series of tests and assessments designed to ascertain whether a training programme has achieved the behavioural objectives specified."
- validity—In the context of training, a valid measurement process is one that measures what it is designed to measure. The most commonly used method for training programs is the content-validity approach. This is a highly subjective approach by which subject matter experts provide opinions regarding whether the process measures what is needed on the job.
- **value**—The net difference between customer-perceived benefits and burdens; it is sometimes expressed as a ratio of benefits to burdens or a ratio of worth to cost.

- **value analysis**—An analytical process that assumes a process, procedure, product, or service is of no value unless proved otherwise. Value analysis assigns a price to every step of a process and then computes the worth-to-cost ratio of that step. *See also* value-added.
- **value engineering (VE)**—An engineering discipline responsible for analyzing the components and processes that create a product, with an emphasis on minimizing costs while maintaining standards required by the customer.
- **value stream**—All activities, both value-added and non-value-added, required to bring a product from a raw material state into the hands of the customer, a customer requirement from order to delivery, or a design from concept to launch.
- value stream analysis (VSA)—An analytical process designed to enhance the benefits of a value delivery system while reducing or eliminating all nonvalue-adding costs associated with value delivery.
- value stream mapping—A technique for following the production path of a product or service from beginning to end while drawing a visual representation of every process in the material and information flows. Subsequently, a future state map is drawn of how value should flow.
- value-added (VA)—The tasks or activities that convert resources into products or services consistent with customer requirements. The customer can be internal or external to the organization. Value-added activities add worth to the product or service from the customer's perspective and typically change form, fit, or function.
- **variables data**—Data resulting from the measurement of a parameter or a variable. The resulting measurements may be recorded on a continuous scale.
- variance inflation factor (VIF)—This measures the increase in the variance of a regression coefficient when predictor variables are correlated. The formula for the VIF is computed as

$$VIF(\beta_j) = \frac{1}{1-R_j^2}; j = 1, 2, ..., k$$

where R_j^2 is the coefficient of multiple determination. It is determined by regressing the predictor variable, x_j , on the other remaining k - 1 predictor variables. Consequently, when R_j^2 is large, $VIF(\beta_j)$ is also large. This causes the variances of the coefficients to become inflated.

VE—See value engineering.

- **verification**—This refers to the design meeting customer requirements and ensures that the design yields the correct product or process.
- **virtual team**—A boundaryless team functioning without a commonly shared physical structure or physical contact, using technology to link the team members. Team members are typically remotely situated and affiliated with a common organization, purpose, or project.

- **visual controls**—The collection of approaches and techniques that permit one to visually determine the status of a system, factory, or process at a glance and to prevent or minimize process variation. To some degree, it can be viewed as a minor form of mistake-proofing. Visual controls are sometimes referred to as the *visual factory*.
- visual factory—See visual controls.
- vital few, useful many—A phrase used by Joseph M. Juran to describe his use of the Pareto principle, which he first defined in 1950. (The principle was used much earlier in economics and inventory control methodologies.) The principle suggests that most effects come from relatively few causes; that is, 80% of the effects come from 20% of the possible causes. The 20% of the possible causes is referred to as the "vital few"; the remaining causes are referred to as the "useful many." When Juran first defined this principle, he was referring to the remaining causes as the "trivial many," but realizing that no problems are trivial in quality assurance, he changed it to "useful many."
- *V*(*L*)—The variance of the number of customers in the queueing system.
- $V(L_q)$ —The variance of the number of customers in the queue.
- VOC—See voice of the customer.
- **voice of the customer (VOC)**—Organizations apply significant effort to hear the voice of the customer by identifying the customers' needs and expectations. Once identified, organizations try to understand these needs and expectations and attempt to provide products and services that truly meet them.
- **voice of the process (VOP)**—The 6σ spread between the upper and lower control limits as determined from an in-control process. The VOP is also known as *natural process variation*.
- VOP—See voice of the process.
- VSA—See value stream analysis.
- *V*(*W*)—The variance of the time a customer spends in the queueing system.
- $V(W_q)$ —The variance of the time a customer spends in the queue.

W

- *W*—The average time a customer spends in the queueing system.
- **waste**—Any activity that consumes resources but does not add value to the product or service a customer receives. Waste is also known as *muda*.
- **work breakdown structure**—A hierarchical decomposition of the work content for a given project.
- **work group**—A group composed of people from one functional area or department who work together on a daily basis and share a common purpose.
- W_q —The average time a customer spends in the queue.

Y

y = f(x)—A foundational concept of Lean Six Sigma, this equation represents the idea that process outputs are a function of process inputs.

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Appendix 26 Glossary of Japanese Terms

В

baka-yoke—A term for a manufacturing technique for preventing mistakes by designing the manufacturing process, equipment, and tools so that an operation literally can not be performed incorrectly. In addition to preventing incorrect operation, the technique usually provides a warning signal of some sort for incorrect performance.

С

chaku-chaku—A term meaning "load-load" in a cell layout where a part is taken from one machine and loaded into the next.

G

gemba visit—The term "gemba" means "place of work" or "the place where the truth can be found." Still others may call it "the value proposition." A gemba visit is a method of obtaining voice of the customer information that requires the design team to visit and observe how the customer uses the product in his or her environment.

Η

- **heijunka**—The act of leveling the variety or volume of items produced at a process over a period of time. Heijunka is used to avoid excessive batching of product types and volume fluctuations, especially at a pacemaker process.
- hoshin kanri—A Japanese term referring to a top-down, bottom-up, systematic and structured strategic planning process that engages all levels of the organization while creating measurable and aligned goals that imbue the concept of continuous improvement through use of the plan-do-check-act cycle. (Note: some sources will cite the PDSA cycle.) Hoshin kanri is also known as *hoshin planning, policy deployment, policy management, policy control,* and *management by policy*
- **hoshin planning**—A term meaning "breakthrough planning." Hoshin planning is a strategic planning process in which a company develops up to four vision statements that indicate where the company should be in the next five years.

Company goals and work plans are developed on the basis of the vision statements. Periodic audits are then conducted to monitor progress. *See* hoshin kanri.

J

jidoka—A method of autonomous control involving the adding of intelligent features to machines to start or stop operations as control parameters are reached, and to signal operators when necessary. Jidoka is also known as *autonomation*.

Κ

- kaikaku—A term meaning a breakthrough improvement in eliminating waste.
- **kaizen**—A term that means gradual, unending improvement by doing little things better and setting and achieving increasingly higher standards.
- **kaizen blitz/event**—An intense team approach to employ the concepts and techniques of continuous improvement in a short time frame (for example, to reduce cycle time, increase throughput).
- **kanban**—A system that signals the need to replenish stock or materials or to produce more of an item (also called a *pull* approach). The system was inspired by Tauchi Ohno's (Toyota) visit to a U.S. supermarket.

Μ

muda—An activity that consumes resources but creates no value; the seven categories are correction, processing, inventory, waiting, overproduction, internal transport, and motion. Alternately, these have been categorized as transportation, inventory, motion, waiting, overproduction, overprocessing, and defects.

Ρ

poka-yoke—A term that means to mistake-proof a process by building safeguards into the system that avoid or immediately find errors. The term comes from the Japanese terms *poka*, which means "error," and *yokeru*, which means "to avoid." *See also* baka-yoke.

S

- **seiban**—The name of a management practice taken from the words *sei*, which means "manufacturing," and *ban*, which means "number." A seiban number is assigned to all parts, materials, and purchase orders associated with a particular customer, job, or project. This enables a manufacturer to track everything related to a particular product, project, or customer and facilitates setting aside inventory for specific projects or priorities. Seiban is an effective practice for project and build-to-order manufacturing.
- **seiketsu**—One of the 5S's that means to conduct seiri, seiton, and seiso at frequent, indeed daily, intervals to maintain a workplace in perfect condition.

- **seiri**—One of the 5S's that means to separate needed tools, parts, and instructions from unneeded materials and to remove the latter.
- seiso—One of the 5S's that means to conduct a cleanup campaign.
- **seiton**—One of the 5S's that means to neatly arrange and identify parts and tools for ease of use.
- **shitsuke**—One of the 5S's that means to form the habit of always following the first four S's.

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Simulated Examination Questions for Parts I–VI

There are 100 multiple-choice questions. You have 2½ hours to complete this examination. Choose the best answer based on the information provided in the book. Good luck!

- 1. Basic control systems must comprise what elements?
 - A. Controller, feedback loop, sensor/transmitter
 - B. Final control element, sensor/transmitter, feedback loop
 - C. Set point, controller, final control element
 - D. Final control element, sensor/transmitter, controller
 - E. None of the above
- 2. What should careful consideration be given to during the deployment of strategic plans?
 - A. Experience in Lean Six Sigma methodologies
 - B. Sequence of deployment, competitive pressure, process capabilities
 - C. Experience in Lean and related tools and techniques
 - D. Quarterly reports to the stockholders and other mandates associated with the Sarbanes–Oxley act
 - E. None of the above
- 3. The cause-and-effect method of project sizing is based on the concept that
 - A. no project should take longer than six months to complete
 - B. Y = f(X)
 - C. no project should take longer than six months to complete and generate less than \$250,000 in cost savings
 - D. no project should generate less than \$250,000 in cost savings
 - E. none of the above
- 4. Governance, in the context of Lean Six Sigma, includes
 - A. the training material used to educate Green Belts and Black Belts
 - B. a new concept adopted by organizations as a result of the Sarbanes–Oxley Act

- C. a specific tool used within the "O" phase of the IDOV methodology
- D. all the processes, procedures, rules, roles, and responsibilities associated with the deployment of Lean Six Sigma.
- E. none of the above
- 5. A common Lean Six Sigma governance structure contains which of the following components?
 - A. Executive steering committee, human resource council, strategic planning council
 - B. Strategic planning council, board of directors, human resource council
 - C. Executive steering committee, quality council, Lean Six Sigma deployment team
 - D. Quality council, Lean Six Sigma deployment team, strategic planning council
 - E. None of the above
- 6. The decomposition method
 - A. assigns a numerical rank to each data point for use with nonparametrical techniques
 - B. removes the signal from the noise in time-series data
 - C. stabilizes the ACF at the zero points
 - D. permits the separation of a time series into trend and seasonal components
 - E. none of the above
- 7. In the general sense, project management is used when
 - A. DMAIC takes too long to complete
 - B. the engineering development organization is resistant
 - C. the cost of DMAIC exceeds the cost of project management
 - D. the solution to the problem is known
 - E. none of the above
- 8. Risk is the probability of not achieving a project goal, schedule, budget, or resource target because
 - A. competitive pressures and market volatility undermine risk mitigation plans
 - B. something did or did not occur
 - C. the expected value computation resulted in an unacceptably high dollar value
 - D. competitive pressure and market dynamics fell into the lower right quadrant of the Kano model
 - E. none of the above

- 9. According to Forrester, systems thinking is a problem-solving approach that
 - A. is based on a wholly contained subsystem linked within and to other subsystems that support both itself and a parent system
 - B. expands the view of something under analysis to determine how it interacts with other elements of the system of which it is a part
 - C. is closed-loop in nature and expands to other subsystems or systems depending on the nature of the input received from outside the system
 - D. permits concepts of different systems to be integrated into a unified hierarchal parent system
 - E. none of the above
- 10. The additive model is used with the decomposition method when
 - A. the trend is not constant and there is no seasonal pattern
 - B. there is either no trend or a constant trend and a constant seasonal pattern
 - C. there is either no trend or a constant trend and no constant seasonal pattern
 - D. the trend is not constant and there is no constant seasonal pattern
- 11. Three potential barriers to a successful Lean Six Sigma implementation include
 - A. funding, leadership, customers
 - B. people, culture, customers
 - C. leadership, people, funding
 - D. technology, people, infrastructure
 - E. none of the above
- 12. What should be considered when planning for change?
 - A. Throttle change, rewards and recognition, goals and objectives, communications plan
 - B. Modifications to the performance appraisal system, enhanced goals and objectives
 - C. Defined scope, communications plan, modifications to the performance appraisal system
 - D. Designated change agents, goals and objectives, modifications to the performance appraisal system
 - E. None of the above
- 13. Valence is a term that refers to
 - A. the effect of monetary rewards on any one individual in a group
 - B. the theory of rewards related to intrinsic motivation

- C. a subordinate theory that supports Covey's fifth habit of "Seek first to understand, then to be understood"
- D. value placed on achievement and reveals the "why" beneath the "what" that is being asked of someone
- E. none of the above
- 14. Some organizations are centralized and some are decentralized. Both of these terms refer to
 - A. the lines of reporting both within a site and to levels higher
 - B. the complexity of reporting introduced through matrix management combined with the dispersed physical locations of many multinational organizations
 - C. how much decision-making authority has been delegated to lower management levels
 - D. the level of empowerment provided to self-directed work teams
- 15. What is a key assumption of a standby system?
 - A. Standby components are identical
 - B. The switch never fails
 - C. Standby components never fail in the standby mode
 - D. *k*-out-of-*n* standby components are required for the system to continue to work
 - E. All of the above
 - F. None of the above
 - G. A and B
 - H. A and C
 - I. A and D
 - J. B and C
 - K. B and D
 - L. C and D
 - M. A, B, and C
 - N. A, B, and D
 - O. B, C, and D
- 16. In activity-based costing, the costs of materials and services are
 - A. allocated based on a percentage of an arbitrary factor such as space, head count, and so on
 - B. allocated by the activity or process by which they are consumed

- C. allocated based on a percentage of general and administrative costs
- D. allocated based on a ratio of production to revenue dollars
- E. none of the above
- 17. The internal rate of return is
 - A. the interest rate used to compute the time value of money for the payback period
 - B. the interest rate used to compute the NPV or the discounted ROI
 - C. that discount rate that normalizes the NPV over the life cycle of the project
 - D. that discount rate that sets the NPV of the costs equal to the NPV of the savings
 - E. none of the above
- 18. Which of the following is not a common customer listening post?
 - A. Observations
 - B. Greeters
 - C. Comment cards
 - D. Mystery shoppers
 - E. None of the above
- 19. Organizational dynamics is a field of study that deals with
 - A. customer–organization–supplier process linkages, conflict management and resolution, organizational structure and alignment
 - B. organizational culture, change management, customer–organization– supplier process linkages
 - C. theories of motivation, strategic planning scenarios, employee morale in a down economy
 - D. leadership styles, organizational culture, change management
 - E. none of the above
- 20. To whom is attributed the identification of five steps for changing an organization's culture to a quality culture?
 - A. Deming
 - B. Juran
 - C. Crosby
 - D. Feigenbaum
 - E. Watson

- 21. A commonly used risk analysis tool is
 - A. value stream analysis
 - B. stakeholder analysis
 - C. failure mode and effects analysis
 - D. fishbone diagram
 - E. none of the above
- 22. The leadership/management theory that is characterized by long-term employment, slow evaluations and promotions, consensus decision making, individual responsibility, informal control with formalized measures, moderately specialized career paths, and a holistic concern for the employee that extends beyond the workplace was developed by
 - A. William Ouchi, theory Z
 - B. Douglas, theory Y
 - C. Drucker, paternalistic theory
 - D. Freedman, theory of development
 - E. none of the above
- 23. Emotional intelligence was first integrated into a leadership model by
 - A. Blanchard
 - B. Pauley and Pauley
 - C. Goleman
 - D. Blake and Mouton
 - E. B. F. Skinner
- 24. The advantage of hoshin kanri over the traditional strategic planning approach is
 - A. hoshin kanri involves less of the organization, thereby costing less
 - B. the traditional approach requires more senior management commitment
 - C. it creates both horizontal and vertical alignment throughout the organization
 - D. hoshin kanri is more widely used than the traditional approach
 - E. none of the above
- 25. Three common forces opposing cultural change are
 - A. social engineering, pandora enigma, moments of truth
 - B. social loafing, propinquity, polarization
 - C. attribution, social engineering, idiosyncrasy credits

- D. executive policies, propinquity, polarization
- E. none of the above
- 26. A split-plot design is used
 - A. when there are two or more factors and it is difficult to randomize one of the factors
 - B. when experimental error is expected to be large in relationship to the main factors
 - C. as a less expensive alternative to the BIBD in special situations
 - D. less and less and has fallen out of favor as computer software simplifies the use of more-useful designs
 - E. none of the above
- 27. When introducing new tools into an organization such as those associated with Lean Six Sigma, they must be introduced in an orderly sequence. In what sequence must they be introduced?
 - A. Culture, tools, practices, principles
 - B. Principles, tools, culture, practices
 - C. Practices, culture, tools, principles
 - D. Principles, culture, practices, tools
 - E. Practices, principles, culture, tools
- 28. Many organizations maintain the need for a specific organization structure long after the need for that structure has passed due to
 - A. infrastructure
 - B. technology
 - C. inertia
 - D. policies
 - E. none of the above
- 29. Heron introduced which two primary types of intervention approaches?
 - A. Conciliatory, facilitative
 - B. Participative, conciliatory
 - C. Facilitative, authoritative
 - D. Peacemaker, authoritative
 - E. None of the above
- 30. The best way to handle risk once it occurs is by
 - A. invoking the contingency plan

- B. budgetary and/or schedule replanning
- C. adding resources in the short term until the risk is mitigated
- D. revising the risk management plan and updating the risk calculations based on current knowledge
- E. none of the above
- 31. Which of the following is not part of the principled negotiation concept?
 - A. Identifying the people with the problem
 - B. Focusing on interests, not position
 - C. Understanding what both sides want to achieve
 - D. Inventing options for mutual gain
 - E. Insisting on objective criteria
- 32. Which of the following is not a dimension of management support?
 - A. Visibility
 - B. Engaged
 - C. Active
 - D. Knowledgeable
 - E. Communicative
 - F. None of the above
 - G. All of the above
- 33. What are several key considerations an organization must address prior to Lean Six Sigma deployment?
 - A. Regulatory influence, competitive organization structures, centralization versus decentralization
 - B. Union environment, availability of Master Black Belts and potential Black Belt talent from within the organization, competitive organization structures, market dynamics
 - C. Organizational placement of the Lean Six Sigma department, union environment, culture and subcultures
 - D. Availability of training material, facility space, agreement on cost savings definitions
 - E. None of the above
- 34. Action plans are also known as
 - A. tactical plans
 - B. operational plans

- C. control plans
- D. contingency plans
- E. none of the above
- 35. Megaprojects are projects
 - A. exceeding \$5M in expense or capital
 - B. requiring eight months or more to complete
 - C. so large they can't be managed effectively
 - D. involving three or more functional organizations
 - E. none of the above
- 36. What key people issues surface when implementing Lean Six Sigma?
 - A. Fear, time constraints, budget limits, morale
 - B. Impatience, rigor, fear, structure
 - C. Morale, budget limits, time constraints, training, peer pressure
 - D. Rigor, impatience, peer pressure
 - E. None of the above
- 37. When reviewing a portfolio of projects, it is important to consider the
 - A. size of each project
 - B. experience level of the Green Belt project leader
 - C. functional department ownership of the project
 - D. predecessor-successor relationships that might exist between projects
 - E. none of the above
- 38. The life cycle phases of a project are
 - A. carrying out the work, initiating, closing, controlling/monitoring
 - B. executing, controlling/monitoring, carrying out the work, planning, celebrating
 - C. organizing and preparing, starting the project, carrying out the work, closing out the work
 - D. reporting, monitoring, executing, closing, planning, controlling
 - E. initiating, closing, monitoring or controlling, executing, planning
- 39. An advantage of automated process control is that
 - A. technological advances have driven implementations significantly downward
 - B. it eliminates or reduces manual intervention, which is slower and more expensive

- C. it achieves product quality levels that might be otherwise unattainable on a manual basis
- D. it accommodates the ability to monitor and react to numerous controlled variables simultaneously
- E. all of the above
- F. none of the above
- G. A and B
- H. A and C
- I. A and D
- J. B and C
- K. B and D
- L. C and D
- M. A, B, and C
- N. A, B, and D
- O. B, C, and D
- 40. A work breakdown structure is
 - A. the production control department's method of expediting work through functional organizations
 - B. a hierarchical decomposition of work content for a given project
 - C. the degeneration of the team composition as it moves from the performing to storming stage
 - D. a methodology for tracking internal cost of quality
 - E. none of the above
- 41. Examples of media used for listening posts include
 - A. mail, telephone, fax
 - B. observations, focus groups, moments of truth
 - C. fax, focus groups, interviews
 - D. interviews, mail, observations
 - E. none of the above
- 42. The value of announcing a project to the organization is that
 - A. it provides the project team with visibility and prestige, which supports many organizations' reward and recognition approaches
 - B. it builds morale within the organization as it permits senior executives to be seen as active, engaged, visible, communicative, and knowledgeable

- C. it minimizes the opportunity for conflicts and places it in a position of authority
- D. it expedites the project completion date since the increased visibility works to garner wider support from other levels of management
- E. none of the above
- 43. Which of the following is not appropriate with regard to team communications?
 - A. All communication should open and honest
 - B. All communication should be without fear of retribution
 - C. All team members should be aware of any project problems or issues before anyone outside of the team
 - D. All team members should be apprised of their own performance
 - E. None of the above
- 44. The bullwhip effect occurs when
 - A. each level of management berates the level immediately below it until the lowest level is achieved and morale is significantly impaired
 - B. suppliers suddenly ship excess inventory at the end of the month to meet revenue demands to achieve goals
 - C. there is a sudden increase in employee attrition as bonus payouts are shifted from a gradient scale to an all-or-nothing scale
 - D. there is a ripple effect on suppliers as customers change schedules or demand for products too frequently
 - E. none of the above
- 45. Before changing the cultural objectives of an organization, the organization must first
 - A. complete its strategic planning process
 - B. develop its human resource strategic plan
 - C. assess its funding capabilities for the time horizon associated with the strategic plan
 - D. define its cultural objectives
 - E. none of the above
- 46. D-optimal designs
 - A. minimize the interaction values across significant factors in fractional factorial designs
 - B. minimize the variance of the model regression coefficients
 - C. stabilize the variance of the model regression coefficients

- D. provide stationarity to the model regression coefficients
- E. none of the above
- 47. Useful tools for helping a project succeed include
 - A. strategic plan, issue log, project charter
 - B. project charter, risk analysis, project plan
 - C. team member recruitment plan, status report, change request log
 - D. list of project assumptions and ground rules, team member requirement plan, list of subject matter experts
 - E. none of the above
- 48. Financially based project selection methods should
 - A. include cash flows, consider the time value of money, ensure that projects are mutually exclusive
 - B. use either the IRR or ROI methods
 - C. use the payback period or ROI methods
 - D. rely strictly on the cost–benefit analysis or IRR methods while ensuring that projects are mutually exclusive
 - E. none of the above
- 49. The value of governance at the project level is that it
 - A. mitigates legal actions and satisfies internal policies
 - B. minimizes employee suits for potentially discriminatory practices and related issues
 - C. provides structure and minimizes the rigor needed to ensure successful project outcomes
 - D. satisfies the amendments made in 2008 to the original Sarbanes–Oxley Act of 2002
 - E. none of the above
- 50. Key components of a time series include
 - A. seasonal effect, stationarity effect, consumer effect
 - B. residual effect, seasonal effect, cyclical effect
 - C. long-term trend, residual effect, stationarity effect, seasonal effect
 - D. cyclical effect, long-term trend, consumer effect
 - E. none of the above
- 51. One of the difficulties associated with budgeting a Lean Six Sigma project is that
 - A. the path from start to completion is unknown

- B. senior executives believe the project savings should pay for the cost of the project
- C. the financial department often disagrees with projected project savings
- D. project savings are often viewed as "funny money"
- E. none of the above
- 52. For savings to be cost (hard) dollar savings,
 - A. the dollars must have been planned and in the budget
 - B. the savings must affect the bottom line
 - C. there must be a prior baseline of spending established
 - D. all of the above
 - E. none of the above
- 53. A direct driver of the human resources strategic plan are the organization's
 - A. critical success factors
 - B. key business drivers
 - C. tactical plans
 - D. budgetary plans
 - E. none of the above
- 54. Before any type of training needs analysis can be done or performance gap analysis conducted, which of the following must be accomplished for the jobs in each target group?
 - A. Performance appraisals
 - B. On-the-job training
 - C. Job descriptions
 - D. Career path guidance
 - E. None of the above
- 55. What are the four common DFSS methodologies?
 - A. IDOV, DMAIC, CMMI, DMADV
 - B. DMEDI, IDOV, DMADOV, DMADV
 - C. CMM, SEI, IDOV, DMADV
 - D. DMAIC, DMEDI, DMADOV, DMADV
 - E. IDOV, DMEDI, DMADV, DMADOV
- 56. Poor performance is often attributed to
 - A. poor hiring practices stemming from human resource policies

- B. talent pools limited by geographic boundaries and budgets
- C. a skill or knowledge deficiency that can be corrected by training when it is not the cause
- D. on again/off again hiring restrictions versus competitive hiring practices
- E. none of the above
- 57. Two recognized methods of paying for training are
 - A. re-charging, overhead
 - B. G&A, overhead
 - C. overhead, traditional
 - D. re-charging, G&A
 - E. none of the above
- 58. Learning gives individuals
 - A. motivation for results
 - B. a feeling of completeness and achievement
 - C. personal reward and self-actualization
 - D. potential to get results
 - E. none of the above
- 59. Which of the following is not another name for hoshin kanri?
 - A. Policy deployment
 - B. Policy execution
 - C. Management by policy
 - D. Policy management
 - E. None of the above
- 60. Many Lean Six Sigma training classes are likely and inadvertently based on Linksman's learning styles, which are
 - A. kinesthetic, visual, tactile, auditory
 - B. logical, visual, kinesthetic, auditory
 - C. visual, auditory, tactile, mathematical
 - D. spatial, visual, logical, mathematical
 - E. none of the above
- 61. Kirkpatrick developed one of the most recognized training effectiveness models. It is based on the following levels of effectiveness:
 - A. ultimate value, functioning, learning, results

- B. results, learning, behavioral, reaction
- C. context, outcome, input, functioning
- D. input, ultimate value, learning, results,
- E. none of the above
- 62. According to Mager, a learning objective principle is
 - A. an objective measure of competency that is input to the job specification
 - B. the structure and shell for developing and communicating a training needs analysis
 - C. a description of performance you want learners to be able to exhibit before you consider them competent
 - D. the structure of the mastery grid for a specific training skill
 - E. none of the above
- 63. Tollgate reviews are conducted by the
 - A. process owner
 - B. Master Black Belt
 - C. project sponsor
 - D. senior executive
 - E. A and C
 - F. A and B
 - G. B and C
 - H. C and D
 - I. A, B, and C
 - J. A, C, and D
 - K. B, C, and D
 - L. A, B, C, and D
 - M. none of the above
- 64. In order to make an integrated VOC and VOP system work, one must ensure that
 - A. key customers routinely receive progress reports
 - B. process managers and marketing managers rotate positions on a 2–3-year basis to ensure cross-functional training and mutual understanding of each others' problems
 - C. regular senior management reviews take place to ensure the visibility, responsibility, and accountability of results

- D. the VOC and VOP processes are internally certified and audited on a regular basis
- E. none of the above
- 65. A project plan and completion date is important because
 - A. it facilitates the portfolio management process
 - B. it provides credibility with the project sponsor
 - C. it adheres to the requirements of the Sarbanes-Oxley Act
 - D. it limits the time requirements of functional group member participation
 - E. none of the above
- 66. One question project sponsors should not ask during tollgate review is
 - A. whether conflicts exist with other projects
 - B. whether resources have been allocated to move to the next phase
 - C. whether there is the right level of engagement from stakeholders
 - D. whether to broaden the scope
- 67. When providing feedback to champions or senior executives, Master Black Belts must
 - A. influence without authority
 - B. choose an effective communication style
 - C. choose an effective interaction style
 - D. deal effectively with conflict
 - E. all of the above
 - F. none of the above
- 68. Which of the following is not a team stage?
 - A. Adjourning
 - B. Performing
 - C. Recognition
 - D. Learning
 - E. None of the above
- 69. The role of the mentor is to
 - A. provide visibility of the mentee to the remainder of the organization
 - B. eliminate organizational obstacles to career progression
 - C. provide guidance, wisdom, and a possible road map to career advancement

- D. enhance the existing skills and capabilities of the mentee
- E. none of the above
- 70. If team failure is imminent,
 - A. the project leader should call in a professional facilitator
 - B. the team coach should intervene
 - C. the team should stop all action and immediately begin a force field analysis on the problem
 - D. the Lean Six Sigma deployment leader should consider replacing the Black Belt or project leader with a stronger, more experienced leader
 - E. none of the above
- 71. When organizations allow executives to achieve belts with less effort than others in the organization, the following effect can occur:
 - A. executives become more active, visible, engaged, communicative, and knowledgeable
 - B. employee demoralization and frustration and the eventual demise of the Lean Six Sigma initiative
 - C. executives become more enthusiastic and begin to take on additional projects even though they may be at the Green Belt level
 - D. employees see their executives as committed nonetheless and rally their support
 - E. none of the above
- 72. In tolerance stack-up analysis, the concept of propagation of errors addresses
 - A. the cumulative stack-up effect tolerance intervals have on the component confidence interval
 - B. the inherent limitations suppliers have when establishing original component specifications
 - C. the psychological tolerance limit organizations face when confronted with an excessive amount of change in too short a period of time
 - D. the cumulative stack-up effect subcomponent specifications have on the total component specifications
 - E. none of the above
- 73. A style of leadership in which the leader views the relationship as one of getting the work done through clear definition of tasks and responsibility and providing whatever resources are needed is known as
 - A. transformational
 - B. autocratic

- C. directive
- D. transactional
- E. none of the above
- 74. Tools and methods to analyze and interpret discrete measurement systems data include
 - A. percent agreement analysis, inter-class correlation coefficient
 - B. percent agreement analysis, control chart method
 - C. kappa statistics, intra-class correlation coefficient
 - D. kappa statistics, control chart method
 - E. all of the above
 - F. none of the above
- 75. Consider the following data: $K_{Overall} = 0.80$, $K_{Category 1} = 0.74$, $K_{Category 2} = 0.78$, $K_{Category 3} = 0.86$. How would you describe the measurement system?
 - A. It requires some attention, but not much
 - B. It's in a gray area depending on its use
 - C. It's excellent; use as is
 - D. It's unacceptable; start over with a new system
 - E. None of the above
- 76. Given the following assumptions, which tool or analysis is most appropriate?

| Tool 1 | Tool 2 |
|---------------------------------------------------------------------|---------------------------------------------------------------------|
| Units must be independent | • The categories are mutually exclusive and collectively exhaustive |
| Some categories can be used more frequently than others | There are consequences for misclassifying |
| There is a need to classify things in a nominal manner | Raters make their classifications independently of other raters |
| • The categories are mutually exclusive and collectively exhaustive | • Ranges are equally distributed, such as: -2, -1, 0, 1, 2 |
| Raters make their classifications independently of other raters | • There is a need to classify things in order, rank, or scale |
| | Units must be independent |

- A. Tool 1 = kappa, tool 2 = ICC
- B. Tool 1 = ICC, tool 2 = percent agreement
- C. Tool 1 = ICC, tool 2 = kappa
- D. Tool 1 = percent agreement, tool 2 = AIAG method

- E. Tool 1 = AIAG method, tool 2 = kappa
- F. None of the above
- 77. The six ICC values were computed as follows: *ICC* = 0.51, *ICC* = 0.62, *ICC* = 0.49, *ICC* = 0.64, *ICC* = 0.55, *ICC* = 0.60. How would you describe the measurement system?
 - A. Requires some attention
 - B. In a gray area depending on its use
 - C. Excellent; use as is
 - D. Subject to interpretation
 - E. None of the above
- 78. Organizational maturity models provide us with information regarding
 - A. whether an organization is moving toward transformation
 - B. the organization's process capabilities and its ability to improve them
 - C. whether an organization's training plan will close its performance gaps
 - D. the degree of correlation between the organization's strategic and human resource plans
 - E. none of the above
- 79. Measures of the accuracy of a fitted time series include
 - A. ACF, MRDR, LDR, MAPE
 - B. MAD, ACF
 - C. MSD, MAPE, ACF
 - D. MAD, MSD, MAPE
 - E. none of the above
- 80. Time-series data are assumed to be stationary. Stationarity occurs
 - A. when the ACF of the time-series data is zero at regular periodic intervals
 - B. in the area of the curve where the Ljung–Box statistic exceeds the chi-square statistics
 - C. when the mean, variance, and autocorrelation structure are constant over the entire range of the data
 - D. only at the zero points of the fitted line
 - E. none of the above
- 81. Project management is important to Lean Six Sigma because
 - A. each phase of DMAIC is likely to see the start and close of many non–Lean Six Sigma projects

- B. project management is universally known, and Lean Six Sigma is no exception
- C. project management has been found to expedite each phase of DMAIC when applied properly
- D. both the CSSBB and CSSMBB bodies of knowledge require it
- E. none of the above
- 82. One way of removing the signal from the noise in time-series data is by
 - A. applying data transformation methods
 - B. applying smoothing techniques
 - C. standardizing the data
 - D. eliminating special cause data points and outliers
 - E. none of the above
- 83. The underlying assumptions of the GLM are that
 - A. residuals are independent and identically distributed as normal with zero mean and variance, σ^2
 - B. the response variable must be continuous, but the regressor variables can be either discrete or continuous
 - C. the relationship between the predictor and response variable is linear
 - D. $\beta_0 \neq \beta_j$ for all $j \ge 1$
 - E. all of the above
 - F. none of the above
 - G. A and B
 - H. A and C
 - I. A and D
 - J. B and C
 - K. B and D
 - L. A, B, and C
 - M. A, B, and D
 - N. B, C, and D
- 84. According to Williams's balanced quadrant approach, managing a project requires balancing
 - A. time, cost, quality, scope
 - B. cost of quality, resources, customer relationships, budget
 - C. scope, cost of quality, on-time delivery, customer relationships

- D. cost, time, budget, scope
- E. none of the above
- 85. The GLM can be used to analyze
 - A. multiple linear regression
 - B. ANCOVA
 - C. nonlinear regression
 - D. continuous MSA
 - E. all of the above
 - F. none of the above
 - G. A and B
 - H. A and C
 - I. A and D
 - J. B and C
 - K. B and D
 - L. A, B, and C
 - M. A, B, and D
 - N. B, C, and D

86. What types of plans comprise the traditional strategic planning approach?

- A. Operational, strategic, tactical
- B. Strategic, human resource, marketing
- C. Human resource, operational, tactical
- D. Marketing, tactical, strategic
- E. None of the above
- 87. Components of variation is useful when the problem can be formulated as a(n)
 - A. multiple linear regression model
 - B. nonlinear regression model
 - C. nested hierarchal structure
 - D. ordinal logistic model
 - E. none of the above
- 88. What key characteristic should one be mindful of when training adult learners?
 - A. Hearing
 - B. Seeing

- C. Learning disabilities
- D. Age-related disabilities
- E. All of the above
- F. None of the above
- 89. An advantage of simulation is that
 - A. it can provide an understanding of the system or component interactions and rapidly explore a variety of system configurations
 - B. once an accurate model is established, a simple run provides an insightful solution
 - C. it can be handed off to Green Belts to develop and run on their own
 - D. simulation time can be run in real time
 - E. none of the above
- 90. Common financial measures used to select and prioritize projects are
 - A. NPV, BAC, IRR, ROI
 - B. EAC, BAC, payback period, NPV
 - C. cost-benefit ratio, IRR, ROI, EAC
 - D. ROI, IRR, NPV, cost-benefit ratio
 - E. none of the above
- 91. Given the following table, which variables will enter and leave the basis?

| | | <i>x</i> ₁ | <i>X</i> ₂ | <i>X</i> ₃ | <i>X</i> ₄ | <i>X</i> ₅ |
|-----------------------|----------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| Ζ | \$52,000 | 0 | 0 | 0.9 | 0.6 | 0 |
| <i>X</i> ₅ | \$84,000 | 0 | 0 | 0.5 | -1 | 1 |
| <i>X</i> ₂ | \$67,000 | 0 | 1 | 0.75 | -0.5 | 0 |
| <i>X</i> ₁ | \$70,000 | 1 | 0 | -0.5 | 1 | 0 |

- A. Enter x_3 , leave x_1
- B. Enter x_4 , leave x_1
- C. Enter x_1 , x_2 , and x_5 , leave x_3 and x_4
- D. Enter x_3 and x_4 , leave x_1 , x_2 , and x_5
- E. None of the above
- 92. Which of the following is a reason for adults to learn?
 - A. To meet expectations
 - B. To build networks
 - C. To seek stimulation

- D. To learn for the sake of learning
- E. All of the above
- F. None of the above
- 93. A VIF = 0.925 indicates that
 - A. high correlation exists between the regressor variables
 - B. very high correlation exists between the predictor variables
 - C. moderate correlation exists between the predictor variables
 - D. no correlation exists between the regressor variables
 - E. none of the above
- 94. Linear programming can be used to solve different types of problems, including
 - A. assignment
 - B. scheduling
 - C. transportation
 - D. mixture
 - E. all of the above
 - F. none of the above
- 95. What is the sequence of projects from creation to completion?
 - A. Identification, qualification, selection, prioritization, assignment, closure
 - B. Qualification, selection, prioritization, assignment, closure, replication
 - C. Qualification, selection, prioritization, assignment, closure, celebration
 - D. Identification, selection, qualification, prioritization, assignment, closure
 - E. None of the above
- 96. A parallel system has three components. The individual component reliabilities are $R_1 = 0.90$, $R_2 = 0.92$, $R_3 = 0.94$. What is the system reliability?
 - A. 0.915527
 - B. 0.77832
 - C. 0.99952
 - D. 0.93268
 - E. None of the above
- 97. Key assumptions underlying mixture experiments include
 - A. the errors are $NID(0, \sigma^2)$
 - B. the response region is continuous over the region being investigated
 - C. the response region must be convex

- D. the response depends on the proportion of the components in the mixture and not on the volume of the mixture itself
- E. all of the above
- F. none of the above
- G. A and B
- H. A and C
- I. A and D
- J. B and C
- K. B and D
- L. C and D
- M. A, B, and C
- N. A, B, and D
- O. B, C, and D
- 98. The five categories of savings are
 - A. efficiency, expense, intermediate, budget, rollover
 - B. budget-impacting, efficiency, revenue growth, cost take-out, working capital/cash flow
 - C. cost take-out, revenue growth, rollover, expense, planned
 - D. baseline, planned, working capital/cash flow, efficiency, budget
 - E. none of the above
- 99. Project closure means that
 - A. the project sponsor has left the project
 - B. funds are no longer available to support the project
 - C. team members have been reassigned
 - D. the project has been completed or is no longer viable
 - E. none of the above
- 100. The primary cause of team failure is
 - A. lack of adequate communication
 - B. lack of attendance by team members
 - C. lack of commitment to complete assigned actions
 - D. lack of interest and overall willingness to provide support
 - E. all of the above
 - F. none of the above

Simulated Examination Answers for Parts I–VI

| 1. | D | 26. A | 51. A | 76. | А |
|-----|---|------------------------------|-------|------|---|
| 2. | В | 27. D | 52. D | 77. | А |
| 3. | В | 28. C | 53. E | 78. | А |
| 4. | D | 29. C | 54. C | 79. | D |
| 5. | С | 30. A | 55. B | 80. | С |
| 6. | D | 31. A | 56. C | 81. | А |
| 7. | D | 32. F | 57. A | 82. | В |
| 8. | В | 33. C | 58. D | 83. | L |
| 9. | В | 34. B | 59. B | 84. | А |
| 10. | В | 35. C | 60. A | 85. | М |
| 11. | D | 36. B | 61. B | 86. | А |
| 12. | А | 37. D | 62. C | 87. | С |
| 13. | D | 38. E | 63. C | 88. | Е |
| 14. | С | 39. O | 64. C | 89. | А |
| 15. | М | 40. B | 65. A | 90. | D |
| 16. | В | 41. A | 66. D | 91. | Е |
| 17. | D | 42. C | 67. E | 92. | Е |
| 18. | В | 43. E | 68. D | 93. | D |
| 19. | D | 44. D | 69. C | 94. | Е |
| 20. | В | 45. E (Chapter 11, page 159) | 70. B | 95. | А |
| 21. | С | 46. B | 71. B | 96. | С |
| 22. | А | 47. B | 72. D | 97. | Ν |
| 23. | С | 48. A | 73. D | 98. | В |
| 24. | С | 49. C | 74. C | 99. | D |
| 25. | В | 50. B | 75. B | 100. | А |
| | | | | | |

Practice Examination Questions for Parts I–VI

Choose the best answer based on the information provided in the book. Good luck!

I. Enterprise-Wide Planning and Deployment

- 1. SWOT is an acronym for
 - A. society, work, technology, opportunity
 - B. threats, strengths, weaknesses, opportunity
 - C. opportunities, strengths, weaknesses, technologies
 - D. weaknesses, threats, opportunities, strengths
 - E. none of the above
- 2. PEST is an acronym for
 - A. environmental, threats, society, political
 - B. threats, politics, social, environmental
 - C. social, economic, political, technological
 - D. political, technological, societal, environmental
 - E. none of the above
- 3. Strategic planning is used by organizations to
 - A. establish long-term goals and objectives and the actions necessary to achieve them
 - B. serve as a countermeasure to global competition
 - C. provide managers with guidelines for performance appraisals
 - D. formulate their advertising and marketing long-range plans
 - E. all of the above
 - F. none of the above
- 4. Lean Six Sigma projects should be aligned with the organization's strategies because

A. it is usually easier to obtain funding

- B. project charters are easier to develop
- C. such alignment advances the organization's strategies
- D. ASQ future studies have shown that most organizations do so
- E. all of the above
- F. none of the above
- 5. RACI is an acronym for
 - A. analytical, rapid, intelligence, comprehensive
 - B. consultation, inform, responsibility, accountability
 - C. inform, analytics, compliance, response
 - D. accountability, responsibility, inform, compliance
 - E. none of the above
- 6. Organizational readiness models provide us with information regarding
 - A. whether an organization is ready to launch Lean Six Sigma
 - B. whether an organization has a fully developed and vetted human resources strategic plan
 - C. the organization's process capabilities and its ability to improve them
 - D. the degree of alignment between the organization's Lean Six Sigma projects and its strategic plan
 - E. all of the above
 - F. none of above
- 7. Which of the following is not a DFSS methodology?
 - A. IDOV
 - B. DMEDI
 - C. BPM
 - D. DMADOV
 - E. None of the above
- 8. Which of the following is not true about business process management?
 - A. There is no unified taxonomy, set of definitions, or business model
 - B. Processes are typically cross-functional
 - C. Typical business process management systems integrate software and the human element
 - D. The field is mature, well-understood, and being adopted rapidly
 - E. None of the above

- 9. What are three sources for identifying Lean Six Sigma projects?
 - A. Senior leaders, potential hiring candidates, trade journals
 - B. Customer complaints, white papers, facilitators
 - C. Lean Six Sigma consultants, board of directors, facilitators
 - D. Action plans, customer complaints, corrective action requests
 - E. None of the above
- 10. The project pipeline refers to all projects
 - A. from identification to closure
 - B. that have been selected or prioritized
 - C. that have been identified, qualified, but not selected
 - D. only active projects
 - E. none of the above
- 11. Project prioritization means that
 - A. a project has been selected and will be independently assessed against specific criteria in order establish a rank score
 - B. executive management agrees the project is important
 - C. a Master Black Belt has been assigned as a coach to support the project
 - D. ranked projects can be assigned to the next available Black Belt as long as the appropriate project team members are available to support the project
 - E. all of the above
 - F. none of the above
- 12. It is crucial that project sponsors attend the following project process meetings:
 - A. identification, assignment, closure
 - B. qualification
 - C. selection, qualification
 - D. prioritization, identification
 - E. none of the above
- 13. It is crucial that senior leaders attend the following project process meetings:
 - A. identification, assignment, closure
 - B. qualification
 - C. selection, qualification

- D. selection, prioritization, identification
- E. none of the above
- 14. Consider the following impact-effort matrix.

| Effort Impact | Low | , | Med | ium | н | igh |
|------------------|------|-----|----------|-----|-----------|-------------|
| High | 1 | 3 | 8 | 10 | 19 , ´ | 6 ⁄ ໌ 11 |
| Medium | 9 | 7 | 5 | , ' | 13 | 15 17 |
| Low | 18 / | ⁄ ́ | 20 14 | 12 | 16 | 2 |

From which cell would you select projects to pursue? The format is: "impact, effort."

- A. Medium, medium
- B. High, high
- C. Low, high
- D. High, low
- E. Low, low
- F. None of the above
- 15. What ways are there to manage risk?
 - A. Transfer, avoidance, legal action
 - B. Transfer, mitigation, avoidance, monitoring
 - C. Legal action, mitigation
 - D. Monitoring, transfer, risk planning, legal action
 - E. None of the above
- 16. Systems thinking was founded in 1956 by
 - A. Jay Forrester
 - B. Jonathan Frazier
 - C. Lars Skyttner
 - D. Emmett T. Brown
 - E. None of the above

- 17. The law of unintended consequences can be grouped as follows:
 - A. expected drawbacks, perverse results, expected benefits
 - B. expected benefits, expected costs, unexpected costs
 - C. expected benefits, unexpected drawbacks, perverse results
 - D. expected costs, expected drawbacks, perverse results
 - E. none of the above
- 18. Three potential barriers to a successful Lean Six Sigma implementation include
 - A. culture, funding, morale
 - B. maturity, culture, leadership
 - C. leadership, technology, culture
 - D. processes, infrastructure, funding
 - E. none of the above
- 19. A force field analysis is a technique for analyzing
 - A. competitive pressure and market forces
 - B. forces that aid or hinder an organization in reaching an objective
 - C. forces that aid an organization in reaching an objective
 - D. forces that hinder an organization in reaching an objective
 - E. global socioeconomic forces and is useful input to the strategic plan
 - F. none of the above
- 20. Expectancy theory was developed by
 - A. Vroom
 - B. Covey
 - C. Kubiak
 - D. Locke and Latham
 - E. none of the above
- 21. What key elements of effective communications are necessary when dealing with management in general?
 - A. Voice intonation and loudness, presentation and delivery, clarity of message
 - B. Clarity of message, adherence to company presentation standards, time limitations
 - C. Knowing your audience, clarity of message, voice intonation and loudness

D. Medium used, knowing your audience, clarity of message

E. None of the above

- 22. What key elements of effective communications are necessary when dealing with senior management in particular?
 - A. Voice intonation and loudness, seek advice from those who have a stake in the outcome, clarity of message
 - B. Clarity of message, adherence to company presentation standards, strict adherence to time limitations
 - C. Knowing your audience, clarity of message, appearance
 - D. Linking strategies to goals and objectives, speaking the language of management, identifying the risks and benefits involved
 - E. None of the above
- 23. Key performance indicators are
 - A. in-line process metrics applied to critical processes
 - B. financial and nonfinancial metrics that reflect an organization's key business drivers
 - C. nonfinancial metrics linked to the organization's action plans
 - D. financial and nonfinancial metrics linked to the organization's tactical plans
 - E. all of the above
 - F. none of the above
- 24. Customer loyalty is a term used to describe customers who
 - A. conduct repeat business, provide referrals, complain in the lowest decile
 - B. complain in the lowest quartile of customer complaints, conduct repeat business
 - C. provide testimonials, are highly satisfied, conduct repeat business
 - D. provide testimonials, complain in the lowest decile, are highly satisfied
 - E. none of the above
- 25. The Sarbanes–Oxley Act of 2002 has established rules for organizations in the areas of
 - A. prevention and detection, record keeping, compliance
 - B. record keeping, analyst guidance, prevention and detection
 - C. compliance, senior executive salary limitations, record keeping, analyst guidance

- D. senior executive salary limitations, prevention and detection, auditor selection
- E. none of the above

II. Cross-Functional Competencies

- 26. Natural process limits are also known as
 - A. natural tolerance, process variation, special process variation
 - B. common cause variation, process variation, natural tolerance
 - C. limiting variation, normal limiting variation, natural limiting variation
 - D. natural process variation, natural tolerance, normal process variation
 - E. none of the above
- 27. When the voice of the process equals the voice of the customer, customers are said to
 - A. be highly satisfied
 - B. provide referrals
 - C. be less likely to complain
 - D. be highly tolerant
 - E. all of the above
 - F. none of the above
- 28. Key design considerations when developing a customer data collection system include
 - A. asking the customer to take surveys to ensure a meaningful sample size
 - B. creating independent sources of data collection
 - C. creating source-linked holistic functional surveys
 - D. ensuring that the analytics team has the ability to transform qualitative data into quantitative data
 - E. all of the above
 - F. none of the above
- 29. Blanchard's situation leadership model is based on
 - A. directive behavior, supportive behavior
 - B. supportive behavior, cognitive behavior
 - C. functional behavior, cognitive behavior
 - D. transformational behavior, functional behavior
 - E. none of the above

- 30. Two recognized types of motivation have been identified. They are
 - A. idiosyncratic, individualistic
 - B. extrinsic, personalistic
 - C. extrinsic, intrinsic
 - D. intrinsic, idiosyncratic
 - E. none of the above
- 31. Transformational leadership is a style whereby a leader
 - A. defines the objective, requests preferred assignments, and follows up frequently
 - B. articulates the vision and values that are necessary for the organization to succeed
 - C. provides a clear-cut system of rewards and recognition that aligns with well-identified goals and objectives
 - D. creates action plans for each employee that provide line of sight to the strategy
 - E. all of the above
 - F. none of the above
- 32. Three common forces opposing cultural change are
 - A. social loafing, propinquity, moments of truth
 - B. social engineering, pandora enigma, polarization
 - C. attribution, social engineering, idiosyncrasy credits
 - D. Abilene paradox, propinquity, social loafing
 - E. none of the above
- 33. Blake and Mouton's management grid plots one's concern for
 - A. people versus production
 - B. effort versus impact
 - C. production versus funding
 - D. revenue versus effort
 - E. none of the above
- 34. The Malcolm Baldrige National Quality Award is based on what type of leadership model?
 - A. Management by fact
 - B. Situational leadership
 - C. Transformational leadership

- D. Participative leadership
- E. None of the above
- 35. Reinforcement theory is based on the concept of antecedent, behavior, and consequences. This theory was developed by
 - A. Victor Vroom
 - B. B. F. Skinner
 - C. John Stacy Adams
 - D. David McClelland
 - E. none of the above
- 36. Heffner contributed one of the most recognized communication styles. Its components are
 - A. confronting, informative, supporting, aggressive
 - B. directive, coaching, informative
 - C. aggressive, passive, assertive
 - D. supporting, passive, neutral
 - E. none of the above
- 37. Which of the following is not appropriate to do when trying to influence others within your organization?
 - A. Seek a win-win outcome
 - B. Go out of your way to assist others
 - C. Don't pull rank
 - D. Assume others understand the situation as well as you do
 - E. Share credit with others
 - F. All of the above
 - G. None of the above
- 38. Which of the following is not an important consideration when attempting to resolve conflict?
 - A. Define the conflict as a mutual problem
 - B. Identify goals for each party
 - C. Find creative alternatives that satisfy all parties
 - D. Ensure that all parties understand their own needs
 - E. All of the above
 - F. None of the above

- 39. An individual who coordinates the various functions and work activities at all levels of a process, has the authority or ability to make changes in the process as required, and manages the entire process cycle so as to ensure performance effectiveness is known as a
 - A. project manager
 - B. project sponsor
 - C. process owner
 - D. Lean Six Sigma leader
 - E. none of the above
- 40. The individual who assumes the ultimate responsibility for the success of a Lean Six Sigma project is the
 - A. project sponsor
 - B. project leader
 - C. Black Belt
 - D. Master Black Belt
 - E. none of the above

III. Project Management

- 41. Minimizing project scope overlap can be accomplished by
 - A. rejecting duplicate projects
 - B. asking project sponsors for input
 - C. adhering to the project selection process
 - D. adhering to the project qualification process
 - E. all of the above
 - F. none of the above
- 42. Which tool is useful in determining the predecessor–successor relationships that might exist between projects?
 - A. Cause-and-effect diagram
 - B. Activity network diagram
 - C. Matrix chart
 - D. Checklist
 - E. All of the above
 - F. None of the above
- 43. According to Portny, the components of a project include
 - A. budget, schedule, scope

- B. time, resources, budget
- C. resources, scope, schedule
- D. quality, cost, schedule
- E. none of the above
- 44. The stages of a project are
 - A. carrying out the work, starting the project, closing the work, organizing and preparing
 - B. reporting, monitoring, executing, closing, planning, controlling
 - C. performing, forming, norming, storming, adjourning, celebrating
 - D. organizing and preparing, closing the work, starting the project, controlling and monitoring
 - E. none of the above
- 45. A communications plan defines
 - A. who will deliver what message to whom, by what means, and how often
 - B. the organization's policy for issuing both internal and external announcements
 - C. the legality of the content that can be included in external announcements
 - D. points of contact within each department for deploying communications downward and throughout the organization
 - E. all of the above
 - F. none of the above
- 46. Two key metrics for measuring project performance are
 - A. KPI, KBD
 - B. CSF, KBD
 - C. CPI, KPI
 - D. SPI, CPI
 - E. KPI, SPI
 - F. none of the above
- 47. The following are recognized measures of project performance:
 - A. CSF, KPI, KBD, ACWP
 - B. BCWP, ACWP, EAC, BAC
 - C. BAC, BCWP, PB, SPI

- D. CV, SV, CPI, KPI
- E. ACWP, KBD, EAC, PB
- F. none of the above
- 48. Which of the following is not a focus of the closing cycle of a project?
 - A. Demonstrating goals as achieved
 - B. Closing all project accounts
 - C. Gathering lessons learned
 - D. Recognizing team members appropriately
 - E. None of the above
- 49. If the CPI is equal to 1 and the SPI is less than 1, then
 - A. CPI is over budget and action is required, SPI is under budget and no action is required
 - B. CPI is under budget and no action is required, SPI is on budget and no action is required
 - C. CPI is on budget and no action is required, SPI is under budget and no action is required
 - D. CPI is over budget and action is required, SPI is over budget and action is required
 - E. none of the above
- 50. Once established, the financial measures associated with pipeline projects
 - A. change over time and should be updated on a regular basis
 - B. should be considered fixed and compared to all future projects as they enter the pipeline
 - C. should be cast out along with the projects that have remained in the pipeline for more than a specified period of time
 - D. should be replaced with new financial measures
 - E. none of the above
- 51. Project prioritization criteria should be reviewed and/or updated regularly because
 - A. Vroom showed there is a natural propensity for people to become disinterested in repetitive actions
 - B. it demonstrates that senior executives are visible, engaged, communicative, active, and knowledgeable
 - C. project sponsors lose interest when their projects are not assigned

- D. the original criteria for prioritizing the projects may no longer apply
- E. all of the above
- F. none of the above
- 52. A system for measuring and managing project and portfolio performance permits
 - A. pipeline projects to be canceled
 - B. active projects to be canceled
 - C. pipeline projects to be re-prioritized
 - D. new projects to be assigned out of the pipeline
 - E. active projects to be provided with additional support
 - F. all of the above
 - G. none of the above
- 53. A project reserve budget is used
 - A. to provide rewards and recognition to project team members
 - B. as contingency dollars to help mitigate project risk
 - C. as expense money for customers and clientele
 - D. as capital dollars associated with the initial software purchase
 - E. all of the above
 - F. none of the above
- 54. The two types of cost-avoidance categories include
 - A. hard, soft
 - B. budget-impacting, soft
 - C. non-budget-impacting, budgeting-impacting
 - D. non-budget-impacting, hard
 - E. none of the above
- 55. Non-budget-impacting cost avoidance results from
 - A. productivity gains
 - B. head count reductions
 - C. revenue growth
 - D. increased working capital
 - E. all of the above
 - F. none of the above

IV. Training Design and Delivery

- 56. A job specification
 - A. posts the job internally prior to release outside the organization
 - B. defines which category of employees are eligible to post for the job
 - C. defines what the job holder should do and be capable of doing
 - D. defines the hours, location, and salary for the job position
 - E. all of the above
 - F. none of the above
- 57. The process phases of the training needs analysis are
 - A. quantitative analysis, analysis wrap-up, subjective analysis, tactical analysis, preliminary analysis
 - B. performance analysis, quantitative analysis, analysis wrap-up, preliminary analysis, strategic analysis
 - C. strategic analysis, wants analysis, operational analysis, subjective analysis, preliminary analysis
 - D. strategic analysis, preliminary analysis, subjective analysis, quantitative analysis, analysis wrap-up
 - E. none of the above
- 58. *Training* and *education* are often used interchangeably. However, which of the following is true?
 - A. Training and education actually describe the same concept
 - B. Training focuses on skill proficiency while education helps individuals learn how to think
 - C. Education focuses on skill proficiency while training helps individuals learn how to think
 - D. Education focuses on skill proficiency while training focuses on broadening an individual's knowledge base and expands thinking processes
 - E. None of the above
- 59. Training plans
 - A. provide credibility and legitimacy to budget plans that support the training wants analysis process
 - B. provide direct input into the human resource strategic plan, thus completing the closed-loop feedback system
 - C. are done at the individual level and updated based on the issuance of actions plans

- D. provide the road map for meeting critical training requirements
- E. all of the above
- F. none of the above
- 60. Training should be designed with regard to
 - A. competitive pressures and market dynamics
 - B. fair and equal opportunities to all employees
 - C. other tools and approaches that are ongoing or planned
 - D. the preponderance of the generation type employed
 - E. all of the above
 - F. none of the above
- 61. OJT is highly useful and practical. However, it must be applied with care or
 - A. inconsistent results and bad habits can occur
 - B. employee morale can suffer for those who are designated as the "trainer"
 - C. temporary pay increases may be incurred due to the extra workload of the "trainer"
 - D. work stoppages can occur in union environments if not handled properly
 - E. all of the above
 - F. none of the above
- 62. When designing training, address the following learning styles and preferences:
 - A. feeling
 - B. doing
 - C. seeing
 - D. hearing
 - E. all of the above
 - F. none of the above
- 63. Above all, training requires
 - A. a sense of confidence and a projection of gravitas
 - B. the ability to transfer knowledge and skills effectively and efficiently
 - C. the ability to develop clear presentations in an articulated and confident manner

- D. the ability to wear multiple hats while instructing and coaching and to change over quickly
- E. none of the above
- 64. The following considerations should be given when establishing training effectiveness measures:
 - A. the performance stated in the objectives and the performance measured in the training should match
 - B. if a measurement can not be developed for an objective, the objective should be reviewed and, if necessary, restated
 - C. the performance conditions stated in the objectives and the performance conditions in the training should match
 - D. all of the above
 - E. none of the above
 - F. A and B only
 - G. A and C only
 - H. B and C only
- 65. Training effectiveness measures should be
 - A. reliable and valid
 - B. verified and reliable
 - C. validated and verified
 - D. stable and valid
 - E. none of the above

V. Mentoring Responsibilities

- 66. Tollgate reviews exist
 - A. to ensure that budgets are not overrun
 - B. to ensure that executive leaders are kept in the communication loop and project sponsors are receiving the support they need
 - C. as a check-and-balance to ensure that teams are ready to transition to the next phase in an orderly manner
 - D. to provide the opportunity to celebrate and reward and recognize achievements
 - E. all of the above
 - F. none of the above

- 67. Poor tollgate reviews are typically characterized by
 - A. incomplete documentation
 - B. champions delegating replacements
 - C. sponsors seeking scope changes
 - D. poor preparations
 - E. all of the above
 - F. none of the above
- 68. What is the first question that should be asked at every tollgate review?
 - A. Is the project still on budget and schedule?
 - B. Is this project still consistent with the goals of the organization?
 - C. Is this project facing any significant risks that must be addressed?
 - D. What has been accomplished since the last tollgate review?
 - E. All of the above
 - F. None of the above
- 69. Which of the following are elements of the dimensional method for sizing projects?
 - A. Demographical
 - B. Geographical
 - C. Hierarchical
 - D. Organizational
 - E. All of the above
 - F. None of the above
 - G. A and B only
 - H. A and C only
 - I. A and D only
 - J. B and C only
 - K. B and D only
 - L. C and D only
 - M. A, B, and C only
 - N. A, B, and D only
 - O. A, C, and D only
 - P. B, C, and D only

- 70. When assigning Black Belts to projects,
 - A. flexibility exists because training is comprehensive and Black Belts are supported by coaches
 - B. only home-grown Black Belts should be given the most complex projects
 - C. only outside consultant Black Belts should be given the most complex projects
 - D. consideration should be given to the skills and experience of the Black Belt and to the complexity of the project
 - E. none of the above
- 71. Key tools for keeping teams focused and moving forward include
 - A. time-based agendas
 - B. reminders of action assignments
 - C. agendas published in advance of the meeting
 - D. updated Gantt charts displaying milestones and dates
 - E. all of the above
 - F. none of the above
- 72. An important piece of Lean Six Sigma deployment infrastructure is
 - A. a belt-level career progression plan
 - B. a reward and recognition procedure linked to the number of projects completed
 - C. the ability to provide salary counteroffers to belts as a retainer incentive
 - D. discretionary Black Belt funding to facilitate a fast-hire process
 - E. all of the above
 - F. none of the above
- 73. A role of the Lean Six Sigma coach is to
 - A. provide input to the Black Belt's performance appraisal
 - B. translate theoretical learning into applied learning
 - C. serve as a backup to the Black Belt
 - D. find suitable job positions when the Black Belt is finishing their rotation in the Lean Six Sigma deployment organization
 - E. all of the above
 - F. none of the above

- 74. The establishment of team norms
 - A. provides clear guidelines regarding what the team will and will not tolerate
 - B. is often seen as a distractor, and many teams' facilitators prefer to dismiss it
 - C. minimizes the feedback loops between stages with large team sizes
 - D. follows the values and culture of the parent organization
 - E. none of the above
- 75. There are many ways to provide general awareness information to non-belt employees. Which of the following is not appropriate?
 - A. Brown-bag lunches
 - B. Presentations at department staff meetings
 - C. E-mail blasts with training material attachments
 - D. One-on-one meetings
 - E. All of the above
 - F. None of the above

VI. Advanced Measurement Methods and Tools

- 76. In response model analysis, the concept of propagation of errors addresses
 - A. the partial derivative of the gradient at local minimum (maximum)
 - B. the effects on *y* when using a simplex in an optimization algorithm with minimal step sizes
 - C. the perpendicularity of the error gradient with respect to the *x_j* values along a ridge line
 - D. how random noise in x_i values affects the predicted value of y
 - E. none of the above
- 77. Which of the following hypotheses are appropriate for Kendall's coefficient of concordance?
 - A. H_0 = There is no association between the appraisers' ratings H_1 = Ratings between appraisers are associated
 - B. H_0 = There is no association between ratings of all appraisers and the known standard
 - H_1 = Ratings by all appraisers are associated with the known standard
 - C. H_0 = There is no association among multiple ratings made by an appraiser
 - H_1 = The ratings are associated with one another

- D. H_0 = There is no association between the ratings of each appraiser and the standard
 - H_1 = Ratings by each appraiser are associated with the standard
- E. All of the above
- F. None of the above
- G. A and B
- H. A and C
- I. A and D
- J. B and C
- K. B and D
- L. C and D
- 78. Which of the following hypotheses are appropriate for Kendall's correlation coefficient?
 - A. H_0 = There is no association among multiple ratings made by an appraiser
 - H_1 = The ratings are associated with one another
 - B. H_0 = There is no association between the appraisers' ratings H_1 = Ratings between appraisers are associated
 - C. H_0 = There is no association between the ratings of each appraiser and the standard
 - H_1 = Ratings by each appraiser are associated with the standard
 - D. H_0 = There is no association between ratings of all appraisers and the known standard
 - H_1 = Ratings by all appraisers are associated with the known standard
 - E. All of the above
 - F. None of the above
 - G. A and B
 - H. A and C
 - I. A and D
 - J. B and C
 - K. B and D
 - L. C and D

| Appraiser | Number inspected | Number matched | Percent |
|-----------|------------------|----------------|---------|
| Tom | 20 | 18 | 90 |
| Sue | 20 | 16 | 80 |
| Bill | 20 | 15 | 75 |
| Frank | 20 | 18 | 90 |

79. Given the data in the following table, what can be concluded?

- A. Tom and Frank matched each other exactly
- B. Bill's performance to standard is the lowest and he may require additional training
- C. A judgement call must be made regarding the acceptability of the measurement system
- D. The volume inspected was quite high
- E. All of the above
- F. None of the above
- G. A and B
- H. A and C
- I. A and D
- J. B and C
- K. B and D
- L. C and D
- M. A, B, and C
- N. A, B, and D
- O. B, C, and D
- 80. Key components of accuracy include
 - A. discrimination, linearity, bias
 - B. stability, bias, linearity
 - C. P/T ratio, stability, discrimination
 - D. linearity, bias, P/T ratio
 - E. none of the above

- 81. Multicollinearity is caused when
 - A. two or more linear regression equations intersect at the mean
 - B. two or more regressor variables are correlated
 - C. the VIF value is less than 1
 - D. the PRESS value is greater than the VIF value
 - E. none of the above
- 82. What does "p" represent in the following equation:

$$R_{adj}^{2} = 1 - \left(\frac{\sum_{i=1}^{n} (y_{i} - \hat{y}_{i})^{2}}{\left(\sum_{i=1}^{n} y_{i} - \overline{y}\right)^{2}}\right) \left(\frac{n-1}{n-p-1}\right)?$$

- A. The number of terms in the multiple regression model including the constant term
- B. The sample statistic for rho, ρ
- C. The number of terms in the multiple regression model excluding the constant term
- D. The degrees of freedom for the error term
- E. None of the above
- 83. In simple linear regression, the $VIF(\beta_j)$ values are
 - A. greater than the $VIF(\beta_i)$ values for multiple linear regression
 - B. close to Mallows's C_p
 - C. not used
 - D. only valid when the β_i are transcendental functions
 - E. all of the above
 - F. none of the above
 - G. A and B
 - H. A and D
 - I. B and D
 - J. A, B, and D

| Model | Variables | Insulation | East | South | North | Time |
|-------|-----------|------------|------|-------|-------|------|
| 1 | 1 | | | | Х | |
| 2 | 1 | Х | | | | |
| 3 | 2 | | | Х | Х | |
| 4 | 2 | | | | Х | Х |
| 5 | 3 | | Х | Х | Х | |
| 6 | 3 | Х | | Х | Х | |
| 7 | 4 | Х | Х | Х | Х | |
| 8 | 4 | Х | | Х | Х | Х |
| 9 | 5 | Х | Х | Х | Х | Х |

84. Given the following best subset table, which model provides the best fit?

- A. 1 or 2
- B. 3 or 4
- C. 5 or 6
- D. 7 or 8
- E. 9
- F. Models with 3 variables or less
- G. Can't tell
- H. None of the above
- 85. The VIF, R_{adj}^2 , PRESS, Mallows's C_p , and MSE
 - A. gauge the level of multicollinearity
 - B. determine the level of residual error
 - C. assess the degree of autocorrelation
 - D. help determine the best fitted model
 - E. none of the above
- 86. A link function
 - A. eliminates autocorrelation in logistic regression models
 - B. is similar to the coefficient of determination in multiple linear regression
 - C. transforms the probabilities of a response variable from a closed interval to an open unbounded interval
 - D. minimizes the equivalent of residual errors in logistic models
 - E. none of the above

- 87. Logistic regression models allow predictor variables to be _____ and response variables to be _____.
 - A. continuous, continuous
 - B. categorical, continuous
 - C. categorical, categorical
 - D. continuous, categorical
 - E. none of the above
- 88. Nonlinear regression models require predictor variables to be _____ or ____ and response variables to be _____.
 - A. continuous, continuous, continuous
 - B. continuous, an indicator variable, continuous
 - C. ordinal, continuous, an indicator variable
 - D. an indicator variable, ordinal, continuous
 - E. none of the above
- 89. In nonlinear regression, the parameters must be solved
 - A. using iterative optimization techniques that may or may not converge
 - B. using a system of linearized nonlinear equations that will converge
 - C. using a Taylor series expansion of parameters
 - D. using a transform function to linearize parameters and then solve using the traditional method of least squares
 - E. none of the above
- 90. To be able to apply nonlinear regression effectively, one must
 - A. have substantial knowledge to determine the functional form of the model
 - B. often rely on subject matter experts to estimate the initial parameter values
 - C. use a system of linearized nonlinear equations that will converge
 - D. restate the model in terms that are solved in a mathematically easier format
 - E. all of the above
 - F. none of the above
 - G. A and B
 - H. A and C

- I. A and D
- J. B and C
- K. B and D
- L. A, B, and C
- M. A, B, and D
- N. B, C, and D
- 91. The general form of a linear programming model comprises
 - A. solution set, implicit constraints, nonnegativity constraints
 - B. objective function, solution set, nonnegativity constraints
 - C. nonnegativity constraints, implicit constraints, objective function
 - D. explicit constraints, implicit constraints, objective function
 - E. none of the above
- 92. When we apply the simplex algorithm, we
 - A. are guaranteed an optimal solution in n –1 steps where n is the number of variables, including slack and surplus, because the polyhedron is convex
 - B. are guaranteed an optimal solution in 2^{n-1} steps where *n* is the number of variables, including slack and surplus, because the polyhedron is convex
 - C. reach an optimal solution or determine that no optimal solution exists
 - D. reach an optimal solution, if the determinant of the initial tableau is nonzero
 - E. none of the above
- 93. Common reliability system configurations include
 - A. standby, parallel, *k*-out-of-2*n*
 - B. parallel, series, *k*-out-of-*n*
 - C. k-out-of-2n, series, standby
 - D. series, *k*-out-of-2*n*, parallel
 - E. none of these
- 94. According to Borror, the following main factors affect a system's reliability:
 - A. preventive maintenance
 - B. operating environment
 - C. design and configuration of the system

- D. reliability of individual components
- E. all of the above
- F. none of the above
- G. A and B
- H. A and C
- I. A and D
- J. B and C
- K. B and D
- L. A, B, and C
- M. A, B, and D
- N. B, C, and D
- 95. The formula

$$R(t) = \sum_{i=k}^{n} {}_{n}C_{i}p^{i}(1-p)^{n-i}$$

describes which type of reliability system configuration?

- A. k-out-of-n
- B. System with *k* standby components
- C. k components in parallel
- D. *k* components in series
- E. None of the above
- 96. The factor relationship diagram is
 - A. a visual tool that helps identify key factors in a principal components analysis
 - B. similar to the cause-and-effect diagram in that it depicts relationships between process components
 - C. a modified form of an affinity diagram
 - D. a visual tool used to assist in the planning of experimental designs
 - E. none of the above
- 97. A design in which the design space is subdivided into blocks in which there are insufficient experimental units available to run a complete replicate of the experiment is called
 - A. RCBD
 - B. IRCBD

- C. BIBD
- D. Greco-Latin square
- E. none of the above
- 98. The basic assumption of the Latin square design is that
 - A. all interactions are limited to the first order
 - B. block factors do not interact with the factor of interest or each other
 - C. the superimposed Greco design does not create interactions
 - D. the superimposed Greco design permits interactions of the first order
 - E. none of the above
- 99. Search techniques for RSM guarantee finding
 - A. local minimums
 - B. global minimum
 - C. local maximum
 - D. global maximum
 - E. all of the above
 - F. none of the above
 - G. A and B
 - H. A and C
 - I. A and D
 - J. B and C
 - K. B and D
 - L. C and D
- 100. Some known advantages of the randomized complete block design include
 - A. some treatments may be replicated more times than others
 - B. no limit on the number of treatments and blocks
 - C. unbiased error can be calculated for specific treatments
 - D. more accurate results than the completely randomized design due to blocking
 - E. all of the above
 - F. none of the above
 - G. A and B
 - H. A and C

- I. A and D
- J. B and C
- K. B and D
- L. C and D
- M. A, B, and C
- N. A, B, and D
- O. B, C, and D

Practice Examination Answers for Parts I–VI

| 1. D | 26. D | 51. | D 76. | D |
|-------|-------|--------------|--------|------------------|
| 2. C | 27. F | 52. | F 77. | Н |
| 3. A | 28. B | 53. | B 78. | L |
| 4. C | 29. A | 54. | C 79. | J |
| 5. B | 30. C | 55. | A 80. | В |
| 6. A | 31. B | 56. | C 81. | В |
| 7. C | 32. E | 57. | B 82. | А |
| 8. D | 33. A | 58. | B 83. | С |
| 9. D | 34. A | 59. | D 84. | G |
| 10. B | 35. B | 60. | C 85. | D |
| 11. A | 36. C | 61. | A 86. | С |
| 12. C | 37. E | 62. | E 87. | С |
| 13. D | 38. B | 63. | B 88. | В |
| 14. D | 39. C | 64. | D 89. | А |
| 15. B | 40. A | 65. | A 90. | G |
| 16. A | 41. D | 66. | C 91. | D |
| 17. C | 42. B | 67. | E 92. | С |
| 18. C | 43. C | 68. | B 93. | В |
| 19. B | 44. A | 69. | N 94. | E |
| 20. A | 45. A | 70. | D 95. | А |
| 21. D | 46. E |) 71. | E 96. | D |
| 22. D | 47. B | 72. | A 97. | С |
| 23. B | 48. E | 73. | B 98. | В |
| 24. C | 49. C | 74. | A 99. | F (saddle point) |
| 25. A | 50. A | 75. | C 100. | E |
| | | | | |